Chapter 6&7: Trigonometric Ratios and Radians

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P1 Chapter 6&7 Solutions

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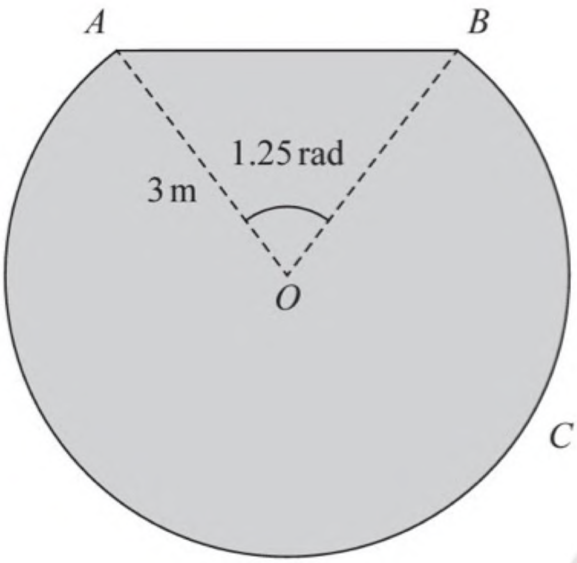


Figure 2

Figure 2 shows the plan view of a design for a garden pond.

The pond consists of a sector, *AOBCA*, of a circle with centre *O*, joined to a triangle *AOB*.

Given $AO = BO = 3$ m and angle $AOB = 1.25$ radians,

(a) find the perimeter of the pond, giving your answer, in metres, to 2 decimal places.
(4)

Given that there is a uniform depth of water in the pond of 1.5 m,

(b) find the volume of water in the pond, in m^3, to one decimal place.
(4)

7. (a) Sketch, on Diagram 1, the graphs of

(i) $y = 2\cos x, \quad -90° \leqslant x \leqslant 360°$

(ii) $y = \tan x, \quad -90° \leqslant x \leqslant 360°$

(4)

(b) Given that $n \in \mathbb{N}$, deduce, in terms of n, the number of real solutions of the equation

(i) $2\cos x = \tan x, \quad -90° \leqslant x \leqslant (360n)°$

(ii) $\tan x = -\frac{3}{2}, \quad -90° \leqslant x \leqslant (360n)°$

(2)

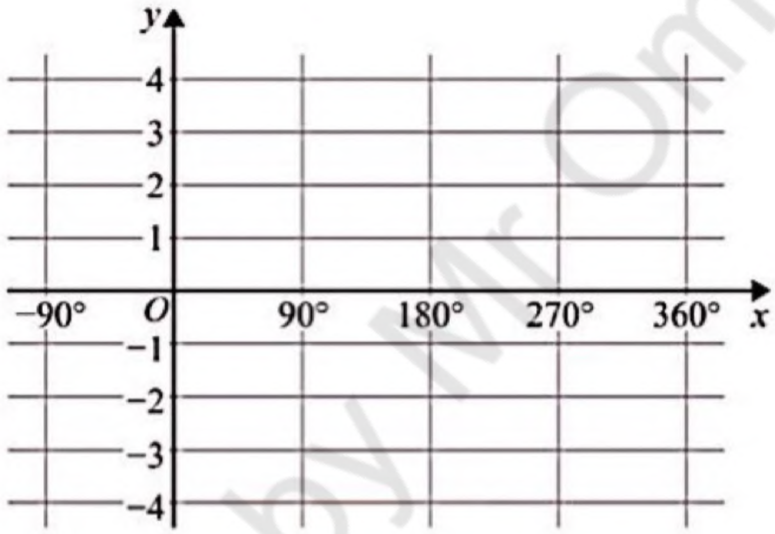
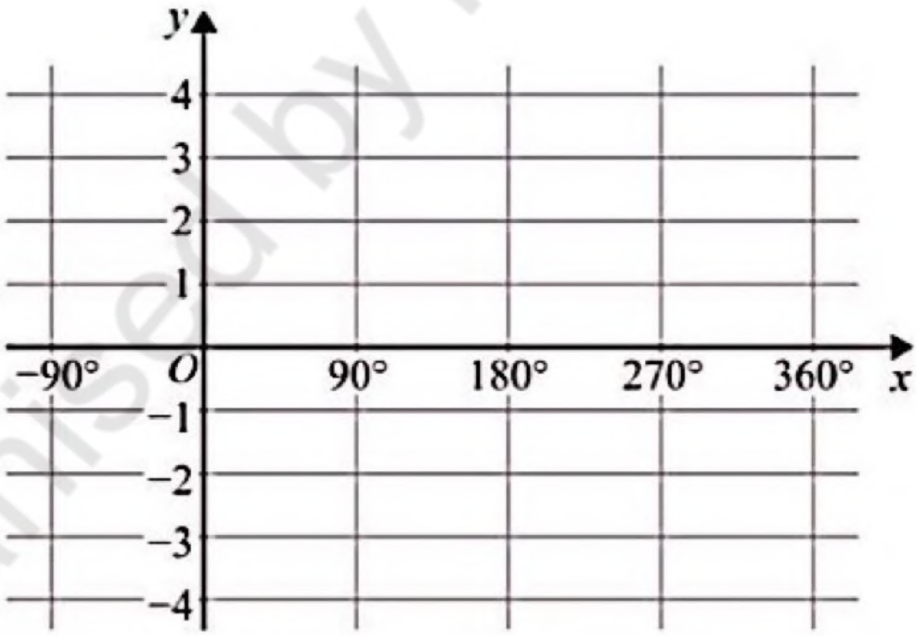


Diagram 1

(If you make an error there is a spare copy of Diagram 1 on the next page.)



Spare copy of Diagram 1

(Only use this diagram if you have made an error on Diagram 1.)

10.

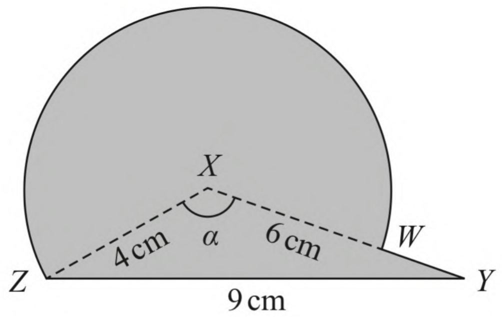


Figure 4

The triangle XYZ in Figure 4 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.

The point W lies on the line XY.

The circular arc ZW, in Figure 4, is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region, shown shaded in Figure 4, is to be used as a design for a logo.

Calculate

(c) the area of the logo

(3)

(d) the perimeter of the logo.

(4)

5.

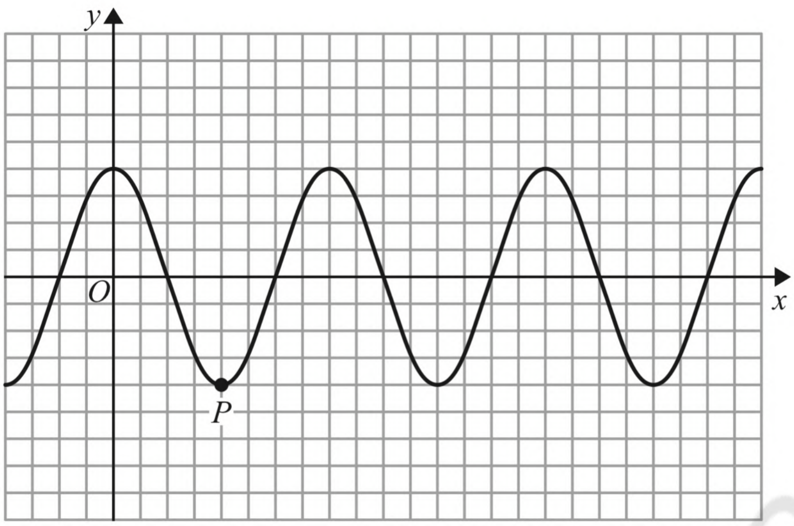


Figure 2

Figure 2 shows a plot of part of the curve with equation $y = \cos 2x$ with x being measured in radians.

The point P, shown on Figure 2, is a minimum point on the curve.

(a) State the coordinates of P.

(2)

A copy of Figure 2, called Diagram 1, is shown at the top of the next page.

