# Chapter 4: Probability

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

**6.** Three events $A$, $B$ and $C$ are such that

$$P(A) = \frac{2}{5} \qquad P(C) = \frac{1}{2} \qquad P(A \cup B) = \frac{5}{8}$$

Given that $A$ and $C$ are mutually exclusive find

(a) $P(A \cup C)$

**(1)**

Given that $A$ and $B$ are independent

(b) show that $P(B) = \frac{3}{8}$

**(4)**

(c) Find $P(A \mid B)$

**(1)**

Given that $P(C' \cap B') = 0.3$

(d) draw a Venn diagram to represent the events $A$, $B$ and $C$

**(5)**

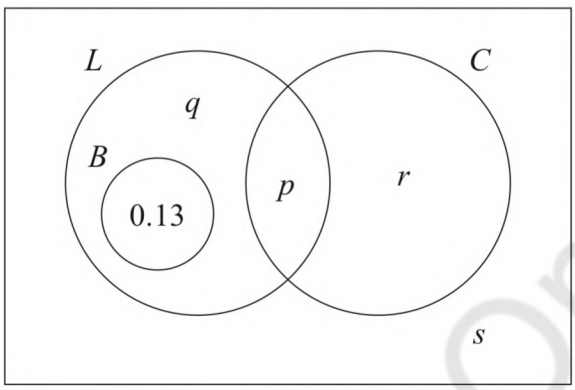
**1.** The Venn diagram shows the probability of a randomly selected student from a school being in the sets $L$, $B$ and $C$, where

$L$ represents the event that the student has instrumental music lessons

$B$ represents the event that the student plays in the school band

$C$ represents the event that the student sings in the school choir

$p$, $q$, $r$ and $s$ are probabilities.




(a) Select a pair of mutually exclusive events from $L$, $B$ and $C$.

**(1)**

Given that $P(L) = 0.4$, $P(B) = 0.13$, $P(C) = 0.3$ and the events $L$ and $C$ are independent,

(b) find the value of $p$,

**(2)**

(c) find the value of $q$, the value of $r$ and the value of $s$.

**(3)**

A student is selected at random from those who play in the school band or sing in the school choir.

(d) Find the exact probability that this student has instrumental music lessons.

**(3)**

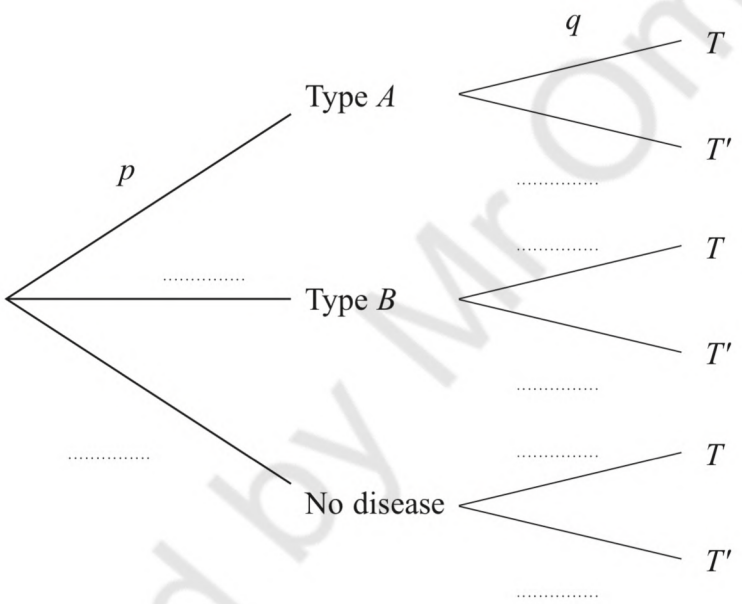
**3.** A certain disease occurs in a population in 2 mutually exclusive types.

It is difficult to diagnose people with type $A$ of the disease and there is an unknown proportion $p$ of the population with type $A$.
It is easier to diagnose people with type $B$ of the disease and it is known that 2% of the population have type $B$.

A test has been developed to help diagnose whether or not a person has the disease. The event $T$ represents a positive result on the test. After a large-scale trial of the test, the following information was obtained.

For a person with type $B$ of the disease the probability of a positive test result is 0.96
For a person who does not have the disease the probability of a positive test result is 0.05
For a person with type $A$ of the disease the probability of a positive test result is $q$

(a) Complete the tree diagram.




**(2)**

The probability of a randomly selected person having a positive test result is 0.169
For a person with a positive test result, the probability that they do not have the disease is $\frac{41}{169}$

(b) Find the value of $p$ and the value of $q$.

**(7)**

A doctor is about to see a person who she knows does not have type $B$ of the disease but does have a positive test result.

(c) (i) Find the probability that this person has type $A$ of the disease.

**(3)**

(ii) State, giving a reason, whether or not the doctor will find the test useful.

**(1)**

**5.** A housing estate consists of 320 houses.

A house on the estate is selected at random.

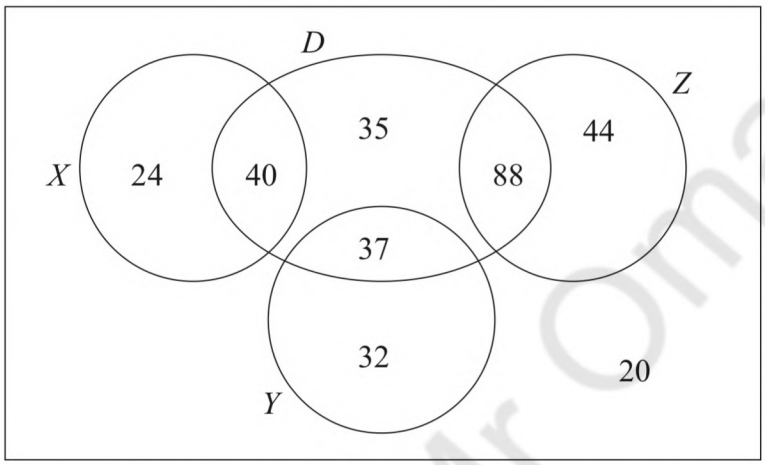
$D$ denotes the event 'the house has a driveway'

$X$ denotes the event 'no cars per household'

$Y$ denotes the event 'exactly 1 car per household'

$Z$ denotes the event 'exactly 2 cars per household'

The Venn diagram shows the number of households in each region for the events $D$, $X$, $Y$ and $Z$.




(a) Find (i) P($D$)

**(1)**

(ii) P($D \cap X'$)

**(1)**

(iii) P($D' \cup Z'$)

**(2)**

Given that the house has a driveway,

(b) find the probability that there are exactly 2 cars in the household.

**(2)**

(c) Write down 2 of the events $D$, $X$, $Y$ and $Z$ that are mutually exclusive.

**(1)**

(d) Determine whether the events $D$ and $X$ are independent. Justify your answer.

**(2)**

(e) Define in the context of this question, the event

(i) $D' \cap Z$

**(1)**

(ii) $D \cap (X \cup Y)$

**(2)**

**2.** A group of 40 families was asked whether their family had a dog, a cat or a rabbit as pets.

No family had a dog and a rabbit
2 families had both a dog and a cat
12 families had a dog
14 families had a cat
11 families had a rabbit
9 families did not have any of these animals as pets

A family from this group is selected at random.

