Chapter 7: Normal Distribution

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7. A machine fills bottles with water. The volume of water delivered by the machine to a bottle is X ml where $X \sim N(\mu, \sigma^2)$

One of these bottles of water is selected at random.

Given that $\mu = 503$ and $\sigma = 1.6$

(a) find

(i) $P(X > 505)$
(ii) $P(501 < X < 505)$

(5)

(b) Find w such that $P(1006 - w < X < w) = 0.9426$

(3)

Following adjustments to the machine, the volume of water delivered by the machine to a bottle is such that $\mu = 503$ and $\sigma = q$

Given that $P(X < r) = 0.01$ and $P(X > r + 6) = 0.05$

(c) find the value of r and the value of q

(7)

3. The weights of women boxers in a tournament are normally distributed with mean 64 kg and standard deviation 8 kg.

(a) Find the probability that a randomly chosen woman boxer in the tournament weighs less than 51 kg.

(3)

In the tournament, women boxers who weigh less than 51 kg are classified as lightweight. Ren weighs 49 kg and she has a match against another randomly selected, lightweight woman boxer.

(b) Find the probability that Ren weighs less than the other boxer.

(4)

In the tournament, women boxers who weigh more than H kg are classified as heavyweight. Given that 10% of the women boxers in the tournament are classified as heavyweight,

(c) find the value of H.

(3)

4. The weights of packages delivered to Susie are normally distributed with a mean of 510 grams and a standard deviation of 45 grams.

(a) Find the probability that a randomly selected package delivered to Susie weighs less than 450 grams.

(3)

The heaviest 5% of packages delivered to Susie are delivered by Rav in his van, the others are delivered by Taruni on foot.

(b) Find the weight of the lightest package that Rav would deliver to Susie.

(3)

Susie randomly selects a package from those delivered by Taruni.

(c) Find the probability that this package weighs more than 450 grams.

(4)

On Tuesday there are 5 packages delivered to Susie.

(d) Find the probability that 4 are delivered by Taruni and 1 is delivered by Rav.

(3)

3. The distance that Jenny throws a javelin is normally distributed with mean 42 m and standard deviation 5 m.

(a) Find the probability that the distance Jenny throws the javelin is less than 40 m.
(3)

Jenny enters a javelin competition. To qualify for the final she has 3 attempts to throw the javelin a distance of more than 40 m. Once she has qualified she does not use any of her remaining attempts.

Given that Jenny qualified for the final and that throws of the javelin are independent,

(b) find the probability that she qualified for the final on her third throw with a distance greater than 45 m.
(5)

6. A machine cuts wood into pieces. The lengths, W metres, of the pieces produced by the machine are normally distributed with mean μ metres and standard deviation σ metres. It is known that

$$P(W < 3.968) = 0.1 \quad \text{and} \quad P(3.968 < W < 4.026) = 0.75$$

(a) Calculate the value of μ and the value of σ

(5)

A second machine cuts wood into logs. The lengths, L cm, of the logs produced by this second machine are normally distributed with $L \sim N(30, 0.5^2)$

An outlier is a value that is greater than $Q_3 + 1.5 \times (Q_3 - Q_1)$ or smaller than $Q_1 - 1.5 \times (Q_3 - Q_1)$

A log is selected at random.

Given that $Q_1 = 29.7$ to 3 significant figures,

(b) find the probability that the length of this log is an outlier.

(5)

5. The random variable X has a normal distribution with mean 10 and standard deviation 6

(a) Find $P(X < 7)$

(3)

(b) Find the value of k such that

$$P(10 - k < X < 10 + k) = 0.60$$

(3)

A single observation x, of X, is to be taken.
A rectangle is drawn on a centimetre grid with vertices having coordinates $(0, 0)$, $(x, 0)$, $(x, x - 3)$ and $(0, x - 3)$

(c) Find the probability that the area of this rectangle is more than $40\,\text{cm}^2$

(8)

3. The distance achieved in a long jump competition by students at a school is normally distributed with mean 3.8 metres and standard deviation 0.9 metres.

Students who achieve a distance greater than 4.3 metres receive a medal.

(a) Find the proportion of students who receive medals.

(3)

The school wishes to give a certificate of achievement or a medal to the 80% of students who achieve a distance of at least *d* metres.

(b) Find the value of *d*.

(3)

Of those who received medals, the $\frac{1}{3}$ who jump the furthest will receive gold medals.

(c) Find the shortest distance, *g* metres, that must be achieved to receive a gold medal.

(4)

A journalist from the local newspaper interviews a randomly selected group of 3 medal winners.

(d) Find the exact probability that there is at least one gold medal winner in the group.

(3)

3. The weights of packages that arrive at a factory are normally distributed with a mean of 18 kg and a standard deviation of 5.4 kg

(a) Find the probability that a randomly selected package weighs less than 10 kg

(3)

The heaviest 15% of packages are moved around the factory by Jemima using a forklift truck.

(b) Find the weight, in kg, of the lightest of these packages that Jemima will move.

(3)

One of the packages **not** moved by Jemima is selected at random.