(b) Sketch, on Diagram 1, the curve with equation $y = \sin x$

(2)

(c) Hence, or otherwise, deduce the number of solutions of the equation

(i) $\cos 2x = \sin x$ that lie in the region $0 \leqslant x \leqslant 20\pi$

(ii) $\cos 2x = \sin x$ that lie in the region $0 \leqslant x \leqslant 21\pi$

(2)

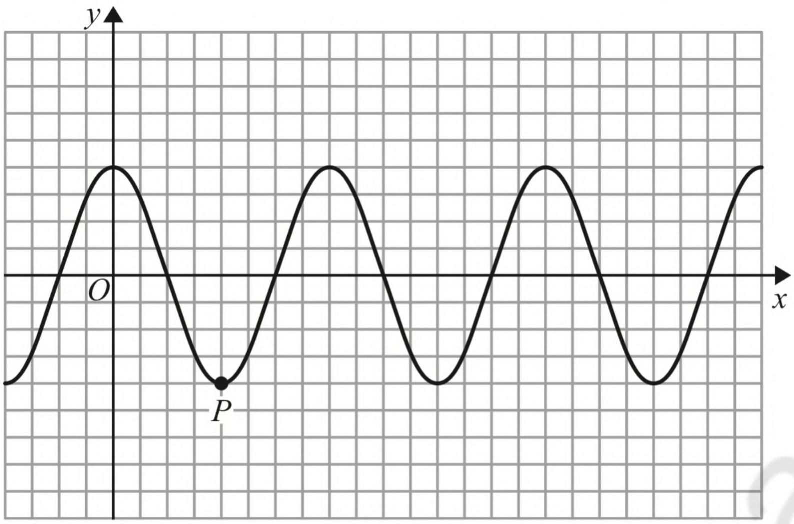
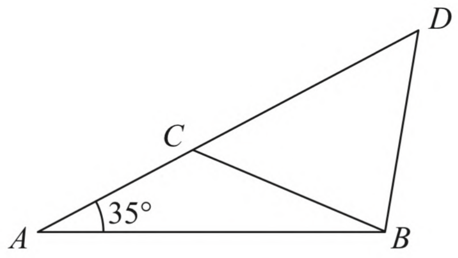


Diagram 1

7.



Not to scale

Figure 3

Figure 3 shows the design for a structure used to support a roof.

The structure consists of four wooden beams, AB, BD, BC and AD.

Given $AB = 6.5\,\text{m}$, $BC = BD = 4.7\,\text{m}$ and angle $BAC = 35°$

(a) find, to one decimal place, the size of angle ACB,

(3)

(b) find, to the nearest metre, the total length of wood required to make this structure.

(3)

7.

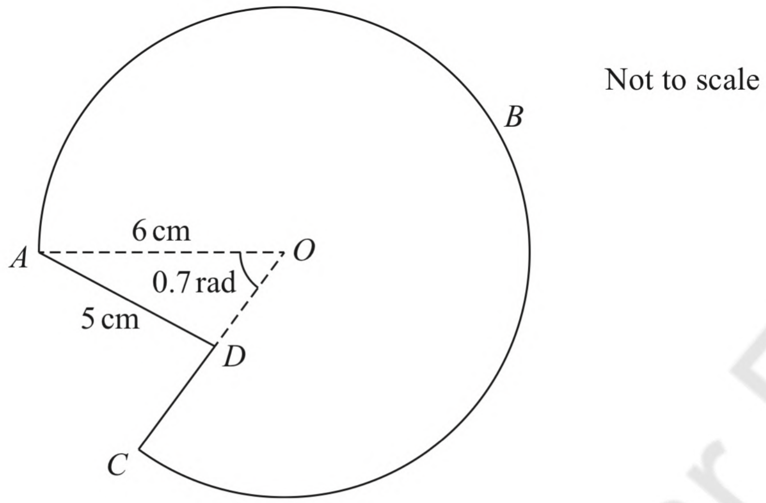


Figure 2

The shape *ABCDA* consists of a sector *ABCOA* of a circle, centre *O*, joined to a triangle *AOD*, as shown in Figure 2.

The point *D* lies on *OC*.

The radius of the circle is 6 cm, length *AD* is 5 cm and angle *AOD* is 0.7 radians.

(a) Find the area of the sector *ABCOA*, giving your answer to one decimal place.
(3)

Given angle *ADO* is obtuse,

(b) find the size of angle *ADO*, giving your answer to 3 decimal places.
(3)

(c) Hence find the perimeter of shape *ABCDA*, giving your answer to one decimal place.
(4)

9.

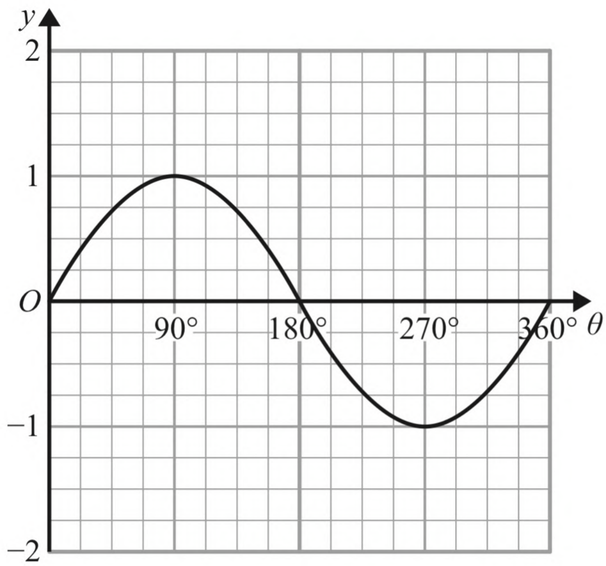


Figure 3

Figure 3 shows a plot of the curve with equation $y = \sin\theta$, $\quad 0 \leqslant \theta \leqslant 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leqslant \theta \leqslant 360°$$

(2)

A copy of Figure 3, called Diagram 1, is shown on the next page.

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta, \quad 0 \leqslant \theta \leqslant 360°$

(ii) $y = \tan\theta, \quad 0 \leqslant \theta \leqslant 360°$

(2)

(c) Hence find the number of solutions of the equation

(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 2160°$

(ii) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 1980°$

(3)

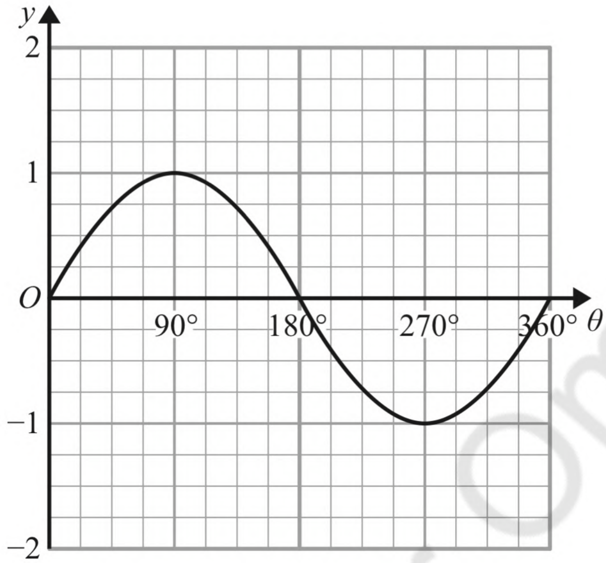


Diagram 1

1.

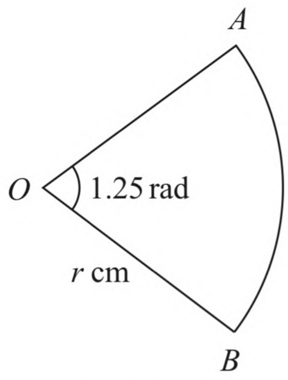


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O* and radius *r* cm.

The angle *AOB* is 1.25 radians.

Given that the area of the sector *AOB* is 15 cm^2

(a) find the exact value of *r*,

(2)

(b) find the exact length of the perimeter of the sector. Write your answer in simplest form.

(3)

4. A parallelogram $ABCD$ has area $40\,\text{cm}^2$

Given that AB has length 10 cm, BC has length 6 cm and angle DAB is obtuse, find

(a) the size of angle DAB, in degrees, to 2 decimal places,

(3)

(b) the length of diagonal BD, in cm, to one decimal place.

(2)

4.

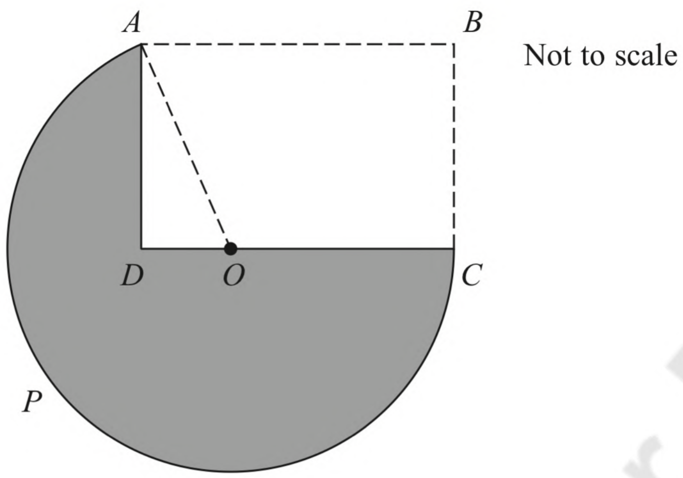


Figure 2

Figure 2 shows the plan view of a house *ABCD* and a lawn *APCDA*.

ABCD is a rectangle with $AB = 16\,\text{m}$.

APCOA is a sector of a circle centre *O* with radius $12\,\text{m}$.

The point *O* lies on the line *DC*, as shown in Figure 2.

(a) Show that the size of angle *AOD* is 1.231 radians to 3 decimal places.

(2)

The lawn *APCDA* is shown shaded in Figure 2.

(b) Find the area of the lawn, in m^2, to one decimal place.

(4)

(c) Find the perimeter of the lawn, in metres, to one decimal place.

(3)

3.