$D$ represents the event that the family has a dog
$C$ represents the event that the family has a cat
$R$ represents the event that the family has a rabbit

(a) Draw a Venn diagram to illustrate this information.

**(4)**

(b) State, giving a reason, a pair of mutually exclusive events from $D$, $C$ and $R$.

**(1)**

(c) Find the probability that the family has exactly 2 of these kinds of animals as pets.

**(1)**

(d) Showing your working clearly, determine whether or not the events $D$ and $C$ are independent.

**(2)**

Sarah's family is in the group and her family has a pet cat.

(e) Find the probability that Sarah's family also has a pet rabbit.

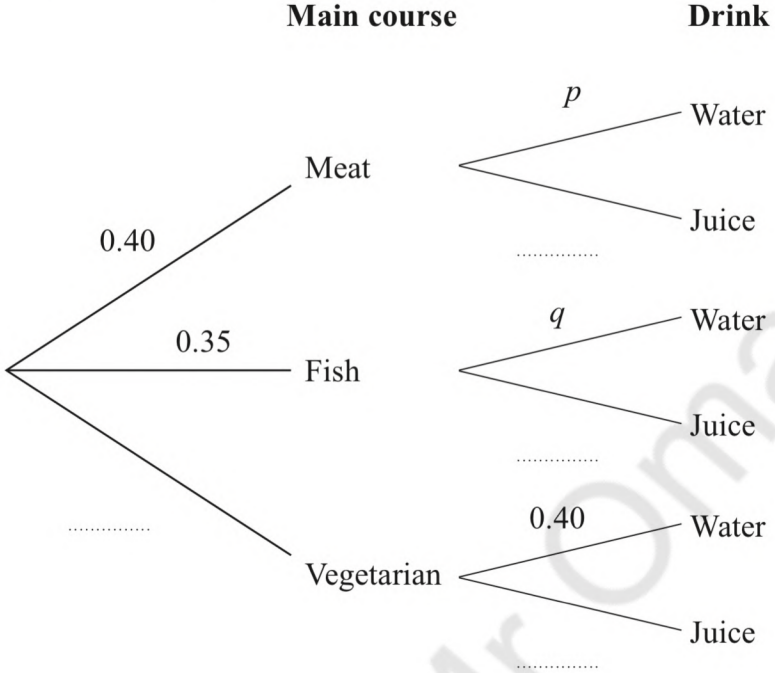
**(2)**

(f) Find the exact value of $P([D \cup R] \mid C')$

**(2)**

**2.** In a school canteen, students can choose from a main course of meat ($M$), fish ($F$) or vegetarian ($V$). They can then choose a drink of either water ($W$) or juice ($J$).

The partially completed tree diagram, where $p$ and $q$ are probabilities, shows the probabilities of these choices for a randomly selected student.




(a) Complete the tree diagram, giving your answers in terms of $p$ and $q$ where appropriate.

**(2)**

(b) Find an expression, in terms of $p$ and $q$, for the probability that a randomly selected student chooses water to drink.

**(1)**

The events "choosing a vegetarian main course" and "choosing water to drink" are independent.

(c) Find a linear equation in terms of $p$ and $q$.

**(2)**

A student who has chosen juice to drink is selected at random. The probability that they chose fish for their main course is $\frac{7}{30}$

(d) Find the value of $p$ and the value of $q$.

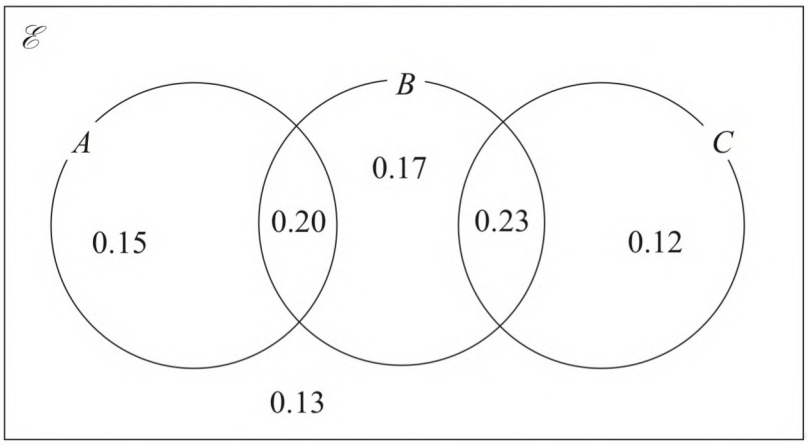
**(5)**

The canteen manager claims that students who choose water to drink are most likely to choose a fish main course.

(e) State, showing your working clearly, whether or not the manager's claim is correct.

**(3)**

**1.** The Venn diagram shows the events $A$, $B$ and $C$ and their associated probabilities.




Find

(a) $P(B')$

**(1)**

(b) $P(A \cup C)$

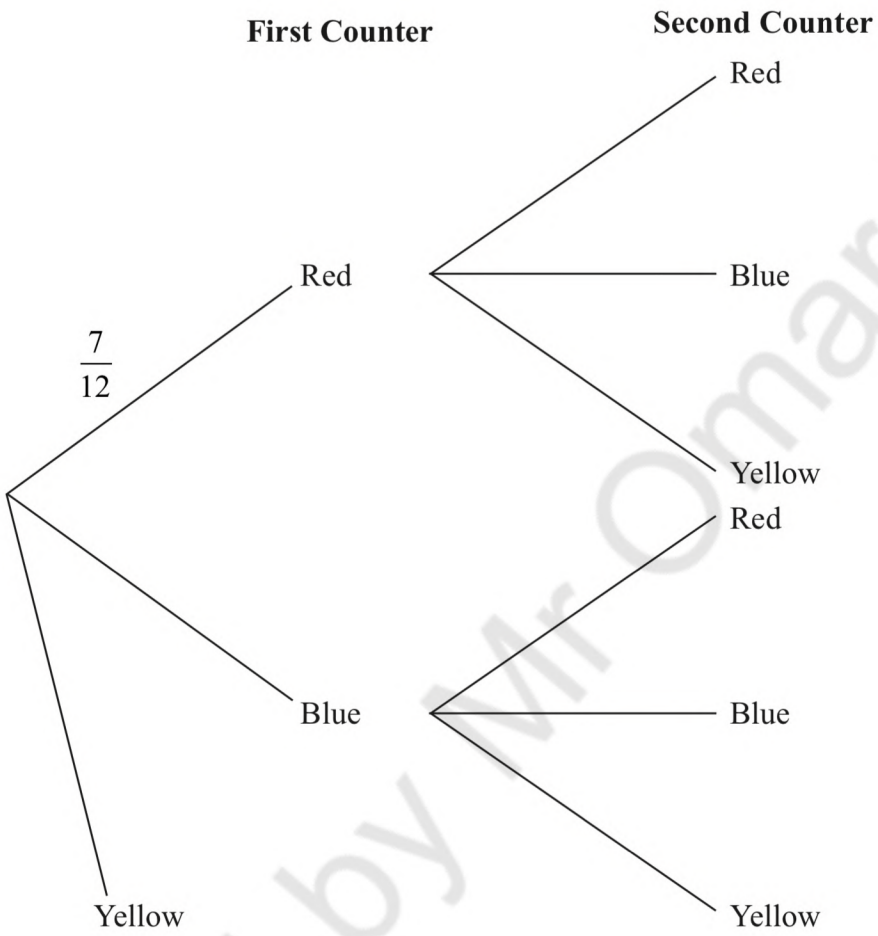
**(2)**

(c) $P(A \mid B')$

**(2)**

**1.** There are 7 red counters, 3 blue counters and 2 yellow counters in a bag. Gina selects a counter at random from the bag and keeps it. If the counter is yellow she does not select any more counters. If the counter is not yellow she randomly selects a second counter from the bag.

