Chapter 5: Straight Line Graphs

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

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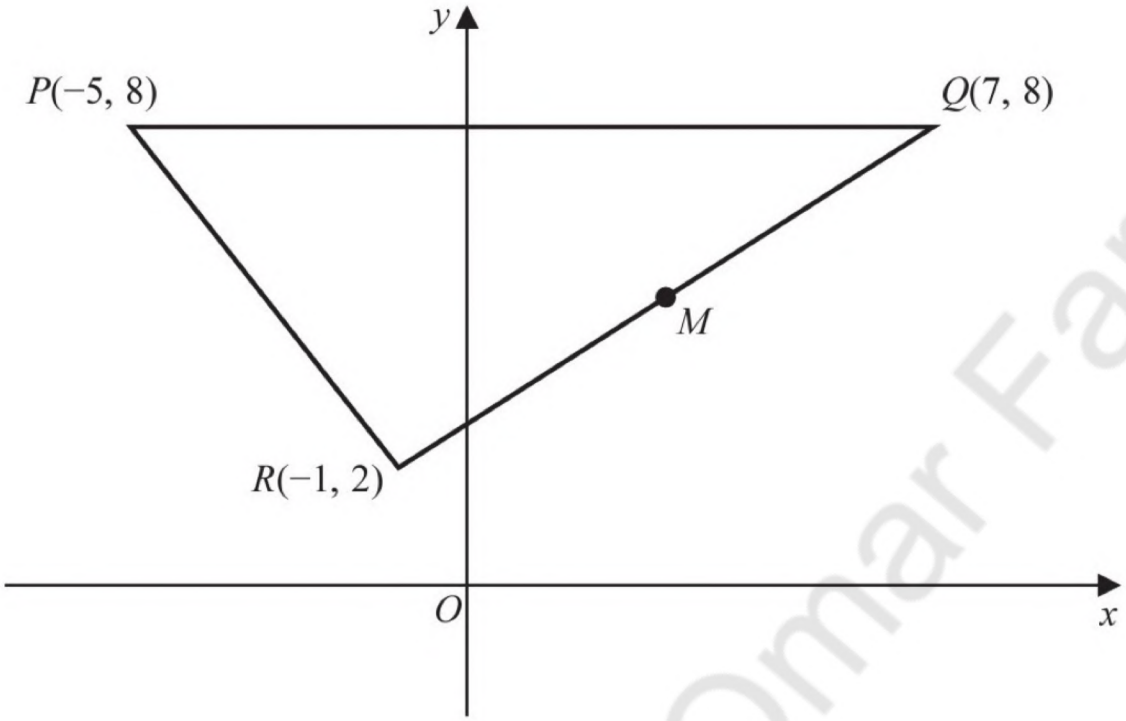

The points $P(-5, 8)$, $Q(7, 8)$ and $R(-1, 2)$ form the vertices of a triangle PQR, as shown in Figure 3. The point M is the midpoint of QR.

The line l passes through M and is parallel to PR.

(a) Find an equation for l, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

The line l cuts PQ at the point N.

(b) Find

(i) the coordinates of N,

(ii) the area of triangle MNQ.

(3)

8.

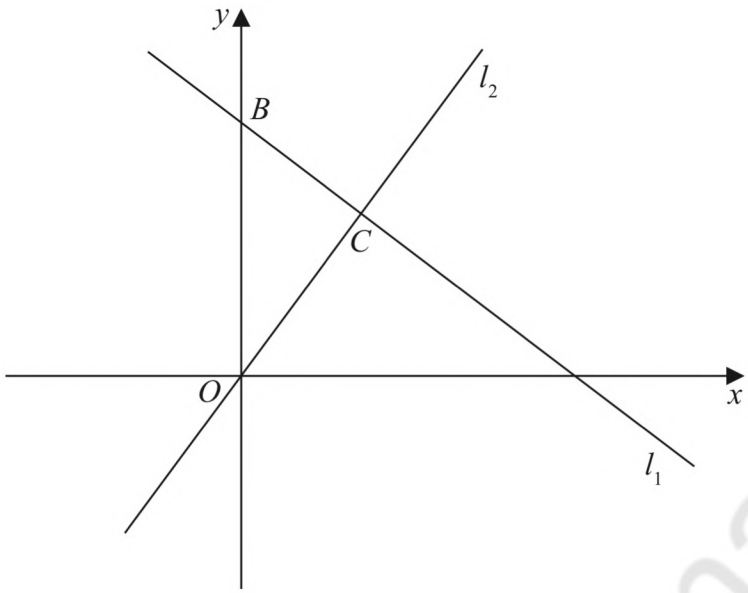

Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

(4)

The line l_2 intersects the line l_1 at the point C. Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC. Give your answer in the form $\frac{a}{b}$, where a and b are integers to be found.