(c) Find the probability that it weighs more than 18 kg

(4)

A delivery of 4 packages is made to the factory.
The weights of the packages are independent.

(d) Find the probability that exactly 2 of them will be moved by Jemima.

(3)

4. Kris works in the mailroom of a large company and is responsible for all the letters sent by the company. The weights of letters sent by the company, W grams, have a normal distribution with mean 165 g and standard deviation 35 g.

(a) Estimate the proportion of letters sent by the company that weigh less than 120 g.

(3)

Kris splits the letters to be sent into 3 categories: heavy, medium and light, with $\frac{1}{3}$ of the letters in each category.

(b) Find the weight limits that determine medium letters.

(4)

A heavy letter is chosen at random.

(c) Find the probability that this letter weighs less than 200 g.

(3)

Kris chooses a random sample of 3 letters from those in the mailroom one day.

(d) Find the probability that there is one letter in each of the 3 categories.

(3)

6. Xiang is designing shelves for a bookshop. The height, H cm, of books is modelled by the normal distribution with mean 25.1 cm and standard deviation 5.5 cm

(a) Show that $P(H > 30.8) = 0.15$

(3)

Xiang decided that the smallest 5% of books and books taller than 30.8 cm would not be placed on the shelves. All the other books will be placed on the shelves.

(b) Find the range of heights of books that will be placed on the shelves.

(3)

The books that will be placed on the shelves have heights classified as small, medium or large.
The numbers of small, medium and large books are in the ratios 2 : 3 : 3

(c) The medium books have heights x cm where $m < x < d$

(i) Show that $d = 25.8$ to 1 decimal place.

(3)

(ii) Find the value of m

(4)

Xiang wants 2 shelves for small books, 3 shelves for medium books and 3 shelves for large books.
These shelves will be placed one above another and made of wood that is 1 cm thick.

(d) Work out the minimum total height needed.

(2)

5. Jia writes a computer program that randomly generates values from a normal distribution.

He sets the mean as 40 and the standard deviation as 2.4

(a) Find the probability that a particular value generated by the computer program is less than 37

(3)

Jia changes the mean to m but leaves the standard deviation as 2.4

The computer program then randomly generates 2 independent values from this normal distribution.

The probability that both of these values are greater than 32 is 0.16

(b) Find the value of m, giving your answer to 2 decimal places.

(4)

Jia now changes the mean to 4 and the standard deviation to 8

The computer program then randomly generates 5 independent values from this normal distribution.

(c) Find the probability that at least one of these values is negative.

(4)

6. A manufacturer fills bottles with oil.
The volume of oil in a bottle, V ml, is normally distributed with $V \sim \mathrm{N}(100, 2.5^2)$

(a) Find $\mathrm{P}(V > 104.9)$

(3)

(b) In a pack of 150 bottles, find the expected number of bottles containing more than 104.9 ml

(2)

(c) Find the value of v, to 2 decimal places, such that $\mathrm{P}(V > v \mid V < 104.9) = 0.2801$

(6)

5. The weights, W grams, of kiwi fruit grown on a farm are normally distributed with mean 80 grams and standard deviation 8 grams.

The table shows the classifications of the kiwi fruit by their weight, where k is a positive constant.

Small		**Large**		
Tiny	**Petite**	**Extra**	**Jumbo**	**Mega**
$w < 66$	$66 \leqslant w < 70$	$70 \leqslant w < 80$	$80 \leqslant w < k$	$w \geqslant k$

One kiwi fruit is selected at random from those grown on the farm.

(a) Find the probability that this kiwi fruit is Large.

(3)

35% of the kiwi fruit are Jumbo.

(b) Find the value of k to one decimal place.

(4)

75% of Tiny kiwi fruit weigh more than y grams.

(c) Find the value of y giving your answer to one decimal place.

(5)

5. The lengths, L mm, of housefly wings are normally distributed with $L \sim N(4.5, 0.4^2)$

(a) Find the probability that a randomly selected housefly has a wing length of less than 3.86 mm.

(3)

(b) Find

(i) the upper quartile (Q_3) of L

(ii) the lower quartile (Q_1) of L

(4)

A value that is greater than $Q_3 + 1.5 \times (Q_3 - Q_1)$ or smaller than $Q_1 - 1.5 \times (Q_3 - Q_1)$ is defined as an outlier.

(c) Find these two outlier limits.

(3)

A housefly is selected at random.

(d) Using standardisation, show that the probability that this housefly is **not** an outlier is 0.993 to 3 decimal places.

(3)

Given that this housefly is **not** an outlier,

(e) showing your working, find the probability that the wing length of this housefly is greater than 5 mm.

(4)

7. A machine squeezes apples to extract their juice. The volume of juice, J ml, extracted from 1 kg of apples is modelled by a normal distribution with mean μ and standard deviation σ

Given that $\mu = 500$ and $\sigma = 25$ use standardisation to

(a) (i) show that $P(J > 510) = 0.3446$

(2)

(ii) calculate the value of d such that $P(J > d) = 0.9192$

(3)

Zen randomly selects 5 bags each containing 1 kg of apples and records the volume of juice extracted from each bag of apples.

