Chapter 4: Graphs and Transformation

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1. The point $P(3, -4)$ lies on the curve with equation $y = f(x)$

State the coordinates of the image of P under the transformation represented by the curve with equation

(a) $y = f(x - 2)$

(1)

(b) $y = -f(x)$

(1)

(c) $2y = f(x)$

(1)

(d) $y = f(x) + 4$

(1)

6.

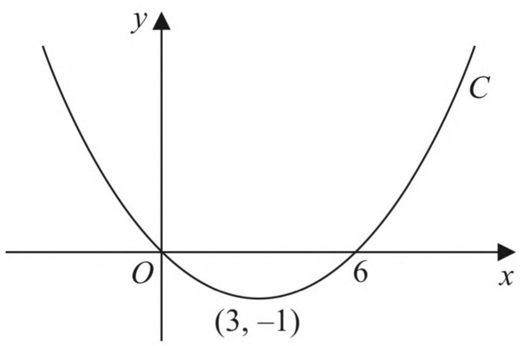

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$

The curve C passes through the origin and through (6, 0)

The curve C has a minimum at the point (3, –1)

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$

(3)

(b) $y = f(x + p)$, where p is a constant and $0 < p < 3$

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

8.

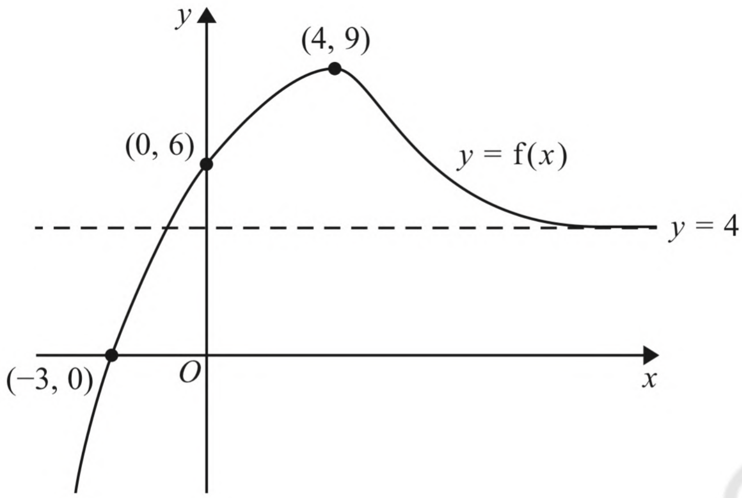

Figure 4

The curve C with equation $y = f(x)$ is shown in Figure 4.

The curve C

- has a single turning point, a maximum at (4, 9)
- crosses the coordinate axes at only two places, (−3, 0) and (0, 6)
- has a single asymptote with equation $y = 4$

as shown in Figure 4.

(a) State the equation of the asymptote to the curve with equation $y = f(-x)$.

(1)

(b) State the coordinates of the turning point on the curve with equation $y = f\left(\frac{1}{4}x\right)$.

(1)

Given that the line with equation $y = k$, where k is a constant, intersects C at exactly one point,

(c) state the possible values for k.

(2)

The curve C is transformed to a new curve that passes through the origin.

(d) (i) Given that the new curve has equation $y = f(x) - a$, state the value of the constant a.

(ii) Write down an equation for another single transformation of C that also passes through the origin.

(2)

11. (a) On Diagram 1 sketch the graphs of

(i) $y = x(3 - x)$

(ii) $y = x(x - 2)(5 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(4)

(b) Show that the x coordinates of the points of intersection of

$$y = x(3 - x) \text{ and } y = x(x - 2)(5 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 13) = 0$

(3)

The point P lies on both curves. Given that P lies in the first quadrant,

(c) find, using algebra and showing your working, the exact coordinates of P.

(5)

10. A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the x-axis at the point P and crosses the x-axis at the point Q.

(a) State the coordinates of the point P.

(1)

The curve with equation $y = f(x + a)$, where a is a constant, passes through the origin O.

(d) State the possible values of a.

