Chapter 8: Differentiation

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3. A curve has equation

$$y = \frac{x^3}{16} - 4\sqrt{x} + \frac{8}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ giving your answer in its simplest form.

(3)

The point $P(4, -2)$ lies on the curve.

(b) Use the answer to part (a) to find the equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

1. Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$,

(3)

9.

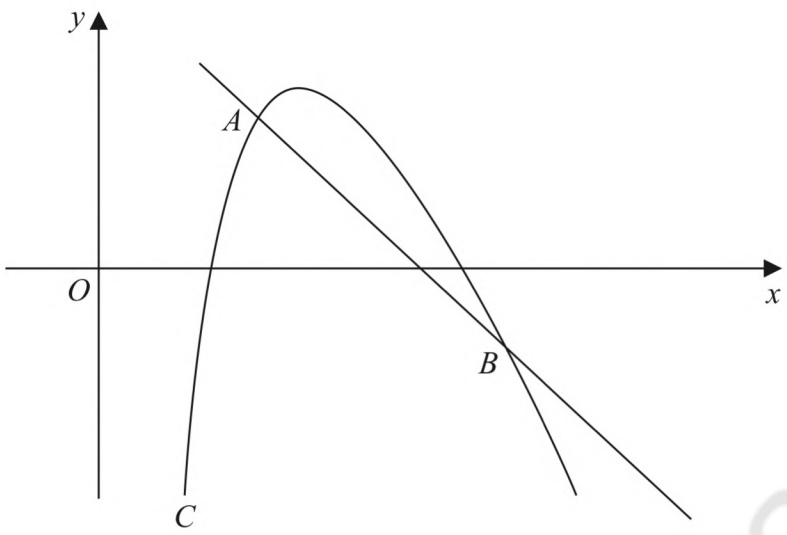

Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point A lies on C and has x coordinate equal to 2

(a) Show that the equation of the normal to C at A is $y = -2x + 7$.

(6)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

6. (*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Given

$$f(x) = 2x^{\frac{5}{2}} - 40x + 8 \qquad x > 0$$

(a) solve the equation $f'(x) = 0$

(4)

(b) solve the equation $f''(x) = 5$

(3)

1. The curve C has equation $y = \frac{1}{8}x^3 - \frac{24}{\sqrt{x}} + 1$

(a) Find $\frac{dy}{dx}$, giving the answer in its simplest form.

(3)

The point $P(4, -3)$ lies on C.

(b) Find the equation of the tangent to C at the point P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

10. A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the x-axis at the point P and crosses the x-axis at the point Q.

(a) State the coordinates of the point P.

(1)

(b) Find $f'(x)$.

(4)

(c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where k is a constant to be found.

(3)

The curve with equation $y = f(x + a)$, where a is a constant, passes through the origin O.

(d) State the possible values of a.

(2)

5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x^2 + 9}{2\sqrt{x}} \qquad x > 0$$

Find the x coordinate of the point on the curve at which $\frac{dy}{dx} = 0$

(6)

8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

1. A curve has equation

$$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

The point P lies on the curve and has x coordinate $\frac{1}{2}$

(b) Find an equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(b) Solve the equation

$$f''(x) = 6$$

(5)

1. The curve C has equation

$$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(4)

The point $P(4, 3)$ lies on C.

(b) Find the equation of the normal to C at the point P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x + 1)(x - 3)^2$$

(b) Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)

7.

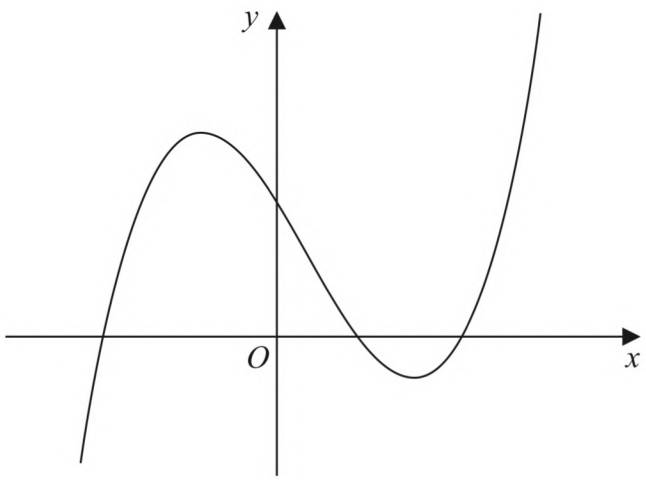

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

(c) Find $f'(x)$.