(6)

3. The line l_1 has equation $3x + 5y - 7 = 0$

(a) Find the gradient of l_1

(2)

The line l_2 is perpendicular to l_1 and passes through the point $(6, -2)$.

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

6. The line l_1 has equation $3x - 4y + 20 = 0$

The line l_2 cuts the x-axis at $R(8,0)$ and is parallel to l_1

(a) Find the equation of l_2, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

The line l_1 cuts the x-axis at P and the y-axis at Q.

Given that $PQRS$ is a parallelogram, find

(b) the area of $PQRS$,

(3)

(c) the coordinates of S.

(2)

6. The point A has coordinates $(-4, 11)$ and the point B has coordinates $(8, 2)$.

(a) Find the gradient of the line AB, giving your answer as a fully simplified fraction.

(2)

The point M is the midpoint of AB. The line l passes through M and is perpendicular to AB.

(b) Find an equation for l, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(4)

The point C lies on l such that the area of triangle ABC is 37.5 square units.

(c) Find the two possible pairs of coordinates of point C.

(5)

4.

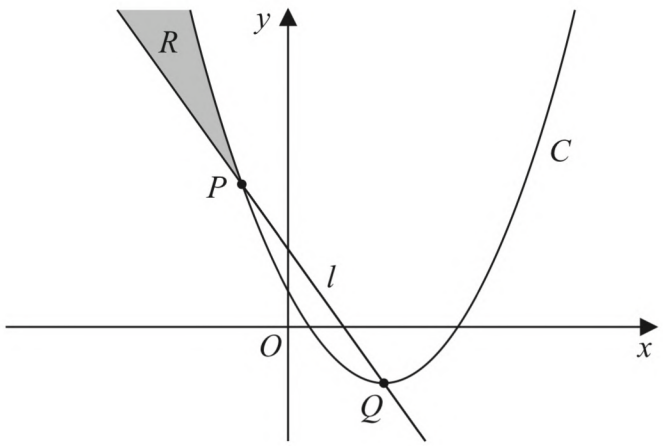

Figure 2

The points P and Q, as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q.

(a) Find an equation for l, writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(3)

7.

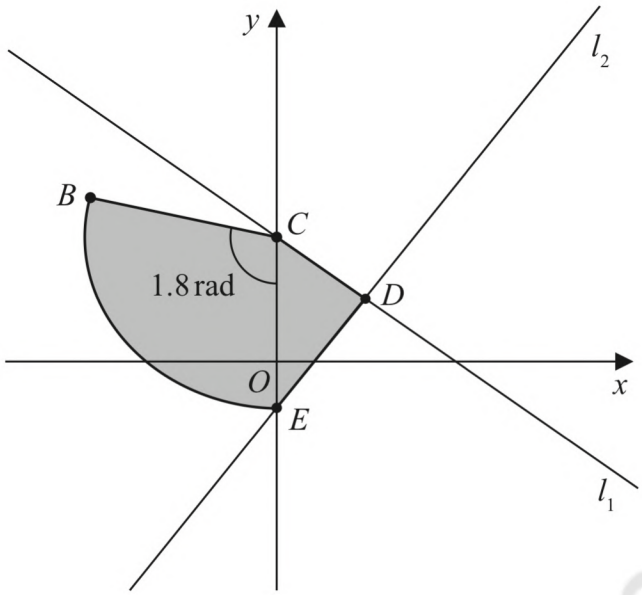

Figure 3

The line l_1 has equation $4y + 3x = 48$

The line l_1 cuts the y-axis at the point C, as shown in Figure 3.

(a) State the y coordinate of C.

(1)

The point $D(8, 6)$ lies on l_1

The line l_2 passes through D and is perpendicular to l_1

The line l_2 cuts the y-axis at the point E as shown in Figure 3.

(b) Show that the y coordinate of E is $-\frac{14}{3}$

(3)

5. The line l_1 has equation $3y - 2x = 30$

The line l_2 passes through the point $A(24, 0)$ and is perpendicular to l_1

Lines l_1 and l_2 meet at the point P

(a) Find, using algebra and showing your working, the coordinates of P.

(5)

Given that l_1 meets the x-axis at the point B,

(b) find the area of triangle BPA.

(3)

8. The line l_1 has equation

$$2x - 5y + 7 = 0$$

(a) Find the gradient of l_1

(1)

Given that

- the point A has coordinates $(6, -2)$
- the line l_2 passes through A and is perpendicular to l_1

(b) find the equation of l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The lines l_1 and l_2 intersect at the point M.

