Chapter 7: Statics of a Particle

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

7.

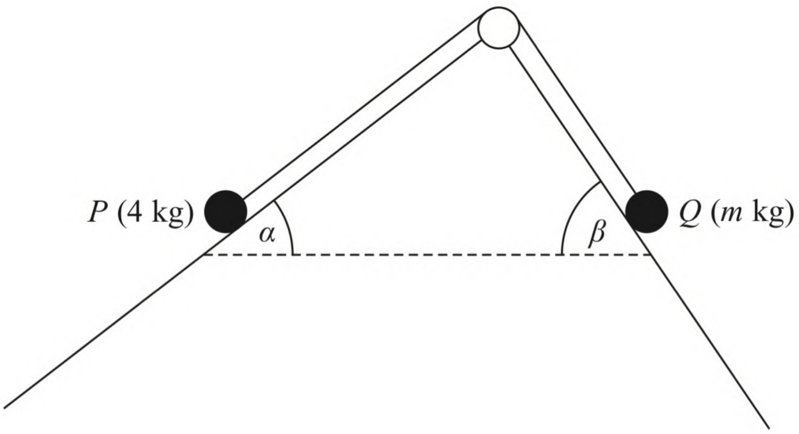

Figure 3

A particle P of mass 4 kg is attached to one end of a light inextensible string. A particle Q of mass m kg is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. The first plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$ and the second plane is inclined to the horizontal at an angle β, where $\tan\beta = \frac{4}{3}$. Particle P is on the first plane and particle Q is on the second plane with the string taut, as shown in Figure 3.

The first plane is rough and the coefficient of friction between P and the plane is $\frac{1}{4}$. The second plane is smooth. The system is in limiting equilibrium.

Given that P is on the point of slipping down the first plane,

(a) find the value of m,

(10)

(b) find the magnitude of the force exerted on the pulley by the string,

(4)

(c) find the direction of the force exerted on the pulley by the string.

(1)

4.

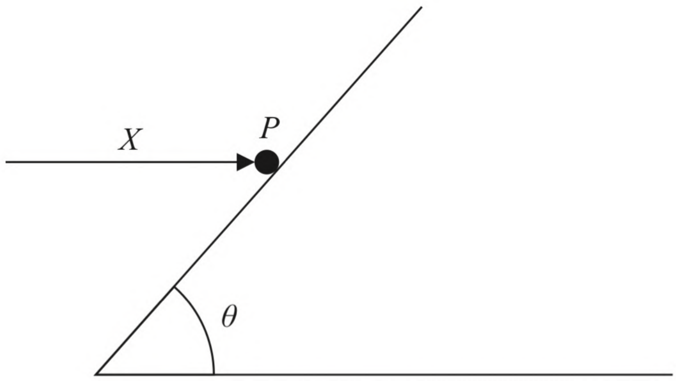

Figure 1

A particle, P, of mass km lies on a fixed rough plane. The plane is inclined to the horizontal at an acute angle θ. A horizontal force of magnitude X acts on P, as shown in Figure 1. The line of action of the force lies in the vertical plane which contains the line of greatest slope of the inclined plane that passes through P. The coefficient of friction between P and the inclined plane is μ.

When $X = mg$, the particle P is in equilibrium and on the point of sliding down the plane.

(a) Show that $\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$

(10)

(b) Deduce that, when $k = 1$, θ must be greater than 45°

(2)

4.