(4)

(d) Hence find the range of values of x for which the gradient of the curve with equation $y = f(x)$ is less than -18

(3)

10.

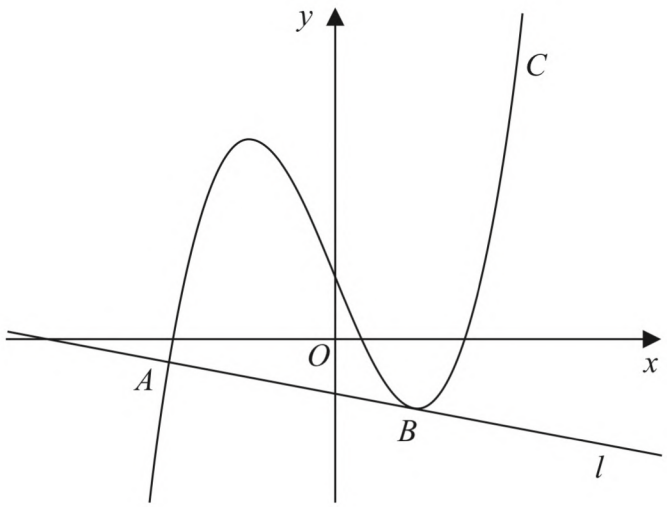

Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The line l, shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B, justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k.

(4)

1. The curve C has equation

$$y = \frac{x^3}{4} - x^2 + \frac{17}{x} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(3)

The point $R\left(2, \frac{13}{2}\right)$ lies on C.

(b) Find the equation of the tangent to C at the point R. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ giving your answer in simplest form.

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

(2)

4. **In this question you must show all stages of your working.**

(a) Write

$$y = \frac{5x^2 + \sqrt{x^3}}{\sqrt[3]{8x}}$$

in the form

$$y = Ax^p + Bx^q$$

where A, B, p and q are constants to be found.

(4)

(b) Hence find $\frac{dy}{dx}$ giving each coefficient in simplest form.

(3)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$

(3)

(b) $\frac{d^2y}{dx^2}$

(2)

3. The curve C has equation

$$y = \frac{5x^3 - 8}{2x^2} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(4)

The point $P(2, 4)$ lies on C.

(b) Find an equation for the tangent to C at P writing your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

7. The curve C has equation $y = f(x)$ where

$$f(x) = 2x^3 - kx^2 + 14x + 24$$

and k is a constant.

(a) Find, in simplest form,

(i) $f'(x)$

(ii) $f''(x)$

(3)

The curve with equation $y = f'(x)$ intersects the curve with equation $y = f''(x)$ at the points A and B.

Given that the x coordinate of A is 5

(b) find the value of k.

(2)

(c) Hence find the coordinates of B.

(3)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$y = x^3 + 96\sqrt{x} + 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

(3)

(b) Find the solution of the equation

$$\frac{d^2y}{dx^2} = 0$$

writing the answer in the form 2^k where k is a constant.

(3)

10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{3}x^3 - 25x - \frac{56}{x} + \frac{194}{3} \qquad x > 0$$

The point P, which lies on C, has coordinates $(2, -8)$

(a) Show that an equation of the tangent to C at P is

$$y = -3x - 2$$

(5)

The point Q also lies on C.

Given that the tangent to C at Q is parallel to the tangent to C at P,

(b) find, using algebra and showing your working, the exact x coordinate of Q.

(5)

3. **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)

9.

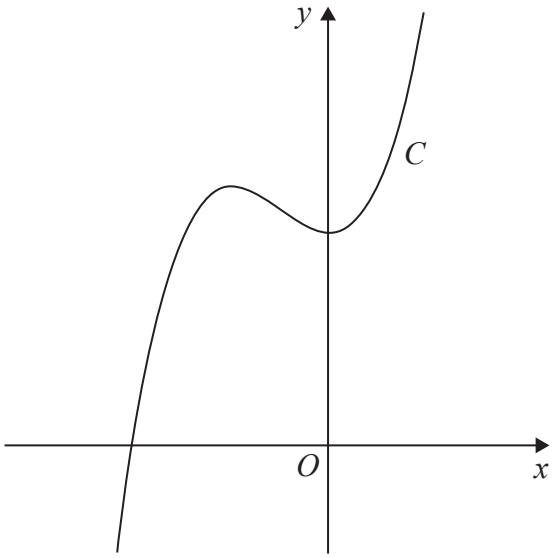

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)