(c) Using algebra and showing all your working, find the coordinates of M.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

Given that the diagonals of a square $ABCD$ meet at M,

(d) find the coordinates of the point C.

(2)

2. The points P, Q and R have coordinates $(-3, 7)$, $(9, 11)$ and $(12, 2)$ respectively.

(a) Prove that angle $PQR = 90°$

(3)

Given that the point S is such that $PQRS$ forms a rectangle,

(b) find the coordinates of S.

(2)

10.

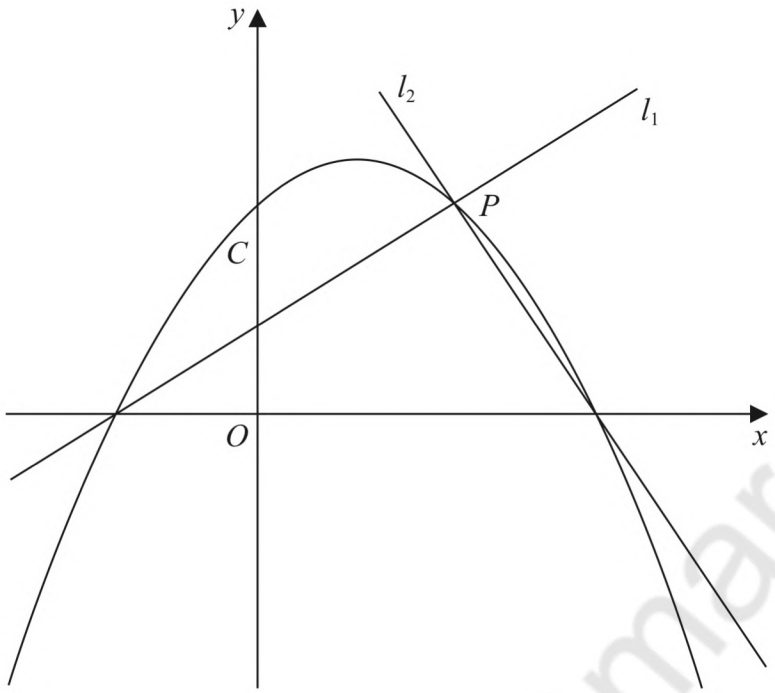

Figure 5

Figure 5 shows a sketch of the quadratic curve C with equation

$$y = -\frac{1}{4}(x+2)(x-b) \qquad \text{where } b \text{ is a positive constant}$$

The line l_1 also shown in Figure 5,

- has gradient $\frac{1}{2}$
- intersects C on the negative x-axis and at the point P

(a) (i) Write down an equation for l_1

(1)

(ii) Find, in terms of b, the coordinates of P

(3)

Given that the line l_2 is perpendicular to l_1 and intersects C on the positive x-axis,

(b) find, in terms of b, an equation for l_2

(2)

Given also that l_2 intersects C at the point P

(c) show that another equation for l_2 is

$$y = -2x + \frac{5b}{2} - 4$$

(2)

(d) Hence, or otherwise, find the value of b

(2)

11.

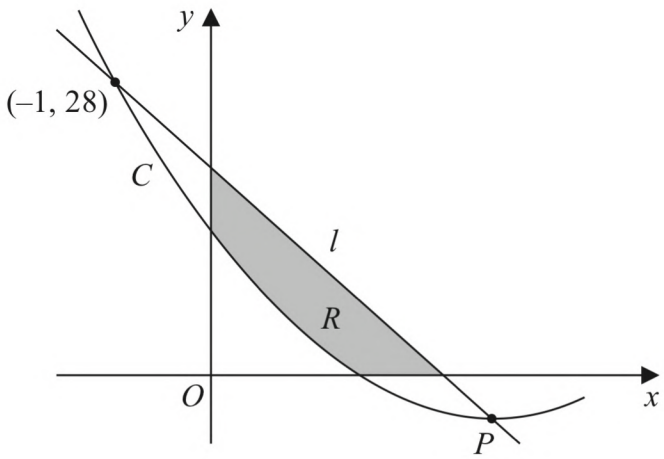

Figure 5

Figure 5 shows part of the curve C with equation $y = f(x)$ where

$$f(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(–1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