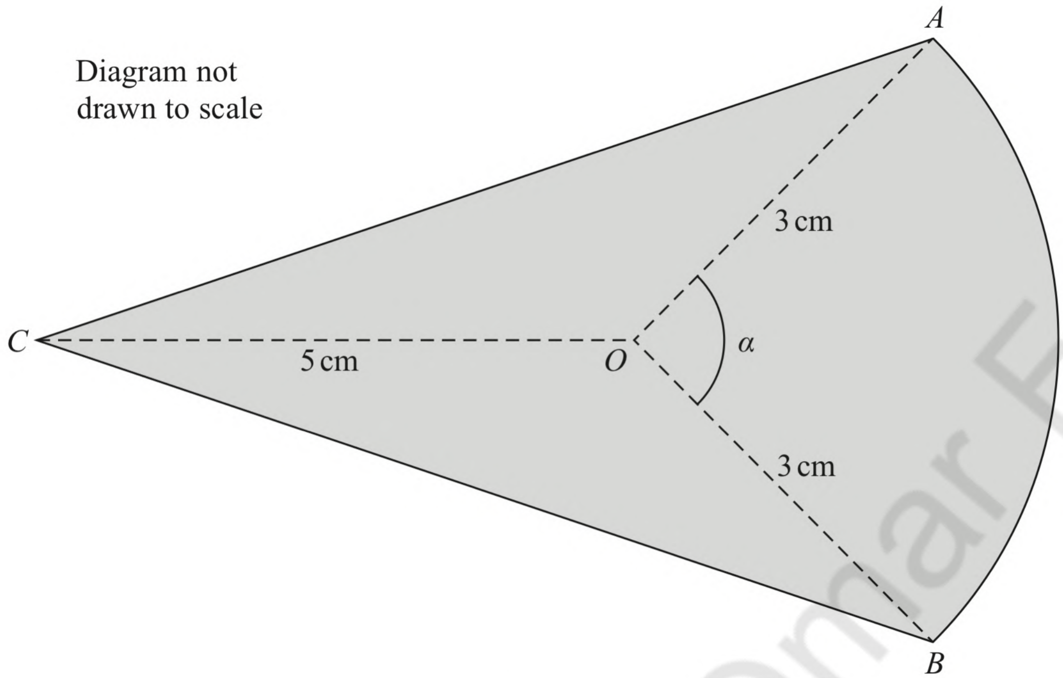


Figure 1

Figure 1 shows the design for a badge.

The design consists of two congruent triangles, *AOC* and *BOC*, joined to a sector *AOB* of a circle centre *O*.

- Angle $AOB = \alpha$
- $AO = OB = 3\,\text{cm}$
- $OC = 5\,\text{cm}$

Given that the area of sector *AOB* is $7.2\,\text{cm}^2$

(a) show that $\alpha = 1.6$ radians.

(2)

(b) Hence find

(i) the area of the badge, giving your answer in cm^2 to 2 significant figures,

(ii) the perimeter of the badge, giving your answer in cm to one decimal place.

(8)

5. (i)

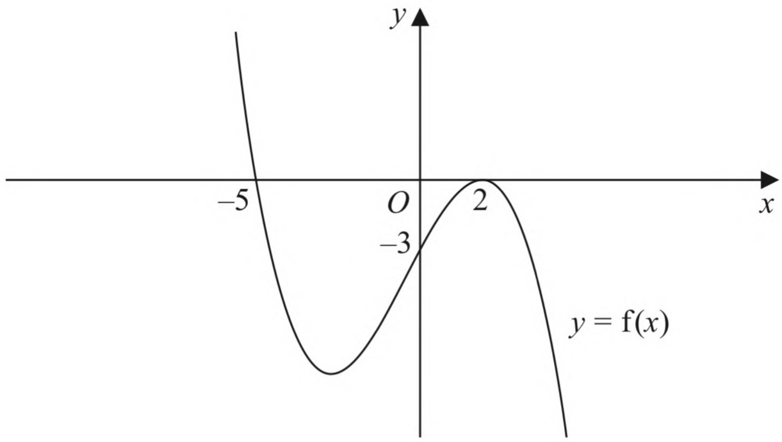


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-5, 0)$ and $(0, -3)$ and touches the x-axis at the point $(2, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 2)$

(b) $y = f(-x)$

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(6)

(ii)

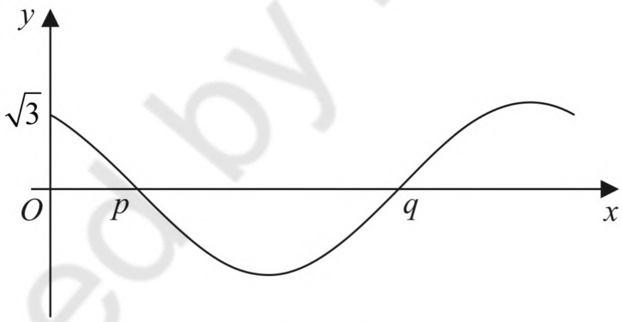


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = k\cos\left(x + \frac{\pi}{6}\right) \qquad 0 \leqslant x \leqslant 2\pi$$

where k is a constant.

The curve meets the y-axis at the point $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

Find

(a) the value of k,

(b) the exact value of p and the exact value of q.

(3)

3.

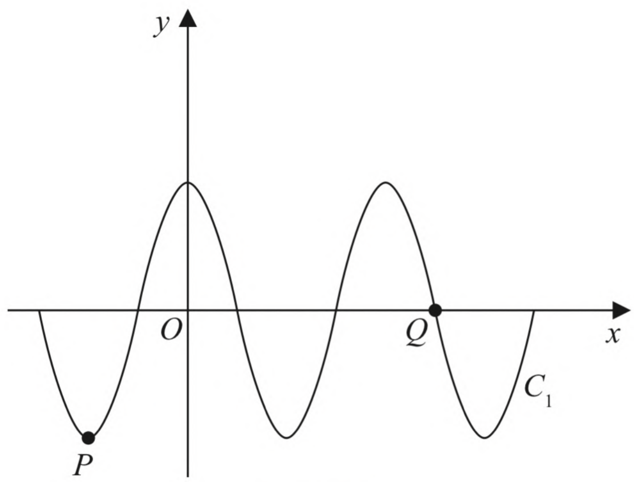


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4\cos x°$

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

The curve C_2 has equation $y = 4\cos x° + k$, where k is a constant.

Curve C_2 has a minimum y value of –1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

5.

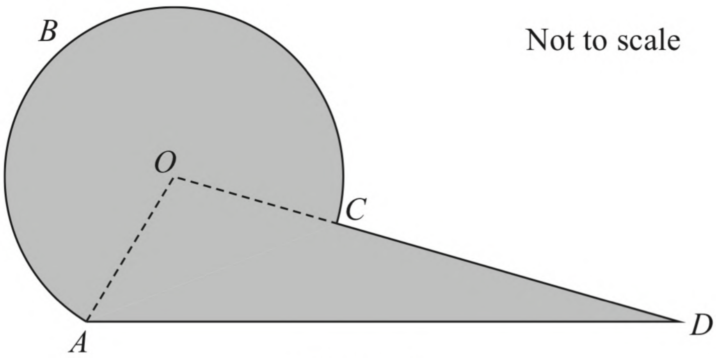


Figure 3

Figure 3 shows the plan view of a viewing platform at a tourist site.

The shape of the viewing platform consists of a sector *ABCOA* of a circle, centre *O*, joined to a triangle *AOD*.

Given that

- $OA = OC = 6\,\text{m}$
- $AD = 14\,\text{m}$
- angle $ADC = 0.43$ radians
- angle *AOD* is an obtuse angle
- *OCD* is a straight line

find

(a) the size of angle *AOD*, in radians, to 3 decimal places,

(3)

(b) the length of arc *ABC*, in metres, to one decimal place,

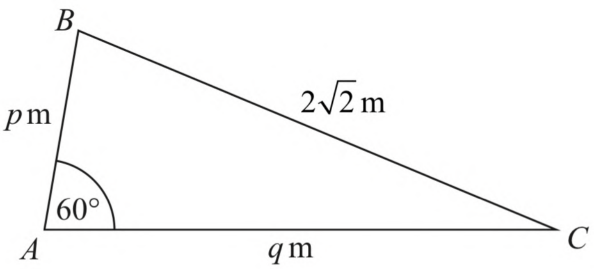
(2)

(c) the total area of the viewing platform, in m^2, to one decimal place.

(4)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.



Not drawn to scale

Figure 1

Figure 1 shows the plan view of a flower bed.
The flowerbed is in the shape of a triangle *ABC* with

- $AB = p$ metres
- $AC = q$ metres
- $BC = 2\sqrt{2}$ metres
- angle $BAC = 60°$

(a) Show that

$$p^2 + q^2 - pq = 8$$

(2)

Given that side *AC* is 2 metres longer than side *AB*, use algebra to find

(b) (i) the exact value of p,

(ii) the exact value of q.

(5)

Using the answers to part (b),

(c) calculate the exact area of the flower bed.

(2)

9.