(a) Complete the tree diagram.




**(2)**

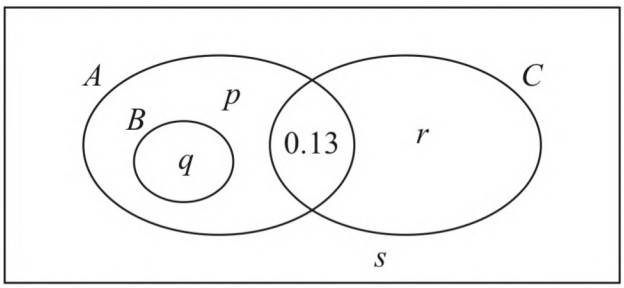
Given that Gina has selected a yellow counter,

(b) find the probability that she has 2 counters.

**(3)**

**2.** In the Venn diagram below, $A$, $B$ and $C$ are events and $p$, $q$, $r$ and $s$ are probabilities.

The events $A$ and $C$ are independent and $P(A) = 0.65$




(a) State which two of the events $A$, $B$ and $C$ are mutually exclusive.

**(1)**

(b) Find the value of $r$ and the value of $s$.

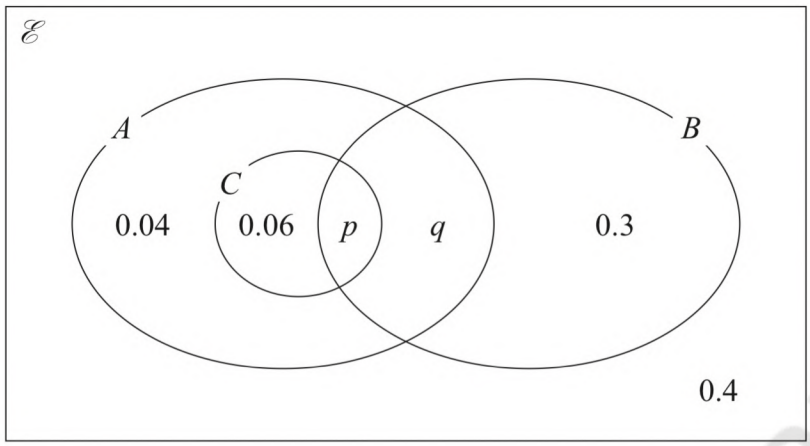
**(5)**

The events $(A \cap C')$ and $(B \cup C)$ are also independent.

(c) Find the exact value of $p$ and the exact value of $q$. Give your answers as fractions.

**(6)**

**1.** The Venn diagram shows the events $A$, $B$ and $C$ and their associated probabilities, where $p$ and $q$ are probabilities.




(a) Find $\mathrm{P}(B)$

**(1)**

(b) Determine whether or not $A$ and $B$ are independent.

**(2)**

Given that $\mathrm{P}(C \mid B) = \mathrm{P}(C)$

(c) find the value of $p$ and the value of $q$

**(3)**

The event $D$ is such that

- $A$ and $D$ are mutually exclusive
- $\mathrm{P}(B \cap D) > 0$

(d) On the Venn diagram show a possible position for the event $D$

**(1)**

**4.** Three bags **A**, **B** and **C** each contain coloured balls.

Bag **A** contains 4 red balls and 2 yellow balls only.
Bag **B** contains 4 red balls and 1 yellow ball only.
Bag **C** contains 6 red balls only.

In a game
Mike takes a ball at random from bag **A**, records the colour and places it in bag **C**.
He then takes a ball at random from bag **B**, records the colour and places it in bag **C**.
Finally, Mike takes a ball at random from bag **C** and records the colour.

(a) Complete the tree diagram on the page opposite, to illustrate the game by adding the remaining branches and all probabilities.

**(3)**

(b) Show that the probability that Mike records a yellow ball exactly twice is $\frac{1}{10}$

**(3)**

Given that Mike records exactly 2 yellow balls,

(c) find the probability that the ball drawn from bag **A** is red.

**(2)**

Mike plays this game a large number of times, each time starting with the bags containing balls as described above. The random variable $X$ represents the number of yellow balls recorded in a single game.

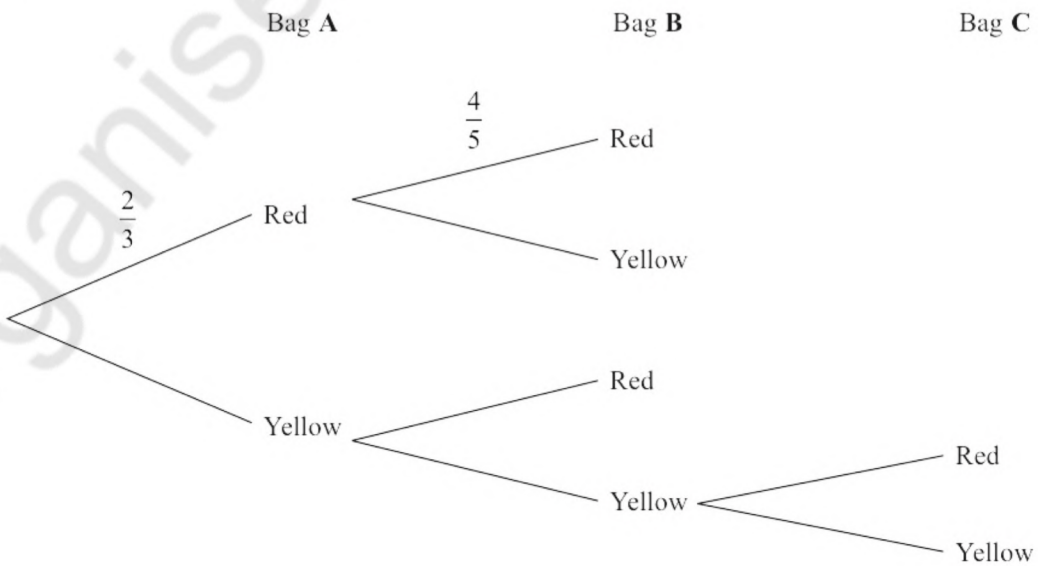
(d) Find the probability distribution of $X$

**(3)**

(e) Find $\mathrm{E}(X)$

**(2)**

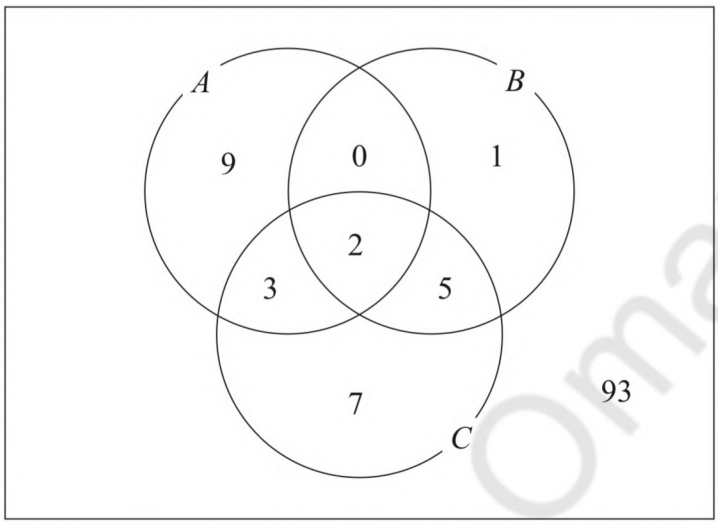
**Question 4 continued**



A factory produces shoes.