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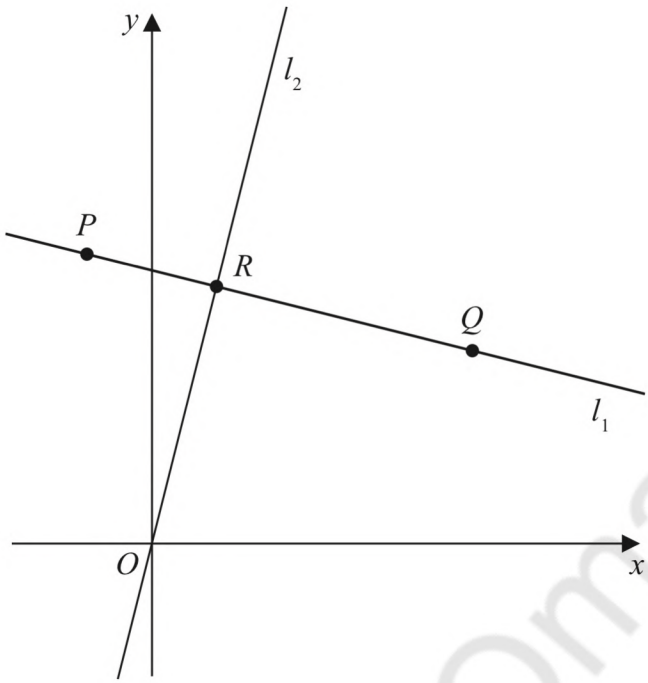

Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The straight line l_1, shown in Figure 1, passes through the points $P(-2, 9)$ and $Q(10, 6)$.

(a) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The straight line l_2 passes through the origin O and is perpendicular to l_1

The lines l_1 and l_2 meet at the point R as shown in Figure 1.

(b) Find the coordinates of R

(4)

(c) Find the exact area of triangle OPQ.

(3)

9. Given that

- the point A has coordinates (4, 2)
- the point B has coordinates (15, 7)
- the line l_1 passes through A and B

(a) find an equation for l_1, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(3)

The line l_2 passes through A and is parallel to the x-axis.

The point C lies on l_2 so that the length of BC is $5\sqrt{5}$

(b) Find both possible pairs of coordinates of the point C.

(4)

(c) Hence find the minimum possible area of triangle ABC.

(2)

7.

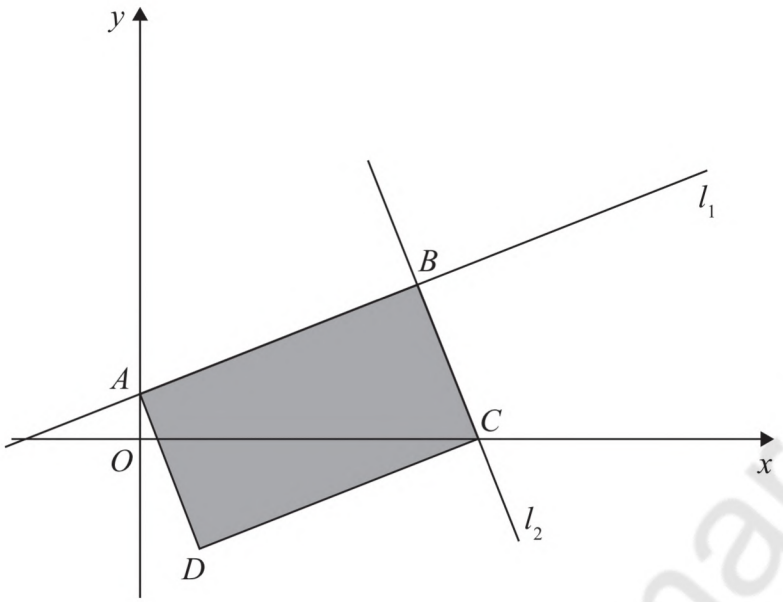

Figure 2

The straight line l_1 shown in Figure 2 has equation $5y = 2x + 10$

The points A and B lie on l_1 such that

- point A lies on the y-axis
- point B has x coordinate 10

(a) Find the distance AB writing your answer as a fully simplified surd.

(3)

The straight line l_2 also shown in Figure 2

- passes through B
- is perpendicular to l_1

(b) Find an equation for l_2 writing your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Line l_2 crosses the x-axis at the point C.

Point D is such that the points A, B, C and D form the vertices of a rectangle, shown shaded in Figure 2.

(c) Find the area of rectangle $ABCD$.

(3)

1. The line l_1 passes through the point $A(-5, 20)$ and the point $B(3, -4)$.

(a) Find an equation for l_1 giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

The line l_2 is perpendicular to l_1 and passes through the midpoint of AB

(b) Find an equation for l_2 giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

(3)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that

- the point A has coordinates $\left(-2\sqrt{3}, 5\right)$
- the point B has coordinates $\left(7\sqrt{3}, 8\right)$
- the straight line l_1 passes through A and B

(a) show that the gradient of l_1 is $p\sqrt{3}$, where p is a rational constant to be found.
You must show each step of your working.

(2)

The straight line l_2 is perpendicular to l_1 and passes through A.

(b) Find the equation of l_2, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)