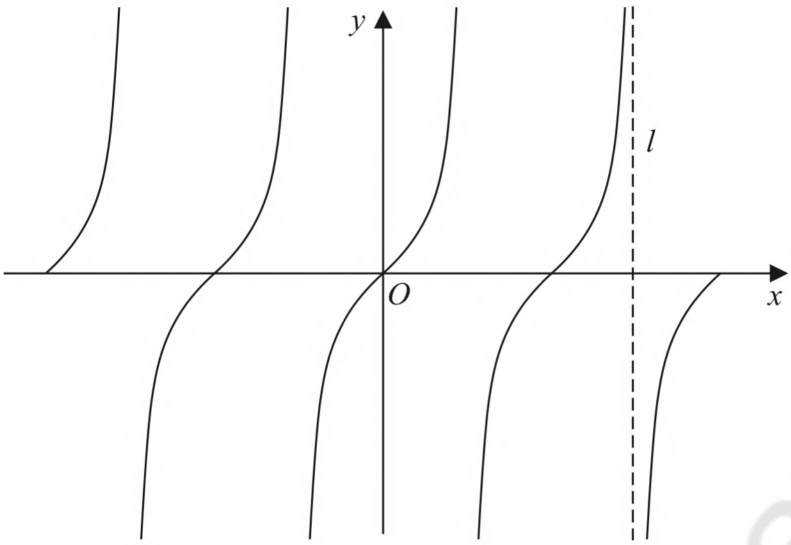


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \frac{5}{2}\pi$

(2)

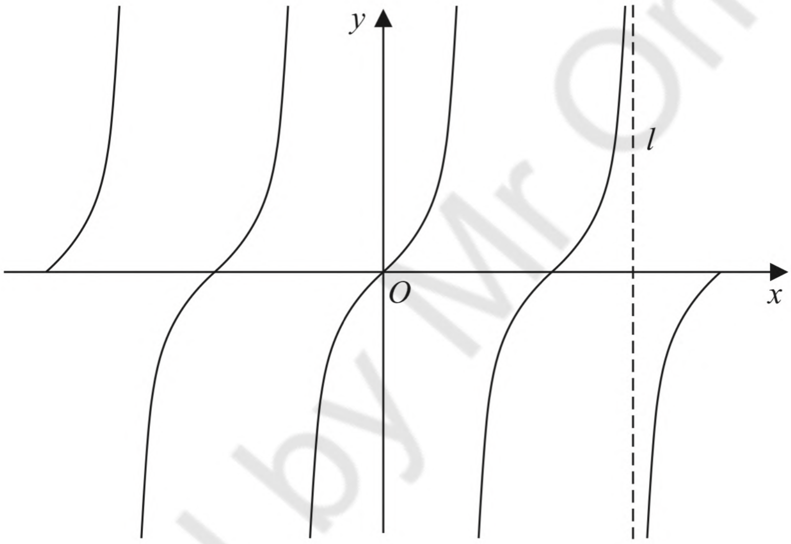


Diagram 1

4.

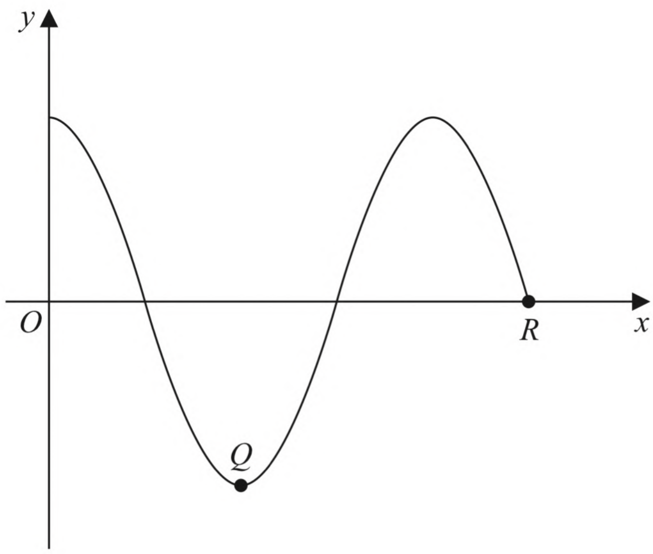


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \cos 2x^{\circ} \qquad 0 \leqslant x \leqslant k$$

The point Q and the point $R(k, 0)$ lie on the curve and are shown in Figure 2.

(a) State

(i) the coordinates of Q,

(ii) the value of k.

(3)

(b) Given that there are exactly two solutions to the equation

$$\cos 2x^{\circ} = p \qquad \text{in the region } 0 \leqslant x \leqslant k$$

find the range of possible values for p.

(2)

7.

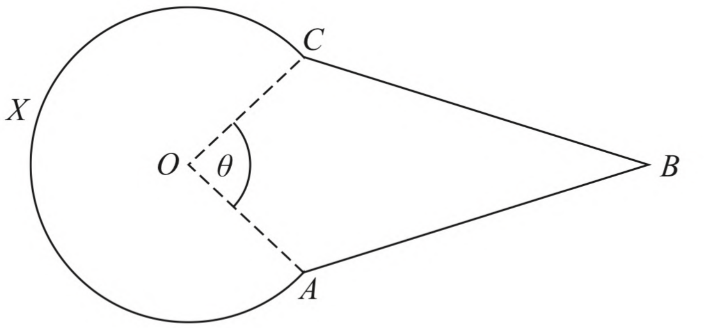


Figure 3

Figure 3 shows the design for a sign at a bird sanctuary.

The design consists of a kite $OABC$ joined to a sector $OCXA$ of a circle centre O.

In the design

- $OA = OC = 0.6\,\text{m}$
- $AB = CB = 1.4\,\text{m}$
- Angle OAB = Angle OCB = 2 radians
- Angle $AOC = \theta$ radians, as shown in Figure 3

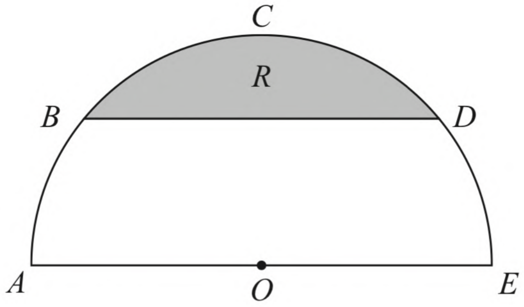
Making your method clear,

(a) show that $\theta = 1.64$ radians to 3 significant figures,
(4)

(b) find the perimeter of the sign, in metres to 2 significant figures,
(2)

(c) find the area of the sign, in m^2 to 2 significant figures.
(4)

5.



Not to scale

Figure 2

Figure 2 shows a plan view of a semicircular garden *ABCDEOA*

The semicircle has

- centre *O*
- diameter *AOE*
- radius 3 m

The straight line *BD* is parallel to *AE* and angle *BOA* is 0.7 radians.

(a) Show that, to 4 significant figures, angle *BOD* is 1.742 radians.

(1)

The flowerbed *R*, shown shaded in Figure 2, is bounded by *BD* and the arc *BCD*.

(b) Find the area of the flowerbed, giving your answer in square metres to one decimal place.

(3)

(c) Find the perimeter of the flowerbed, giving your answer in metres to one decimal place.

(3)

9.

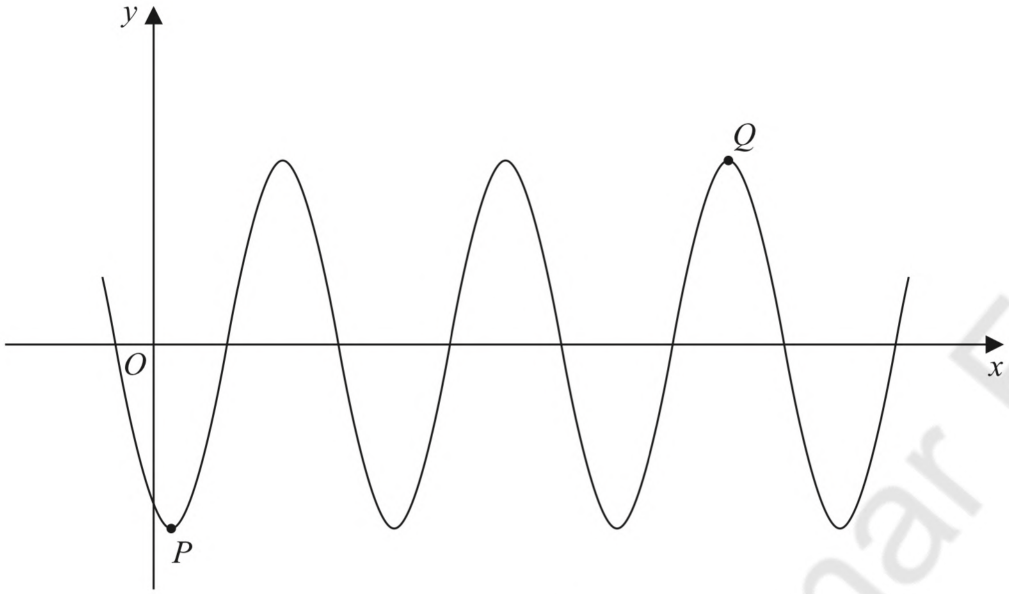


Figure 4

Figure 4 shows part of the curve with equation

$$y = A\cos(x - 30)°$$

where A is a constant.

The point P is a minimum point on the curve and has coordinates $(30, -3)$ as shown in Figure 4.

(a) Write down the value of A.