A quality control inspector at the factory checks a sample of 120 shoes for each of three types of defect. The Venn diagram represents the inspector's results.

$A$ represents the event that a shoe has defective stitching
$B$ represents the event that a shoe has defective colouring
$C$ represents the event that a shoe has defective soles




One of the shoes in the sample is selected at random.

(a) Find the probability that it does **not** have defective soles.

**(1)**

(b) Find $P(A \cap B \cap C')$

**(1)**

(c) Find $P(A \cup B \cup C')$

**(2)**

(d) Find the probability that the shoe has at most one type of defect.

**(2)**

(e) Given the selected shoe has at most one type of defect, find the probability it has defective stitching.

**(2)**

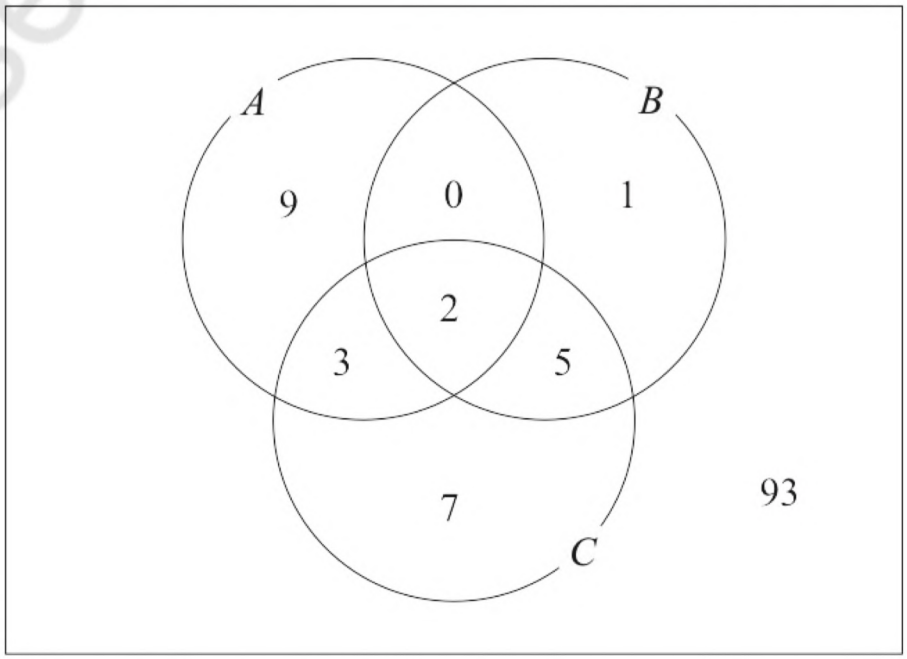
The random variable $X$ is the number of the events $A$, $B$, $C$ that occur for a randomly selected shoe.

(f) Find $E(X)$

**(3)**

**Question 1 continued**

**This is a copy of the Venn diagram for this question.**



**4.** The events $H$ and $W$ are such that

$$P(H) = \frac{3}{8} \qquad P(H \cup W) = \frac{3}{4}$$

Given that $H$ and $W$ are independent,

(a) show that $P(W) = \frac{3}{5}$

**(4)**

The event $N$ is such that

$$P(N) = \frac{1}{15} \qquad P(H \cap N) = P(N)$$

(b) Find $P(N' \mid H)$

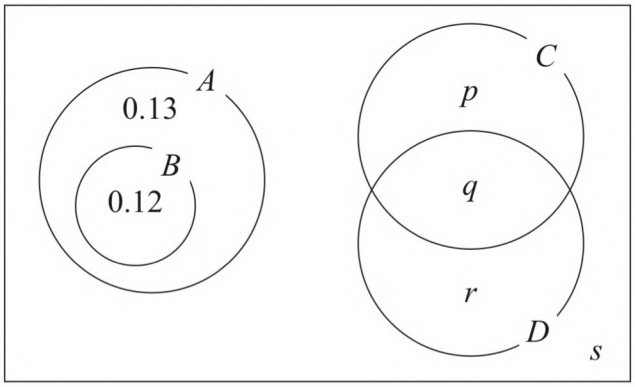
**(2)**

Given that $W$ and $N$ are mutually exclusive,

(c) draw a Venn diagram to represent the events $H$, $W$ and $N$ giving the exact probabilities of each region in the Venn diagram.

**(5)**

**6.** The Venn diagram shows the events $A$, $B$, $C$ and $D$, where $p$, $q$, $r$ and $s$ are probabilities.




(a) Write down the value of

(i) $P(A)$

(ii) $P(A|B)$

(iii) $P(A|C)$

**(3)**

Given that $P(B' \cap D') = \frac{7}{10}$ and $P(C|D) = \frac{3}{5}$

(b) find the exact value of $q$ and the exact value of $r$

**(6)**

Given also that $P(B \cup C') = \frac{5}{8}$

(c) find the exact value of $s$

**(2)**

**2.** Two bags, ***X*** and ***Y***, each contain green marbles (G) and blue marbles (B) only.

- Bag ***X*** contains 5 green marbles and 4 blue marbles
- Bag ***Y*** contains 6 green marbles and 5 blue marbles

A marble is selected at random from bag ***X*** and placed in bag ***Y***
A second marble is selected at random from bag ***X*** and placed in bag ***Y***
A third marble is then selected, this time from bag ***Y***

(a) Use this information to complete the tree diagram shown on page 7

**(3)**

(b) Find the probability that the 2 marbles selected from bag ***X*** are of different colours.

**(2)**

(c) Find the probability that all 3 marbles selected are the same colour.