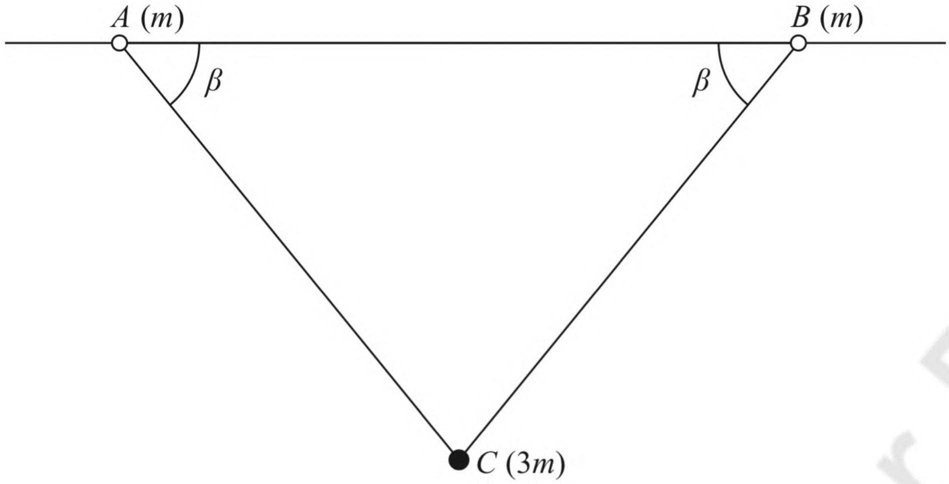

Figure 2

Two identical small rings, A and B, each of mass m, are threaded onto a rough horizontal wire. The rings are connected by a light inextensible string. A particle C of mass $3m$ is attached to the midpoint of the string. The particle C hangs in equilibrium below the wire with angle $BAC = \beta$, as shown in Figure 2.

The tension in each of the parts, AC and BC, of the string is T

(a) By considering particle C, find T in terms of m, g and β

(2)

(b) Find, in terms of m and g, the magnitude of the normal reaction between the wire and A.

(3)

The coefficient of friction between each ring and the wire is $\frac{4}{5}$

The two rings, A and B, are on the point of sliding along the wire towards each other.

(c) Find the value of $\tan\beta$

(5)

3.

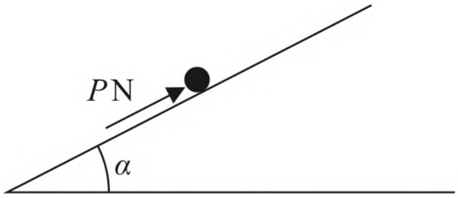

Figure 1

A particle of mass 10 kg is placed on a fixed rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The particle is held in equilibrium by a force of magnitude P newtons, which acts up the plane, as shown in Figure 1. The line of action of the force lies in a vertical plane that contains a line of greatest slope of the plane. The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

(a) Find the normal reaction between the particle and the plane.
(2)

(b) Find the greatest possible value of P.
(4)

(c) Find the least possible value of P.
(2)

3.

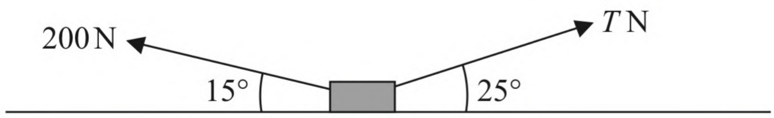

Figure 1

A parcel of mass 20 kg is at rest on a rough horizontal floor. The coefficient of friction between the parcel and the floor is 0.3

Two forces, both acting in the same vertical plane, of magnitudes 200 N and T N are applied to the parcel. The line of action of the 200 N force makes an angle of 15° with the horizontal and the line of action of the T N force makes an angle of 25° with the horizontal, as shown in Figure 1. The parcel is modelled as a particle P.

Find the smallest value of T for which P remains in equilibrium.

(9)

5.

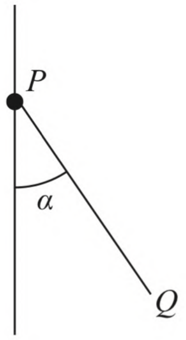

Figure 2

A small bead of mass 0.2 kg is attached to the end P of a light rod PQ. The bead is threaded onto a fixed vertical rough wire.

The bead is held in equilibrium with the rod PQ inclined to the wire at an angle α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

The thrust in the rod is T newtons.

The bead is modelled as a particle.