(1)

The point Q is shown in Figure 4 and is a maximum point.

(b) Find the coordinates of Q.

(3)

2. In the triangle ABC,

- $AB = 21\text{ cm}$
- $BC = 13\text{ cm}$
- angle $BAC = 25°$
- angle $ACB = x°$

(a) Use the sine rule to find the value of $\sin x°$, giving your answer to 4 decimal places.
(2)

Given also that AB is the longest side of the triangle,

(b) find the value of x, giving your answer to 2 decimal places.
(3)

8.

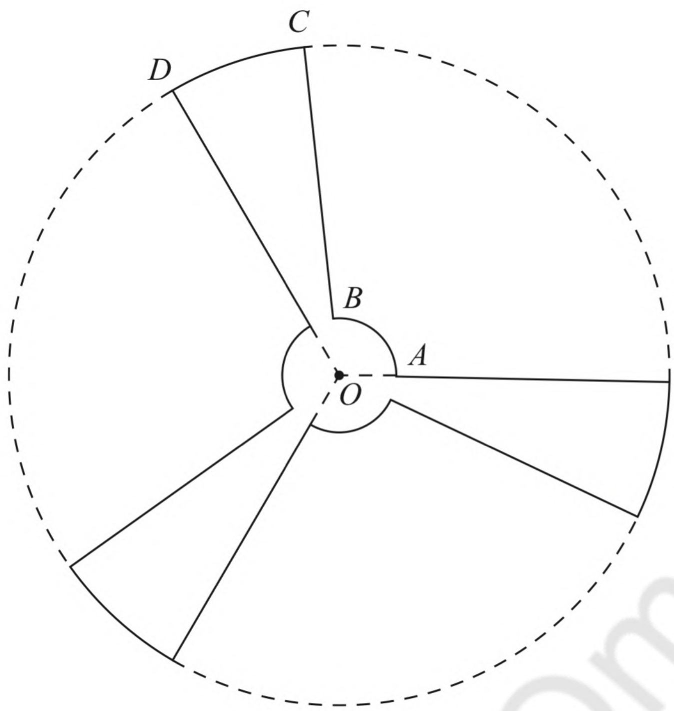


Figure 3

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to *OABCDO*, shown in Figure 3, where

- *OABO* is a sector of a circle with centre *O* and radius 9 cm
- *OBCDO* is a sector of a circle with centre *O* and radius 84 cm
- angle $AOD = \frac{2\pi}{3}$ radians

Given that the length of the arc *AB* is 15 cm,

(a) show that the length of the arc *CD* is 35.9 cm to one decimal place.
(3)

The face of the fan is modelled to be a flat surface.

Find, according to the model,

(b) the perimeter of the face of the fan, giving your answer to the nearest cm,
(2)

(c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear.
(5)

9.

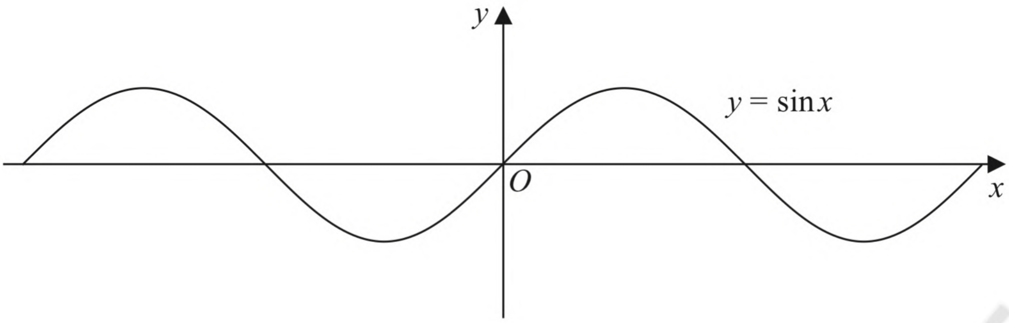


Figure 4

Figure 4 shows part of the graph of the curve with equation $y = \sin x$

Given that $\sin\alpha = p$, where $0 < \alpha < 90°$

(a) state, in terms of p, the value of

(i) $2\sin(180° - \alpha)$

(ii) $\sin(\alpha - 180°)$

(iii) $3 + \sin(180° + \alpha)$

(3)

A copy of Figure 4, labelled Diagram 1, is shown on page 27.

On Diagram 1,

(b) sketch the graph of $y = \sin 2x$

(2)

(c) Hence find, in terms of α, the x coordinates of any points in the interval $0 < x < 180°$ where

$$\sin 2x = p$$

(3)

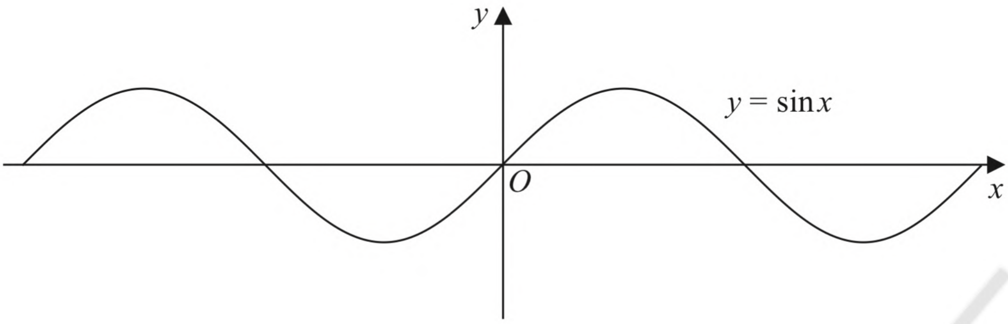


Diagram 1

8.

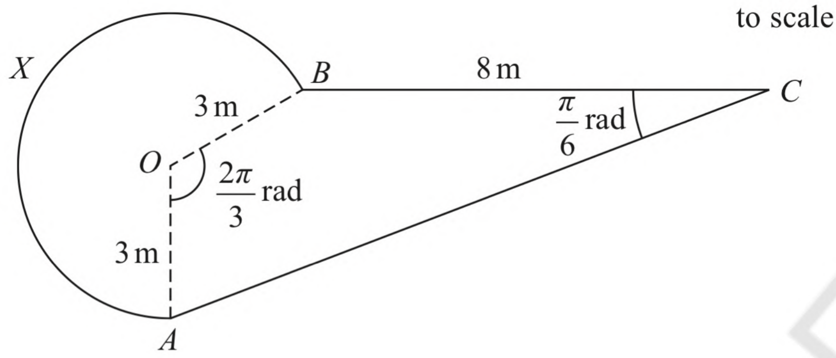


Figure 2

Figure 2 shows the plan view of a design for a pond.

The design consists of a sector *AOBX* of a circle centre *O* joined to a quadrilateral *AOBC*.

- $BC = 8\text{ m}$
- $OA = OB = 3\text{ m}$
- angle *AOB* is $\frac{2\pi}{3}$ radians
- angle *BCA* is $\frac{\pi}{6}$ radians

(a) Calculate (i) the exact area of the sector *AOBX*,

(ii) the exact perimeter of the sector *AOBX*.

(5)

(b) Calculate the exact area of the triangle *AOB*.

(2)

(c) Show that the length *AB* is $3\sqrt{3}$ m.

(2)

(d) Find the total surface area of the pond. Give your answer in m^2 correct to 2 significant figures.

(5)

6.

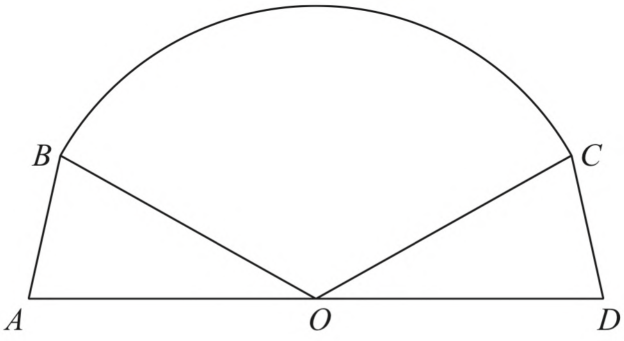


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the plan view for the design of a stage.

The design consists of a sector *OBC* of a circle, with centre *O*, joined to two congruent triangles *OAB* and *ODC*.

Given that

- angle $BOC = 2.4$ radians
- area of sector $BOC = 40\,\text{m}^2$
- *AOD* is a straight line of length 12.5 m

(a) find the radius of the sector, giving your answer, in m, to 2 decimal places,
(2)

(b) find the size of angle *AOB*, in radians, to 2 decimal places.
(1)

Hence find

(c) the total area of the stage, giving your answer, in m^2, to one decimal place,
(3)

(d) the total perimeter of the stage, giving your answer, in m, to one decimal place.
(4)

9.