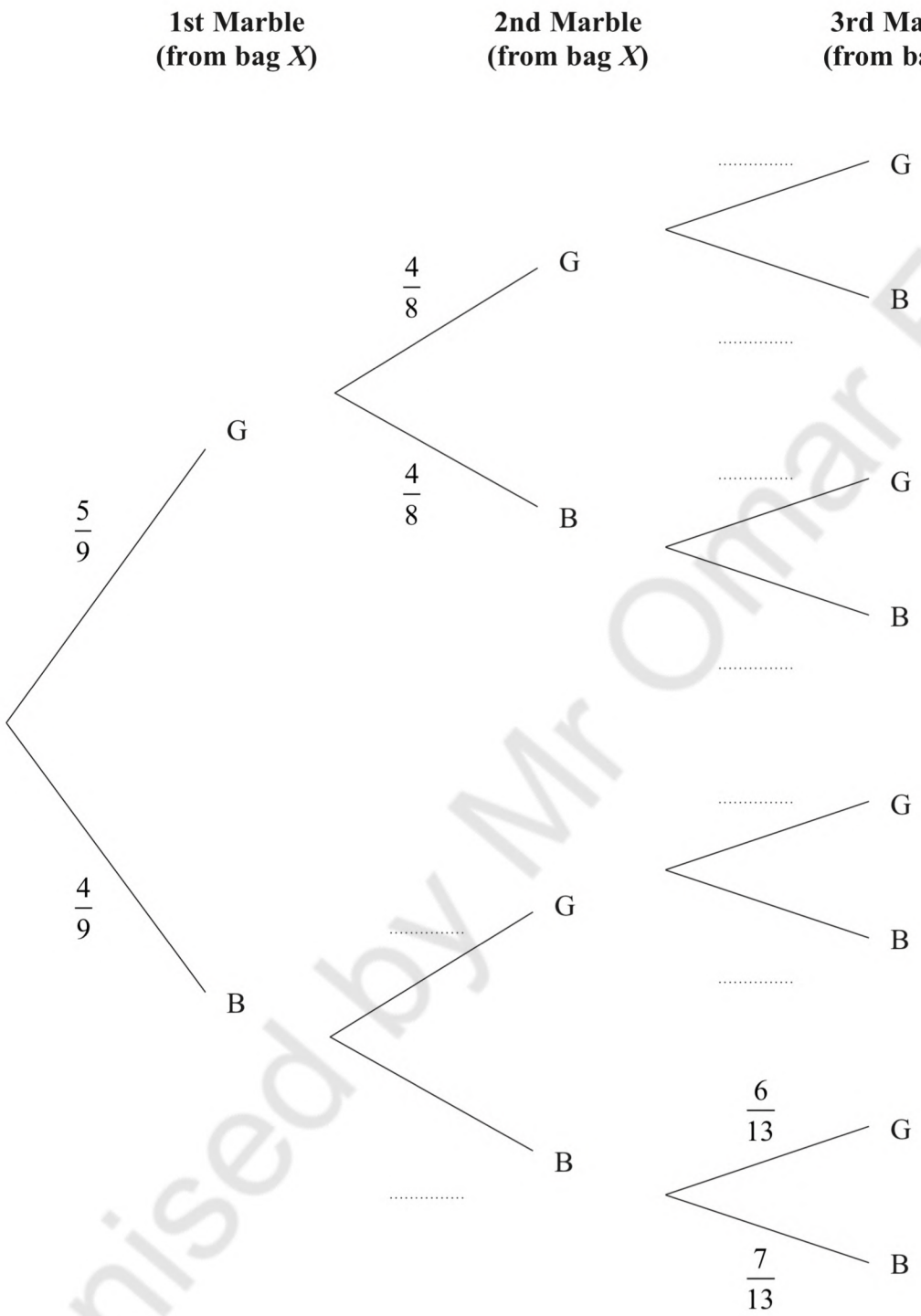
**(2)**

Given that all three marbles selected are the same colour,

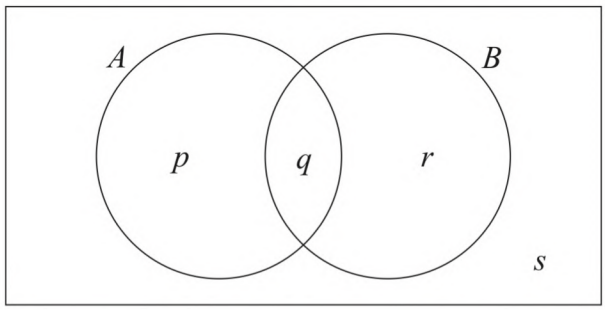
(d) find the probability that they are all green.

**(3)**

**Question 2 continued**



**4.** (i) In the Venn diagram below, $A$ and $B$ represent events and $p$, $q$, $r$ and $s$ are probabilities.




$$P(A) = \frac{7}{25} \qquad P(B) = \frac{1}{5} \qquad P\left[\left(A \cap B'\right) \cup \left(A' \cap B\right)\right] = \frac{8}{25}$$

(a) Use algebra to show that $2p + 2q + 2r = \frac{4}{5}$

**(4)**

(b) Find the value of $p$, the value of $q$, the value of $r$ and the value of $s$

**(5)**

(ii) Two events, $C$ and $D$, are such that

$$P(C) = \frac{x}{x+5} \qquad P(D) = \frac{5}{x}$$

where $x$ is a positive constant.

By considering $P(C) + P(D)$ show that $C$ and $D$ **cannot** be mutually exclusive.

**(4)**

**4.** A bag contains a large number of coloured counters. Each counter is labelled A, B or C

30% of the counters are labelled A
45% of the counters are labelled B
The rest of the counters are labelled C

It is known that

2% of the counters labelled A are red
4% of the counters labelled B are red
6% of the counters labelled C are red

One counter is selected at random from the bag.

(a) Complete the tree diagram on the opposite page to illustrate this information.

**(2)**

(b) Calculate the probability that the counter is labelled A and is not red.

**(2)**

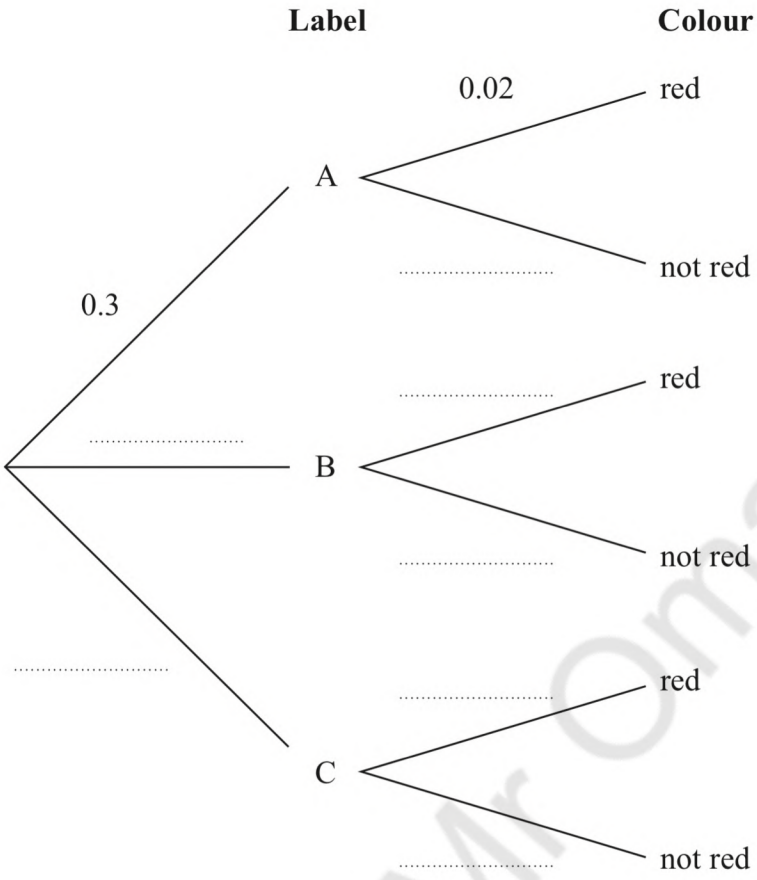
(c) Calculate the probability that the counter is red.