(2)

6. The curve C has equation $y = \frac{4}{x} + k$, where k is a positive constant.

(a) Sketch a graph of C, stating the equation of the horizontal asymptote and the coordinates of the point of intersection with the x-axis.

(3)

The line with equation $y = 10 - 2x$ is a tangent to C.

(b) Find the possible values for k.

(5)

10.

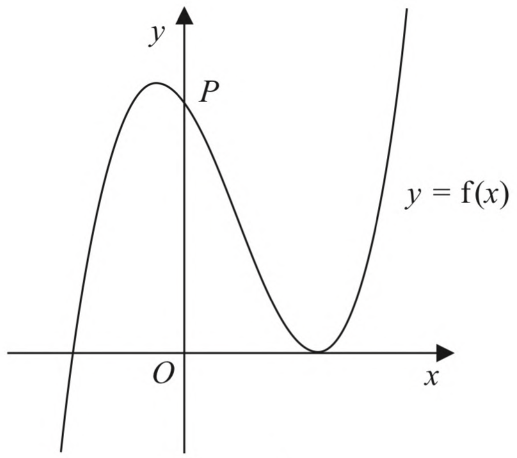

Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

(a) Deduce the values of x for which $f(x) \leqslant 0$

(2)

The curve crosses the y-axis at the point P, as shown.

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

(c) Hence, or otherwise, find

(i) the coordinates of P,

(ii) the gradient of the curve at P.

(2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

(d) (i) Find $g(x)$, giving your answer in a simplified factorised form.

(ii) Hence state the y intercept of the curve with equation $y = g(x)$.

(3)

10. The curve C_1 has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (4x - 3)(x - 5)^2$$

(a) Sketch C_1 showing the coordinates of any point where the curve touches or crosses the coordinate axes.

(3)

(b) Hence or otherwise

(i) find the values of x for which $\mathrm{f}\left(\frac{1}{4}x\right) = 0$

(ii) find the value of the constant p such that the curve with equation $y = \mathrm{f}(x) + p$ passes through the origin.

(2)

A second curve C_2 has equation $y = \mathrm{g}(x)$, where $\mathrm{g}(x) = \mathrm{f}(x + 1)$

(c) (i) Find, in simplest form, $\mathrm{g}(x)$. You may leave your answer in a factorised form.

(ii) Hence, or otherwise, find the y intercept of curve C_2

(3)

5. (i)

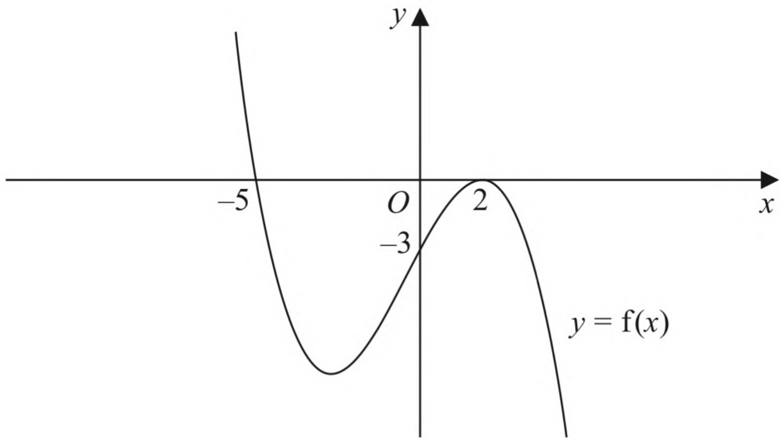

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-5, 0)$ and $(0, -3)$ and touches the x-axis at the point $(2, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 2)$

(b) $y = f(-x)$

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(6)

7. The curve C has equation

$$y = \frac{1}{2 - x}$$

(a) Sketch the graph of C. On your sketch you should show the coordinates of any points of intersection with the coordinate axes and state clearly the equations of any asymptotes.

(3)

The line l has equation $y = 4x + k$, where k is a constant.