(a) Find the magnitude and direction of the friction force acting on the bead when $T = 2.5$

(3)

The coefficient of friction between the bead and the wire is μ.

Given that the greatest possible value of T is 6.125

(b) find the value of μ.

(7)

1.

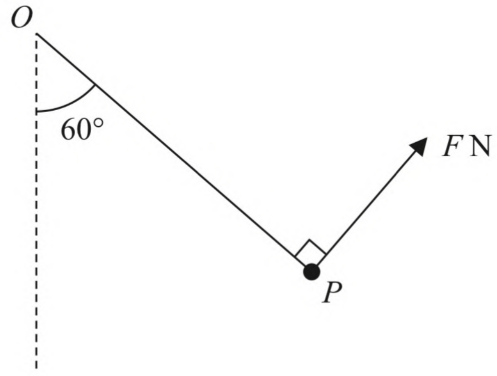

Figure 1

A particle P of weight 5 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle P is held in equilibrium by a force of magnitude F newtons. The direction of this force is perpendicular to the string and OP makes an angle of 60° with the vertical, as shown in Figure 1.

Find

(a) the value of F

(3)

(b) the tension in the string.

(3)

5.

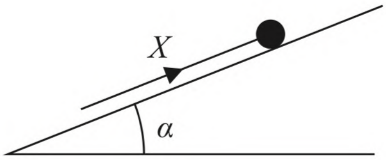

Figure 3

A particle of mass m rests in equilibrium on a fixed rough plane under the action of a force of magnitude X. The force acts up a line of greatest slope of the plane, as shown in Figure 3.

The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the particle and the plane is μ.

- When $X = 2P$, the particle is on the point of sliding up the plane.
- When $X = P$, the particle is on the point of sliding down the plane.

Find the value of μ.

(8)

3.

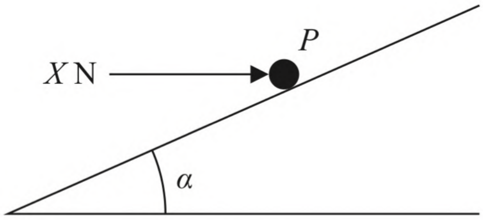

Figure 2

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$

A particle P of mass 2 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 2. The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

(a) Show that when $X = 14.7$ there is no frictional force acting on P

(3)

The coefficient of friction between P and the plane is 0.5

(b) Find the smallest possible value of X.

(8)

5.

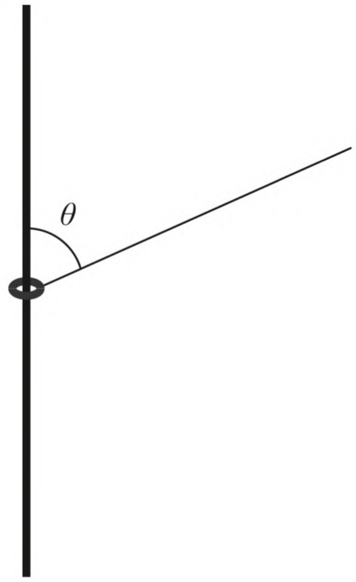

Figure 3

A small ring of mass 0.2 kg is attached to one end of a light inextensible string.

The ring is **threaded** onto a fixed rough vertical rod.

The string is taut and makes an angle θ with the rod, as shown in Figure 3, where $\tan\theta = \frac{12}{5}$

Given that the ring is in equilibrium and that the tension in the string is 10 N,

(a) find the magnitude of the frictional force acting on the ring,

(3)

(b) state the direction of the frictional force acting on the ring.

(1)

The coefficient of friction between the ring and the rod is $\frac{1}{4}$

Given that the ring is in equilibrium, and that the tension in the string, T newtons, can now vary,

(c) (i) find the minimum value of T

(ii) find the maximum value of T

(8)

1.