**(2)**

(d) Given that the counter is red, find the probability that it is labelled C

**(3)**

**Question 4 continued**



**6.** Three events $A$, $B$ and $C$ are such that

$$P(A) = 0.1 \qquad P(B|A) = 0.3 \qquad P(A \cup B) = 0.25 \qquad P(C) = 0.5$$

Given that $A$ and $C$ are mutually exclusive

(a) find $P(A \cup C)$

**(1)**

(b) Show that $P(B) = 0.18$

**(3)**

Given also that $B$ and $C$ are independent,

(c) draw a Venn diagram to represent the events $A$, $B$ and $C$ and the probabilities associated with each region.

**(5)**

**1.** Sally plays a game in which she can either win or lose.

A turn consists of up to 3 games. On each turn Sally plays the game up to 3 times. If she wins the first 2 games or loses the first 2 games, then she will not play the 3rd game.

- The probability that Sally wins the first game in a turn is 0.7
- If Sally wins a game the probability that she wins the next game is 0.6
- If Sally loses a game the probability that she wins the next game is 0.2

(a) Use this information to complete the tree diagram on page 3

**(3)**

(b) Find the probability that Sally wins the first 2 games in a turn.

**(2)**

(c) Find the probability that Sally wins exactly 2 games in a turn.

**(2)**

Given that Sally wins 2 games in a turn,

(d) find the probability that she won the first 2 games.

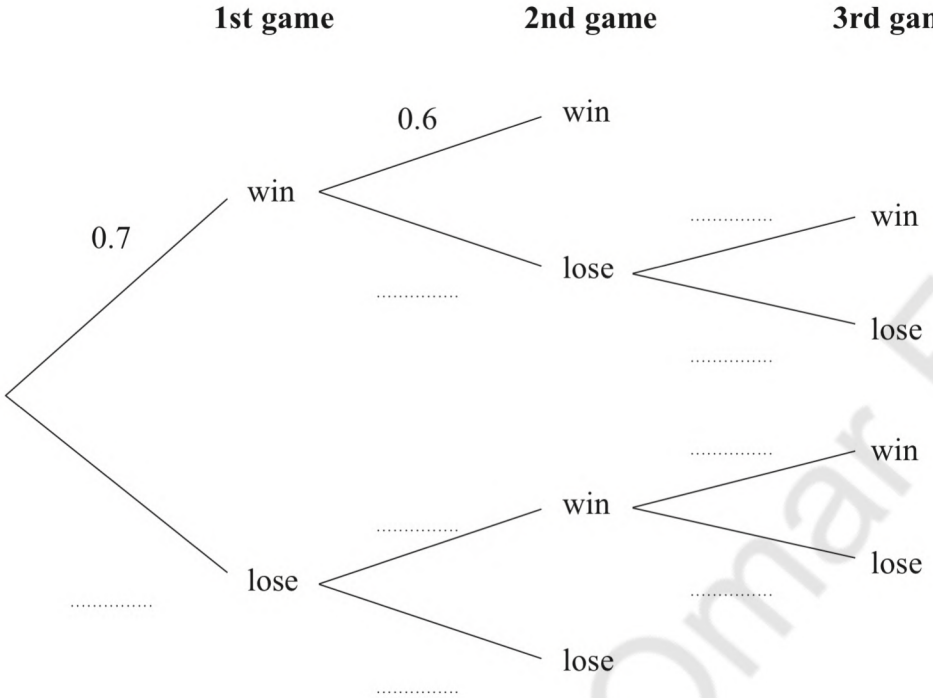
**(2)**

Given that Sally won the first game in a turn,

(e) find the probability that she won 2 games.

**(2)**

**Question 1 continued**



**3.** (i) Bob shops at a market each week. The event that

Bob buys carrots is denoted by $C$

Bob buys onions is denoted by $O$

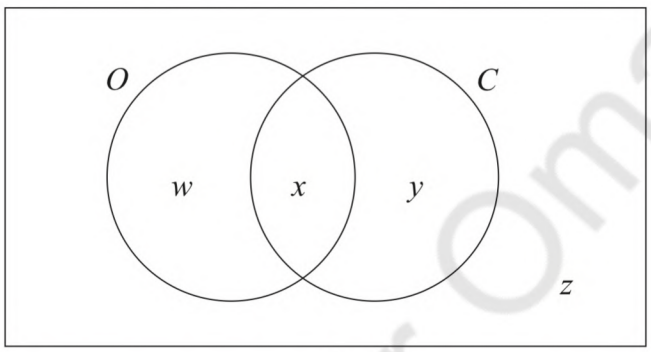
At each visit, Bob may buy neither, or one, or both of these items. The probability that

Bob buys carrots is 0.65

Bob does **not** buy onions is 0.3

Bob buys onions but not carrots is 0.15

The Venn diagram below represents the events $C$ and $O$




where $w$, $x$, $y$ and $z$ are probabilities.

(a) Find the value of $w$, the value of $x$, the value of $y$ and the value of $z$

**(4)**

For one visit to the market,

(b) find the probability that Bob buys either carrots or onions but not both.

**(1)**

(c) Show that the events $C$ and $O$ are **not** independent.

**(2)**

(ii) $F$, $G$ and $H$ are 3 events. $F$ and $H$ are mutually exclusive. $F$ and $G$ are independent.

Given that

$$\mathrm{P}(F) = \frac{2}{7} \qquad \mathrm{P}(H) = \frac{1}{4} \qquad \mathrm{P}(F \cup G) = \frac{5}{8}$$

(a) find $\mathrm{P}(F \cup H)$

**(1)**

(b) find $\mathrm{P}(G)$

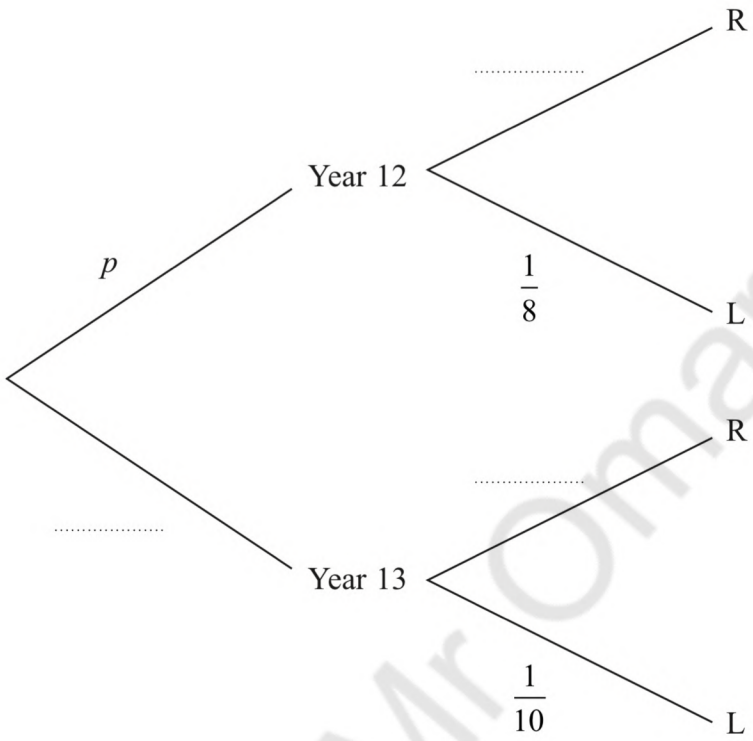
**(3)**

(c) find $\mathrm{P}(F \cap G)$

**(1)**

**3.** In a sixth form college each student in Year 12 and Year 13 is either left-handed (L) or right-handed (R).

The partially completed tree diagram, where $p$ is a probability, gives information about these students.