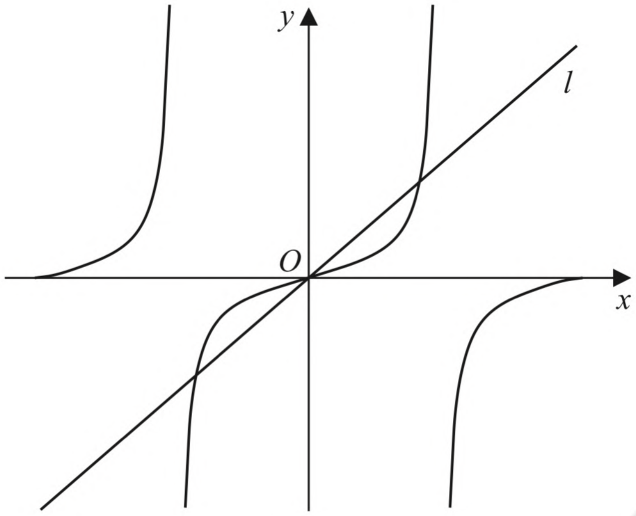


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan x$
- the straight line l with equation $y = \pi x$

in the interval $-\pi < x < \pi$

(a) State the period of $\tan x$

(1)

(b) Write down the number of roots of the equation

(i) $\tan x = (\pi + 2)x$ in the interval $-\pi < x < \pi$

(1)

(ii) $\tan x = \pi x$ in the interval $-2\pi < x < 2\pi$

(1)

(iii) $\tan x = \pi x$ in the interval $-100\pi < x < 100\pi$

(1)

5.

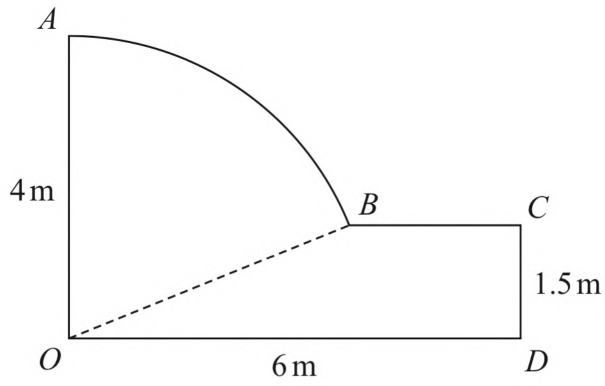


Figure 1

Figure 1 shows the plan for a garden.

In the plan

- OA and CD are perpendicular to OD
- AB is an arc of the circle with centre O and radius 4 metres
- BC is parallel to OD
- OD is 6 metres, OA is 4 metres and CD is 1.5 metres

(a) Show that angle AOB is 1.186 radians to 4 significant figures.

(2)

(b) Find the perimeter of the garden, giving your answer in metres to 3 significant figures.

(4)

(c) Find the area of the garden, giving your answer in square metres to 3 significant figures.

(4)

9. (i)

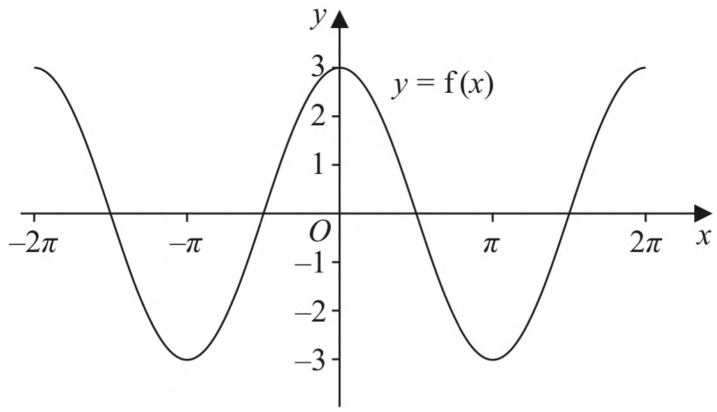


Figure 3

Figure 3 shows part of the graph of the trigonometric function with equation $y = \mathrm{f}(x)$

(a) Write down an expression for $\mathrm{f}(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = \mathrm{f}\left(x + \frac{\pi}{4}\right)$

Show clearly the coordinates of all the points where the curve intersects the coordinate axes.

(3)

(ii)

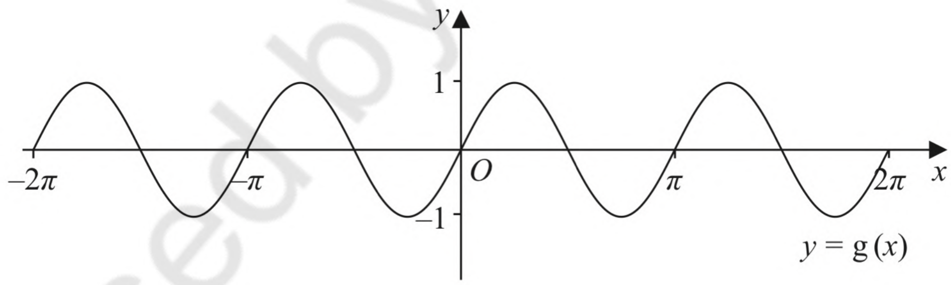


Figure 4

Figure 4 shows part of the graph of the trigonometric function with equation $y = \mathrm{g}(x)$

(a) Write down an expression for $\mathrm{g}(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = \mathrm{g}(x) - 2$

Show clearly the coordinates of the y intercept.

(2)

9.

Diagram **NOT** accurately drawn

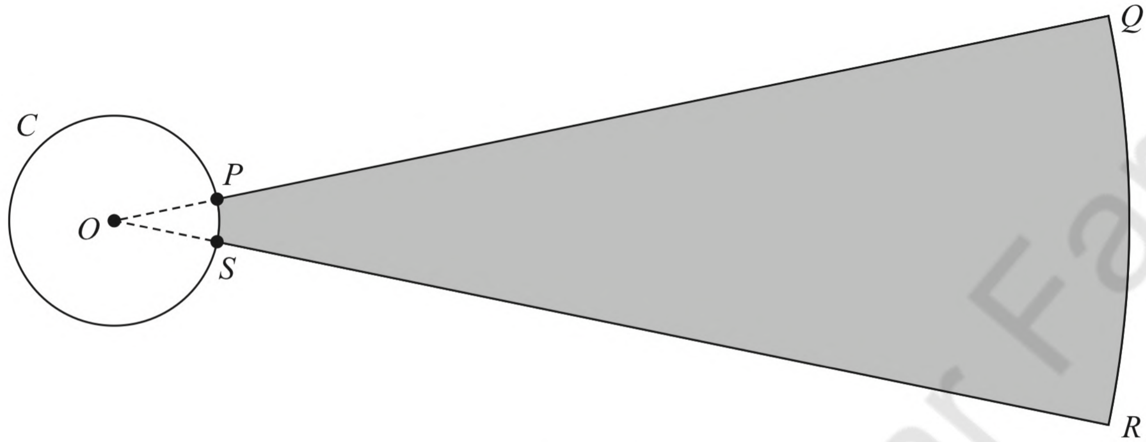


Figure 3

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle C and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle C has centre O
- P and S are points on C
- $OPQRSO$ is a sector of a circle with centre O
- the length of arc PS is 0.72 m
- the size of angle POS is 0.6 radians

(a) show that $OP = 1.2\text{ m}$

(1)

Given also that

- the target area, $PQRS$, is 90 m^2
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

(3)

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

(3)

10.

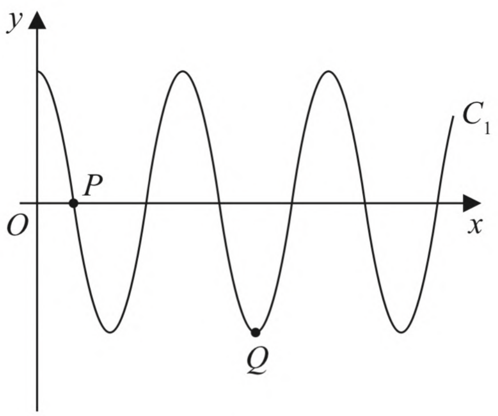


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3\cos\left(\frac{x}{n}\right)^{\circ} \qquad x \geqslant 0$$

where n is a constant.

The curve C_1 cuts the positive x-axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of n

(ii) State the period of C_1

(2)

The point Q, shown in Figure 4, is a minimum point of C_1

(b) State the coordinates of Q.

(2)

The curve C_2 has equation $y = 2\sin x^{\circ} + k$, where k is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on C_2

Given that a is a constant less than 90

(c) find the value of k.

(2)

6.

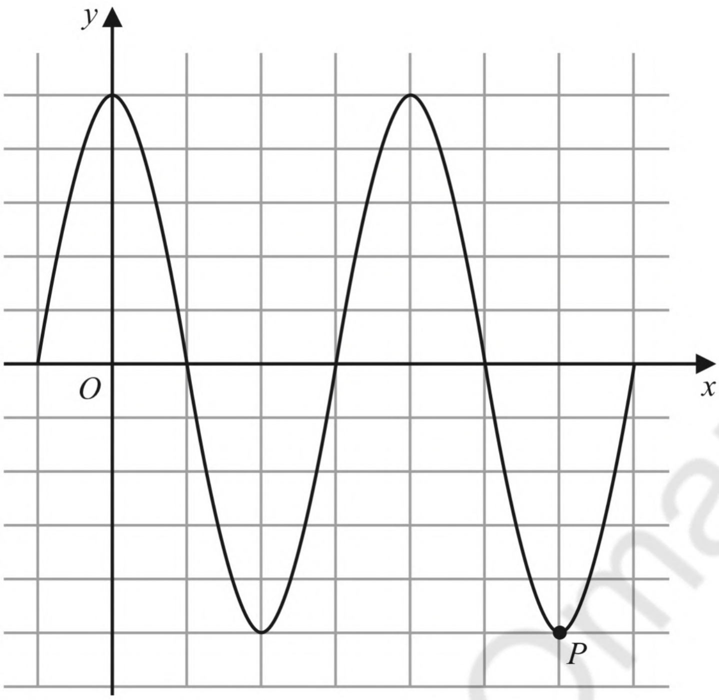


Figure 2

Figure 2 shows a plot of part of the curve C_1 with equation

$$y = 5\cos x$$

with x being measured in degrees.