(b) Calculate the probability that each of the 5 bags of apples produce less than 510 ml of juice.

(2)

Following adjustments to the machine, the volume of juice, R ml, extracted from 1 kg of apples is such that $\mu = 520$ and $\sigma = k$

Given that $P(R < r) = 0.15$ and $P(R > 3r - 800) = 0.005$

(c) find the value of r and the value of k

(7)

5. The weights, X grams, of a particular variety of fruit are normally distributed with

$$X \sim \mathrm{N}(210, 25^2)$$

A fruit of this variety is selected at random.

(a) Show that the probability that the weight of this fruit is less than 240 grams is 0.8849

(2)

(b) Find the probability that the weight of this fruit is between 190 grams and 240 grams.

(2)

(c) Find the value of k such that $\mathrm{P}(210 - k < X < 210 + k) = 0.95$

(3)

A wholesaler buys large numbers of this variety of fruit and classifies the lightest 15% as small.

(d) Find the maximum weight of a fruit that is classified as small.
You must show your working clearly.

(3)

The weights, Y grams, of a second variety of this fruit are normally distributed with

$$Y \sim \mathrm{N}(\mu, \sigma^2)$$

Given that 5% of these fruit weigh less than 152 grams and 40% weigh more than 180 grams,

(e) calculate the mean and standard deviation of the weights of this variety of fruit.

(5)

5. The distance an athlete can throw a discus is normally distributed with mean 40 m and standard deviation 4 m

(a) Using standardisation, show that the probability that this athlete throws the discus less than 38.8 m is 0.3821

(2)

This athlete enters a discus competition.
To qualify for the final, they have 3 attempts to throw the discus a distance of more than 38.8 m
Once they qualify, they do not use any of their remaining attempts.

Given that they qualified for the final and that throws are independent,

(b) find the probability that this athlete qualified for the final on their second throw with a distance of more than 44 m

(5)

8. The random variable X is normally distributed with mean μ and variance 36

Given that

$$P(\mu - 2k < X < \mu + 2k) = 0.6$$

(a) find the value of k

(4)

The random variable Y is normally distributed with mean μ and standard deviation σ

Given that

$$2\mu = 3\sigma^2 \qquad \text{and} \qquad P\left(Y > \frac{3}{2}\mu\right) = 0.0668$$

(b) find the value of μ and the value of σ

(5)

5. A competition consists of two rounds.

The time, in minutes, taken by adults to complete round one is modelled by a normal distribution with mean 15 minutes and standard deviation 2 minutes.

(a) Use standardisation to find the proportion of adults that take less than 18 minutes to complete round one.

(2)

Only the fastest 60% of adults from round one take part in round two.

(b) Use standardisation to find the longest time that an adult can take to complete round one if they are to take part in round two.

(3)

The time, T minutes, taken by adults to complete round two is modelled by a normal distribution with mean μ

Given that $P(\mu - 10 < T < \mu + 10) = 0.95$

(c) find $P(T > \mu - 5 \mid T > \mu - 10)$

(5)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The distances, m miles, a motorbike travels on a full tank of petrol can be modelled by a normal distribution with mean 170 miles and standard deviation 16 miles.

(a) Find the probability that, on a randomly selected journey, the motorbike could travel at least 190 miles on a full tank of petrol.

(2)

The probability that, on a randomly selected journey, the motorbike could travel at least d miles on a full tank of petrol is 0.9

(b) Find the value of d

(3)

8. An orchard produces apples.

The weights, A grams, of its apples are normally distributed with mean μ grams and standard deviation σ grams.

It is known that

$$P(A < 162) = 0.1 \text{ and } P(162 < A < 175) = 0.7508$$

(a) Calculate the value of μ and the value of σ

(5)

A second orchard also produces apples.

The weights, B grams, of its apples have distribution $B \sim N(215, 10^2)$

An outlier is a value that is

$$\text{greater than } Q_3 + 1.5 \times (Q_3 - Q_1) \text{ or smaller than } Q_1 - 1.5 \times (Q_3 - Q_1)$$

An apple is selected at random from this second orchard.

Using $Q_3 = 221.74$ grams,

(b) find the probability that this apple is an outlier.

(5)

5. A recycling centre measures the weight of glass deposited by the public each day.

The weight of glass, S kg, deposited at the recycling centre in a day during the summer can be modelled by $S \sim N(700, 50^2)$

(a) Using standardisation and showing your working, find the probability that, in one randomly selected day during the summer,

(i) more than 640 kg of glass is deposited at the recycling centre,

(2)

(ii) 700 kg of glass, correct to the nearest 50 kg, is deposited at the recycling centre.

(5)

The weight of glass, W kg, deposited at the recycling centre in a day during the winter can be modelled by $W \sim N(\mu, \sigma^2)$

(b) Given that $P(W > 680) = 0.0668$ and $P(W < 599) = 0.3$

(i) find **two** equations in terms of μ and σ

(3)

(ii) Hence, showing your working, find the value of μ and the value of σ

(3)