(a) Complete the tree diagram, in terms of $p$ where necessary.

**(1)**

The probability that a student is left-handed is 0.11

(b) Find the value of $p$

**(3)**

(c) Find the probability that a student selected at random is in Year 12 and left-handed.

**(2)**

Given that a student is right-handed,

(d) find the probability that the student is in Year 12

**(2)**

**6.** The events $A$ and $B$ satisfy

$$P(A) = x \qquad P(B) = y \qquad P(A \cup B) = 0.65 \qquad P(B|A) = 0.3$$

(a) Show that

$$14x + 20y = 13$$

**(3)**

The events $B$ and $C$ are mutually exclusive such that

$$P(B \cup C) = 0.85 \qquad P(C) = \frac{1}{2}x + y$$

(b) (i) Find a second equation in $x$ and $y$

(ii) Hence find the value of $x$ and the value of $y$

**(4)**

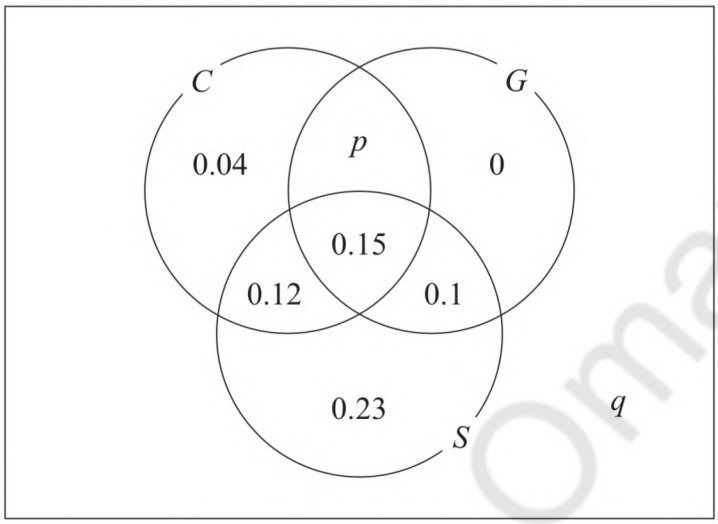
(c) Determine whether or not $A$ and $B$ are statistically independent.
You must show your working clearly.

**(2)**

**6.** The Venn diagram shows the probabilities related to teenagers playing 3 particular board games.

$C$ is the event that a teenager plays Chess
$S$ is the event that a teenager plays Scrabble
$G$ is the event that a teenager plays Go

where $p$ and $q$ are probabilities.




(a) Find the probability that a randomly selected teenager plays Chess but does not play Go.

**(1)**

Given that the events $C$ and $S$ are independent,

(b) find the value of $p$

**(4)**

(c) Hence find the value of $q$

**(2)**

(d) Find (i) $P((C \cup S) \cap G')$

**(1)**

(ii) $P(C \mid (S \cap G))$

**(2)**

A youth club consists of a large number of teenagers.
In this youth club 76 teenagers play Chess and Go.

(e) Use the information in the Venn diagram to estimate how many of the teenagers in the youth club do not play Scrabble.

**(3)**

**3.** A group of 200 adults were asked whether they read cooking magazines, travel magazines or sport magazines.
Their replies showed that

- 29 read only cooking magazines
- 33 read only travel magazines
- 42 read only sport magazines
- 17 read cooking magazines and sport magazines but **not** travel magazines
- 11 read travel magazines and sport magazines but **not** cooking magazines
- 22 read cooking magazines and travel magazines but **not** sport magazines
- 32 **do not** read cooking magazines, travel magazines or sport magazines

(a) Using this information, complete the Venn diagram on page 11

**(3)**

One of these adults was chosen at random.

(b) Find the probability that this adult,

(i) reads cooking magazines and travel magazines and sport magazines,

**(1)**

(ii) does **not** read cooking magazines.

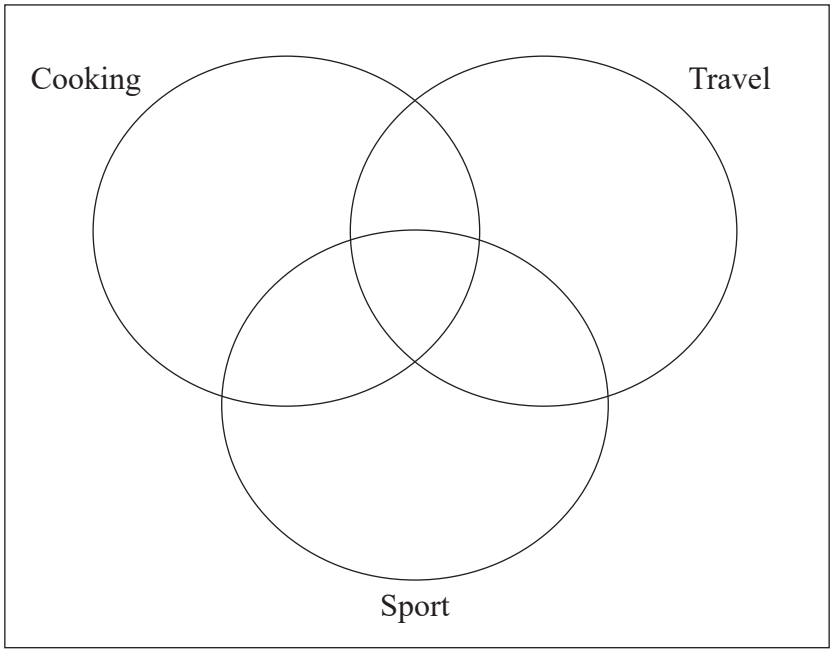
**(2)**

Given that this adult reads travel magazines,

(c) find the probability that this adult also reads sport magazines.