The point P, shown in Figure 2, is a minimum point on C_1

(a) State the coordinates of P

(2)

The point Q lies on a different curve C_2

Given that point Q

- is a maximum point on the curve
- is the maximum point with the **smallest** x coordinate, $x > 0$

(b) find the coordinates of Q when

(i) C_2 has equation $y = 5\cos x - 2$

(ii) C_2 has equation $y = -5\cos x$

(4)

8.

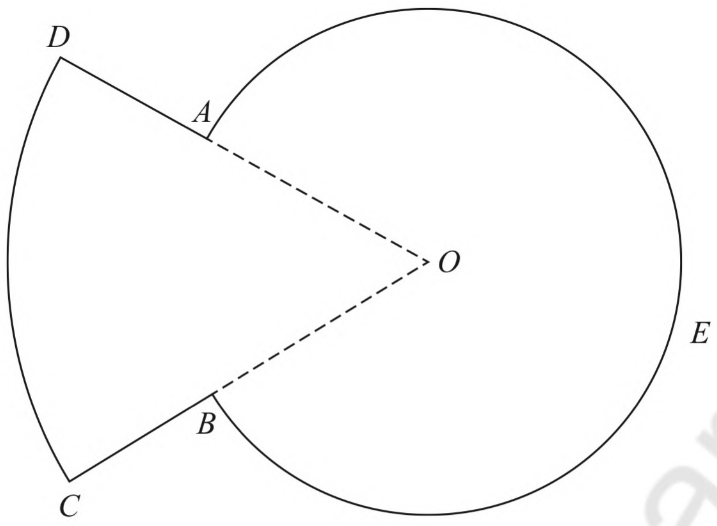


Figure 3

Figure 3 shows a sketch of the plan view of a platform.

The plan view of the platform consists of a sector DOC of a circle centre O joined to a sector $AOBEA$ of a different circle, also with centre O.

Given that

- angle $AOB = 0.8$ radians
- arc length $CD = 9\,\text{m}$
- $DA : AO = 3 : 5$

(a) show that $AO = 7.03\,\text{m}$ to 3 significant figures.

(3)

(b) Find the perimeter of the platform, in m, to 3 significant figures.

(3)

(c) Find the total area of the platform, giving your answer in m^2 to the nearest whole number.

(3)

5.

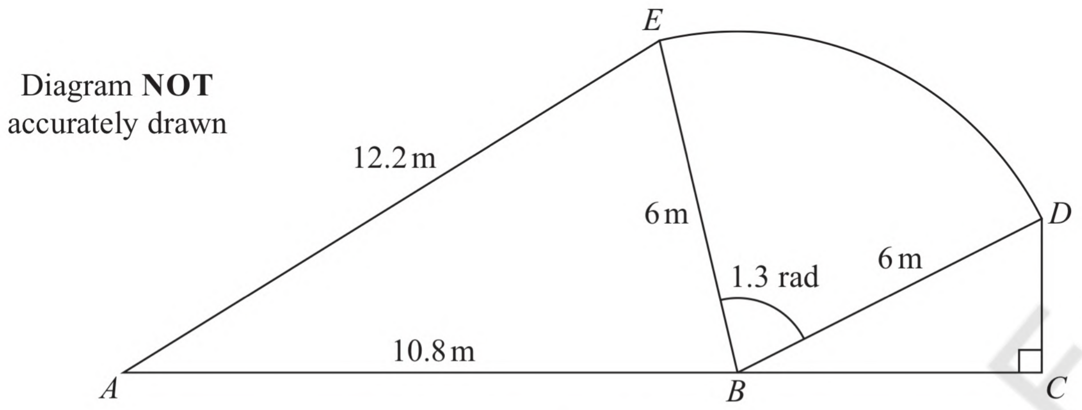


Figure 2

Figure 2 shows the plan view of a garden.

The shape of the garden *ABCDEA* consists of a triangle *ABE* and a right-angled triangle *BCD* joined to a sector *BDE* of a circle with radius 6 m and centre *B*.

The points *A*, *B* and *C* lie on a straight line with $AB = 10.8$ m

Angle $BCD = \frac{\pi}{2}$ radians, angle $EBD = 1.3$ radians and $AE = 12.2$ m

(a) Find the area of the sector *BDE*, giving your answer in m^2

(2)

(b) Find the size of angle *ABE*, giving your answer in radians to 2 decimal places.

(2)

(c) Find the area of the garden, giving your answer in m^2 to 3 significant figures.

(3)

11.

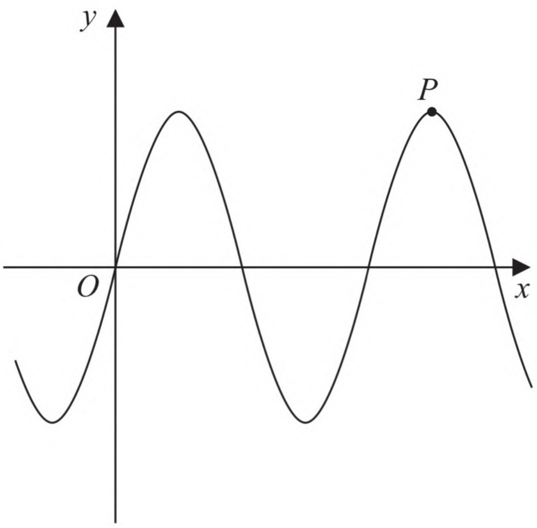


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P.

(2)

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

6.

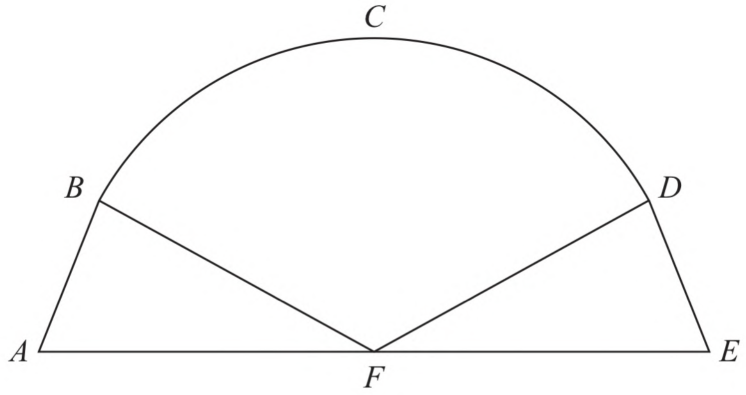


Diagram not drawn to scale

Figure 1

Figure 1 shows a sketch of the entrance to a tunnel.

The shape of the entrance consists of a sector *BCDF*, of a circle centre *F*, joined to two congruent (identical) triangles *ABF* and *EDF*.

Given that

AFE is a straight line

$AF = FE = 6.4\text{ m}$

$FB = FD = 6.2\text{ m}$

angle $BFD = 2.275$ radians

(a) Show that angle $AFB = 0.433$ radians to 3 decimal places.

(1)

(b) Find the perimeter of the entrance to the tunnel, *ABCDEFA*, in metres, to one decimal place.

(4)

(c) Find the cross-sectional area of the entrance to the tunnel, *ABCDEFA*, in m^2, to one decimal place.

(4)

9.

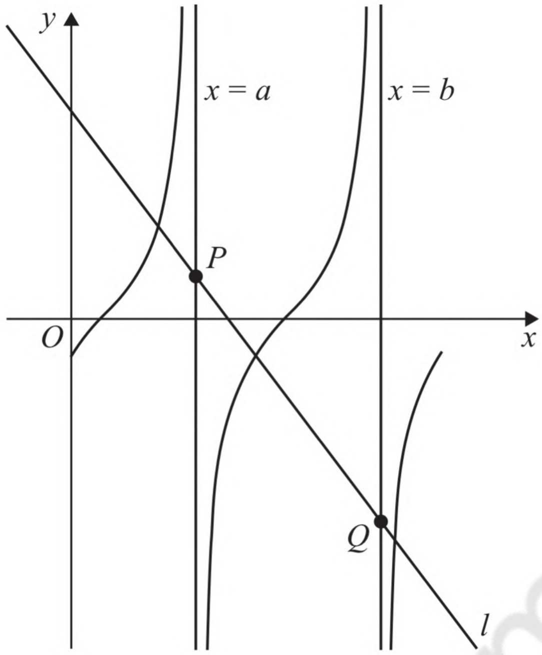


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan\left(x - \frac{\pi}{6}\right)$ for $0 \leqslant x \leqslant 2\pi$
- part of the straight line l with equation $y = \pi - x$

(a) State the number of solutions of the equation

(i) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 2\pi$

(ii) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 100\pi$

(iii) $\tan\left(x - \frac{\pi}{6}\right) = \pi + x$ in the interval $0 \leqslant x \leqslant 2\pi$

(3)

The line with equation $x = a$, shown in Figure 3, is the asymptote to the curve with the smallest positive x coordinate.