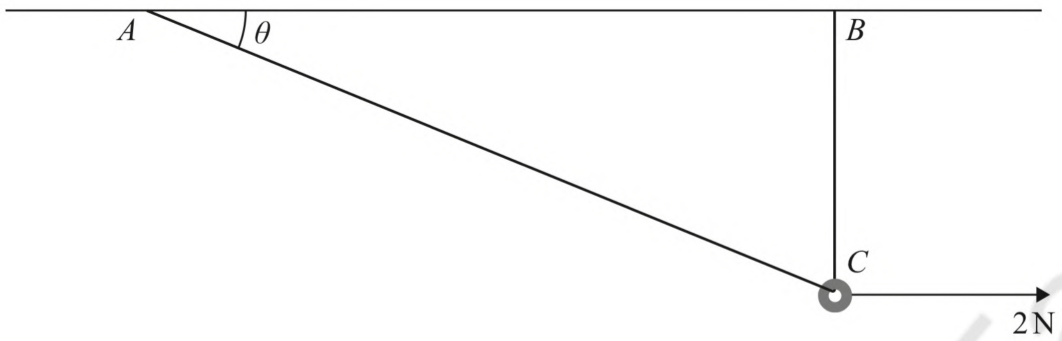

Figure 1

Figure 1 shows a small smooth ring **threaded** onto a light inextensible string.

One end of the string is attached to a fixed point A on a horizontal ceiling and the other end of the string is attached to a fixed point B on the ceiling.

A horizontal force of magnitude 2N acts on the ring so that the ring rests in equilibrium at a point C, vertically below B, with the string taut.

The line of action of the horizontal force and the string both lie in the same vertical plane.

The angle that the string makes with the ceiling at A is θ, where $\tan\theta = \dfrac{3}{4}$

The tension in the string is T newtons. The mass of the ring is M kg.

(a) Find the value of T

(3)

(b) Find the value of M

(3)

6.

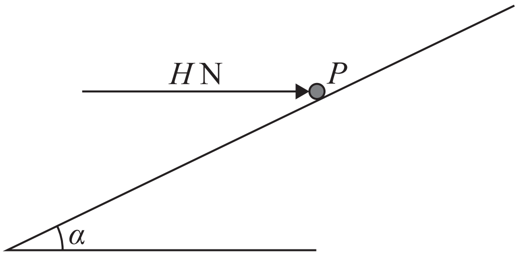

Figure 4

A particle P of mass 5 kg lies on the surface of a rough plane.

The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The particle is held in equilibrium by a horizontal force of magnitude H newtons, as shown in Figure 4.

The horizontal force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the smallest possible value of H.

(6)

The horizontal force is now removed, and P starts to slide down the slope.

In the first T seconds after P is released from rest, P slides 1.5 m down the slope.

(b) Find the value of T.

(6)

6.

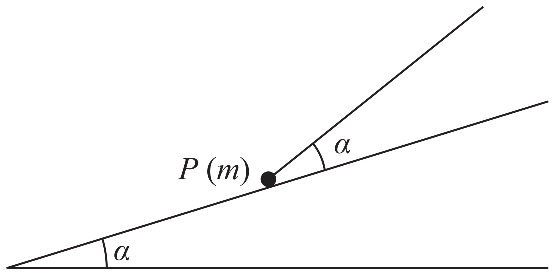

Figure 3

A particle P of mass m is held in equilibrium on a fixed rough inclined plane by a light inextensible string.

The plane is inclined at an angle α to the horizontal, where $\alpha < 45°$
The string is inclined to the plane at angle α, as shown in Figure 3.

The string lies in a vertical plane that contains a line of greatest slope of the inclined plane.

When the tension in the string is $0.75mg$, P is on the point of moving up the plane.

(a) Find an expression for the magnitude of the frictional force acting on P, giving your answer in terms of m, g and α.

(3)

The coefficient of friction between P and the plane is $\frac{1}{2}$

(b) Show that

$$\tan\alpha = \frac{2}{5}$$

(6)

The string breaks.

(c) Determine whether P remains at rest. You must justify your reasoning.

(3)