**(2)**

**Question 3 continued**

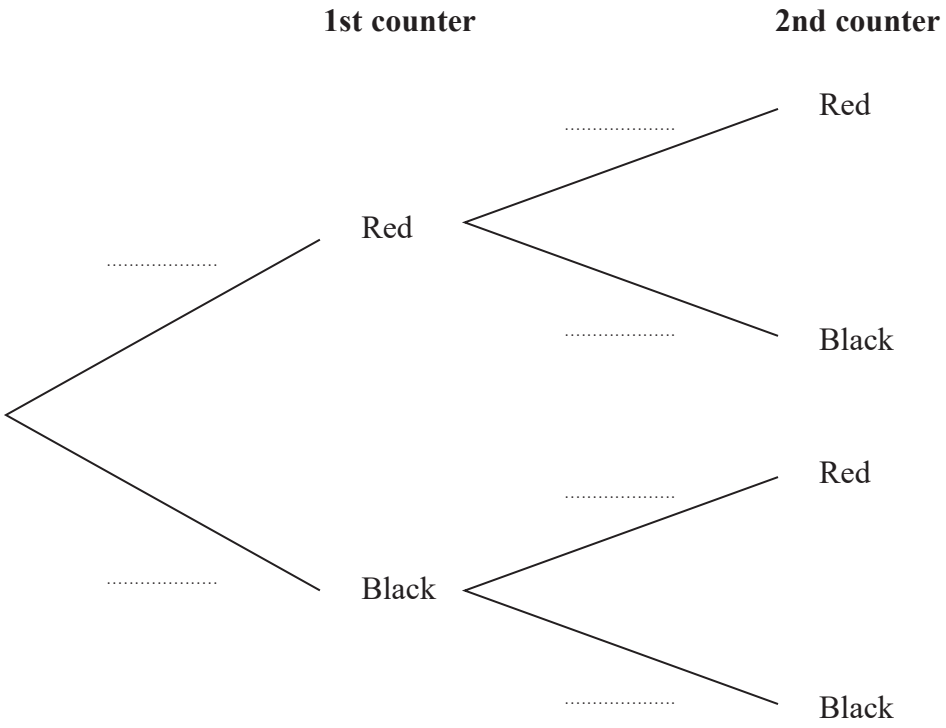


**7.** A box contains only red counters and black counters.

There are $n$ red counters and $n + 1$ black counters.

Two counters are selected at random, one at a time **without** replacement, from the box.

(a) Complete the tree diagram for this information. Give your probabilities in terms of $n$ where necessary.




**(3)**

(b) Show that the probability that the two counters selected are different colours is

$$\frac{n+1}{2n+1}$$

**(2)**

The probability that the two counters selected are different colours is $\frac{25}{49}$

(c) Find the total number of counters in the box before any counters were selected.

**(2)**

Given that the two counters selected are different colours,

(d) find the probability that the 1st counter is black.
You must show your working.

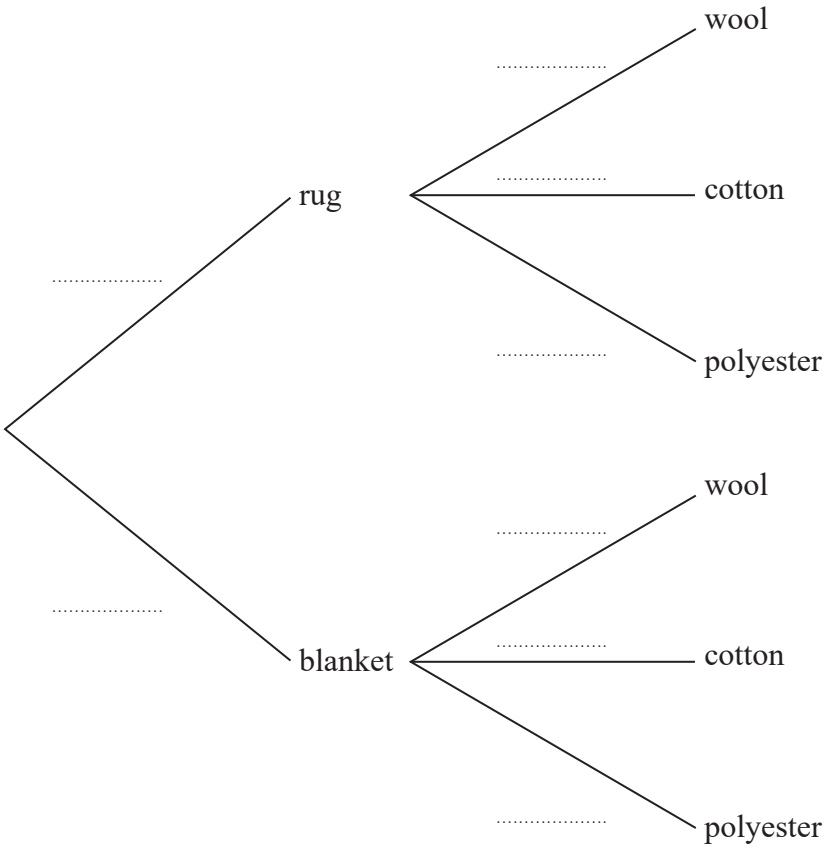
**(2)**

**3.** A factory makes rugs and blankets from wool **or** cotton **or** polyester.

- 30% of the items made are rugs and 70% of the items made are blankets
- 60% of the rugs are made from wool
- 35% of the blankets are made from wool
- 10% of the rugs are made from cotton
- 20% of the blankets are made from cotton
- the rest of the rugs and blankets are made from polyester

An item made by the factory is selected at random.

(a) Complete the tree diagram below to illustrate this information.




**(2)**

(b) (i) Find the probability that the item selected is **not** made from wool.

**(2)**

(ii) Given that this item is **not** made from wool, find the probability that it is a blanket.

**(2)**

**4.** A manufacturer of pet food gives samples of three new flavours of cat food, $A$, $B$ and $C$, to each of 100 customers who each own one cat. These customers are surveyed to find out whether or not their cat likes each flavour with the following results

40 cats like $A$

45 cats like $B$

50 cats like $C$

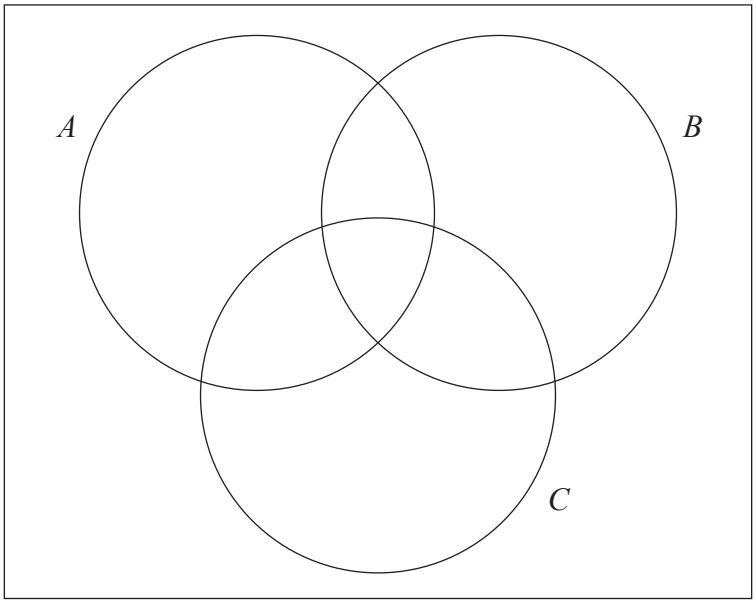
19 cats like both $A$ and $B$

20 cats like both $A$ and $C$

22 cats like both $B$ and $C$

12 cats like all three of these flavours.

(a) Using these results complete the Venn diagram below.




**(4)**

For these cats,

(b) determine whether or not a cat liking flavour $A$ is independent of it liking flavour $C$

**(2)**

(c) One of these 100 cats is chosen at random.

(i) Find the probability that it likes only one of the three flavours.

**(1)**

(ii) Given that it likes flavour $C$, find the probability that it also likes flavour $B$

**(2)**