(b) State the value of a

(1)

The line with equation $x = b$, also shown in Figure 3, is the asymptote to the curve with the second smallest positive x coordinate.

The line l meets $x = a$ at point P and meets $x = b$ at point Q as shown in Figure 3.

(c) Find the midpoint of the line segment PQ.

(4)

5. A plot of land OAB is in the shape of a sector of a circle with centre O.

Given

- $OA = OB = 5\,\text{km}$
- angle $AOB = 1.2$ radians

(a) find the perimeter of the plot of land.

(2)

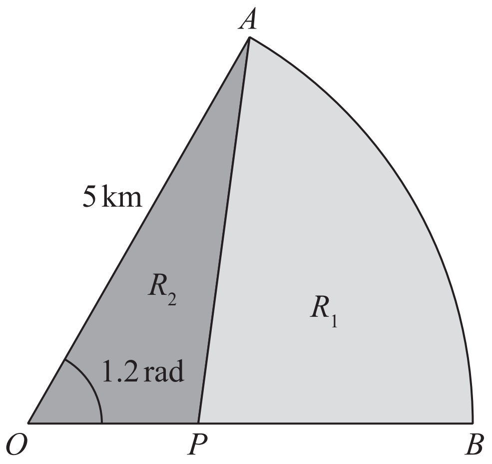


Diagram **NOT** accurately drawn

Figure 2

A point P lies on OB such that the line AP divides the plot of land into two regions R_1 and R_2 as shown in Figure 2.

Given that

$$\text{area of } R_1 = 3 \times \text{area of } R_2$$

(b) show that the area of $R_2 = 3.75\,\text{km}^2$

(3)

(c) Find the length of AP, giving your answer to the nearest 100 m.

(4)

7.

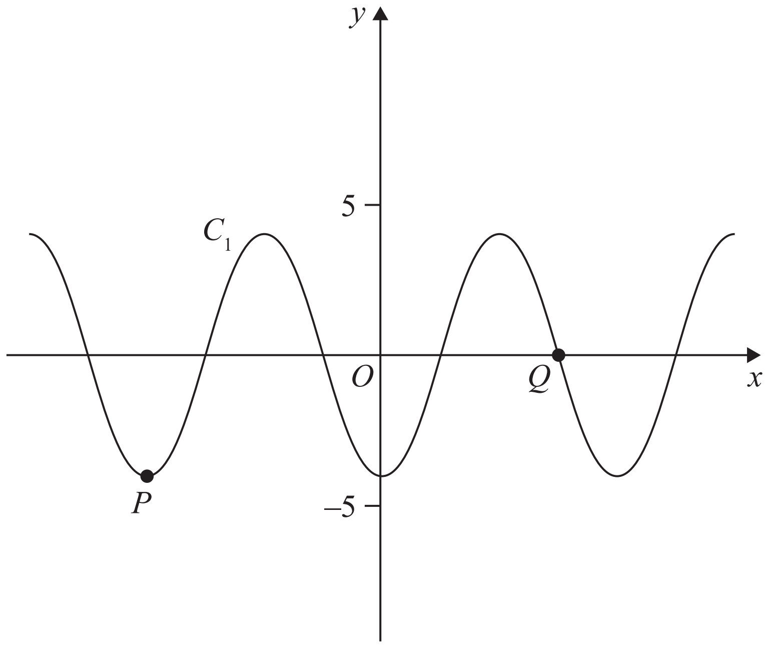


Figure 3

Figure 3 shows a plot of part of the curve C_1 with equation

$$y = -4\cos x$$

where x is measured in radians.

Points P and Q lie on the curve and are shown in Figure 3.

(a) State

(i) the coordinates of P

(ii) the coordinates of Q

(3)

The curve C_2 has equation $y = -4\cos x + k$ where x is measured in radians and k is a constant.

Given that C_2 has a maximum y value of 11

(b) (i) state the value of k

(ii) state the coordinates of the minimum point on C_2 with the smallest positive x coordinate.

(3)

On the opposite page there is a copy of Figure 3 labelled Diagram 1.

(c) Using Diagram 1, state the number of solutions of the equation

$$-4\cos x = 5 - \frac{10}{\pi}x$$

giving a reason for your answer.

(2)

Question 7 continued

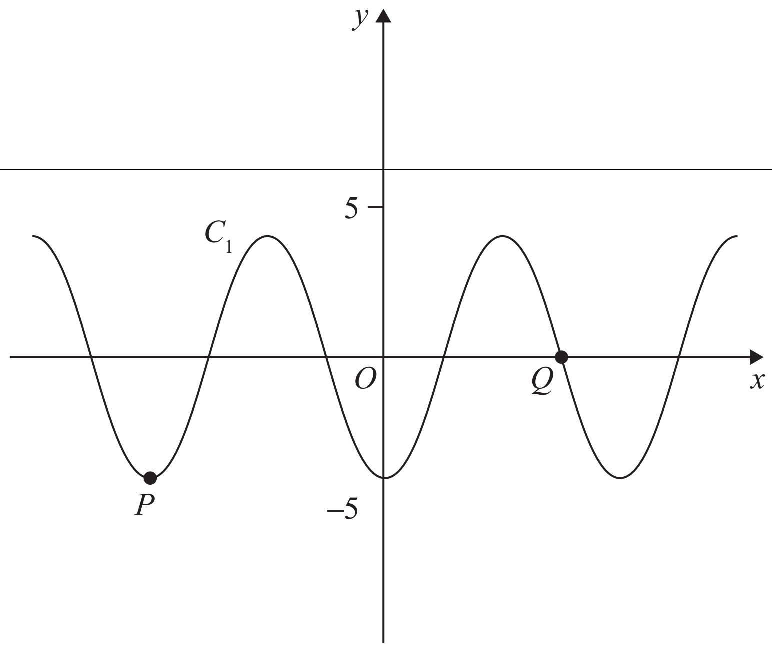


Diagram 1

(Total for Question 7 is 8 marks)

8.

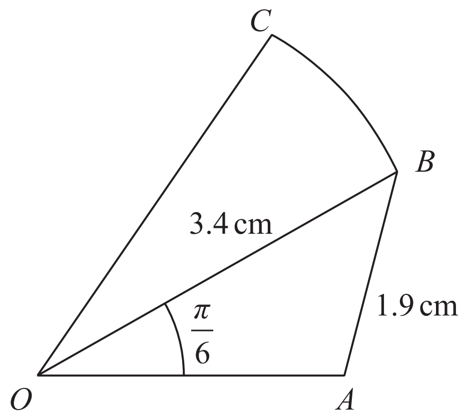


Figure 1

Figure 1 shows a sketch of a design for a badge.
The design consists of a triangle OAB joined to a sector OBC of a circle with centre O
In the design

- $OB = 3.4\,\text{cm}$
- $AB = 1.9\,\text{cm}$
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

(a) find the size of angle OAB, giving your answer in radians to 4 significant figures,
(3)

(b) find the area of triangle OAB, in cm^2, giving your answer to 3 significant figures.
(2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is $3:2$

(c) show that angle BOC is 0.462 radians to 3 significant figures.
(3)

(d) Hence find the perimeter of the badge, in cm, to the nearest integer.
(5)

9.

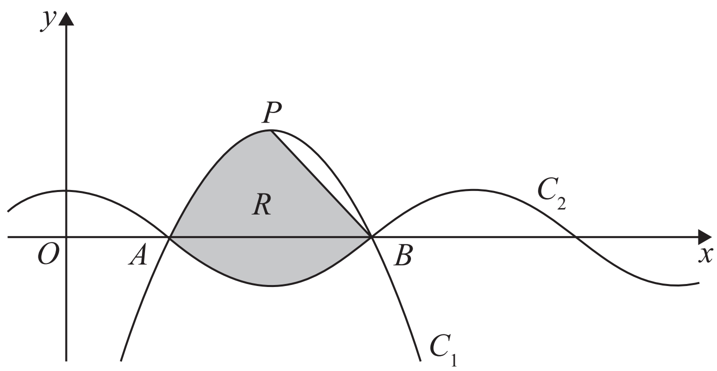


Figure 2

(a) Express $6x - \frac{27}{4} - x^2$ in the form $a + b(x + c)^2$ where a, b and c are constants to be found.

(3)

Figure 2 shows part of a sketch of curve C_1 with equation

$$y = 6x - \frac{27}{4} - x^2$$

Given that the point P is the maximum point on C_1

(b) state the coordinates of P

(2)

Figure 2 also shows part of a sketch of curve C_2 with equation

$$y = \cos(kx)$$

where k is a constant and x is measured in radians.

Given that C_1 and C_2 intersect on the x-axis at point A and at point B, as shown in Figure 2,

(c) (i) state the x coordinate of B

(ii) state the value of k

(iii) state the period of C_2

(3)

The line segment L joins P and B.
The region R, shown shaded in Figure 2, is bounded by L, C_1 and C_2

(d) Use inequalities to define R.

(5)