Given that l meets C at two distinct points,

(b) show that

$$k^2 + 16k + 48 > 0$$

(4)

(c) Hence find the range of possible values for k.

(4)

6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

(2)

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

(3)

(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)

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8.

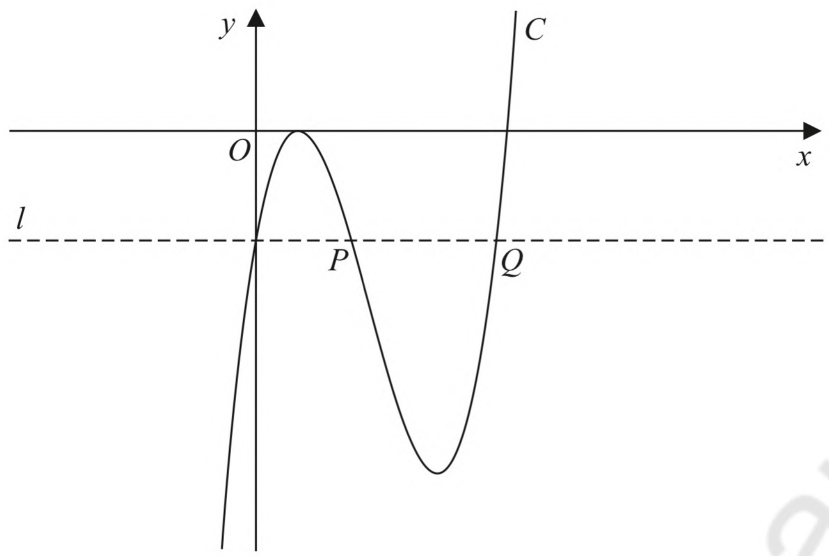


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (3x - 2)^2 (x - 4)$$

(a) Deduce the values of x for which $\mathrm{f}(x) > 0$

(1)

(b) Expand $\mathrm{f}(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

The line l, also shown in Figure 4, passes through the y intercept of C and is parallel to the x-axis.

The line l cuts C again at points P and Q, also shown in Figure 4.

(c) Using algebra and showing your working, find the length of line PQ. Write your answer in the form $k\sqrt{3}$, where k is a constant to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

The point P is the minimum point of C_1

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at –4, –2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

6.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x + 1)(x - 3)^2$$

(a) Sketch a graph of C.

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write f(x) in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

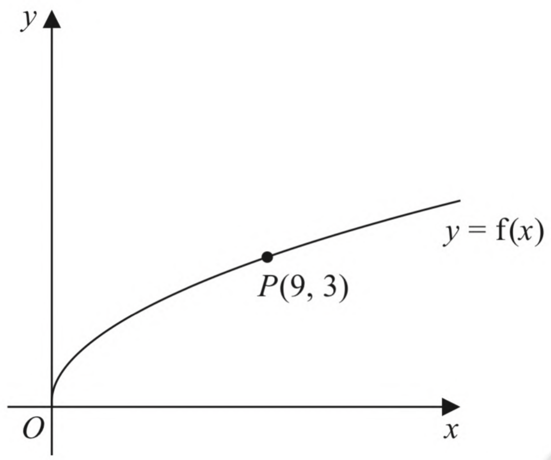


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \sqrt{x} \qquad x > 0$$

The point $P(9, 3)$ lies on the curve and is shown in Figure 5.

On the next page there is a copy of Figure 5 called Diagram 1.

(a) On Diagram 1, sketch and clearly label the graphs of

$$y = \mathrm{f}(2x) \quad \text{and} \quad y = \mathrm{f}(x) + 3$$

Show on each graph the coordinates of the point to which P is transformed.

(3)

The graph of $y = \mathrm{f}(2x)$ meets the graph of $y = \mathrm{f}(x) + 3$ at the point Q.

(b) Show that the x coordinate of Q is the solution of

$$\sqrt{x} = 3\left(\sqrt{2} + 1\right)$$

(3)

(c) Hence find, in simplest form, the coordinates of Q.

(3)

Question 9 continued

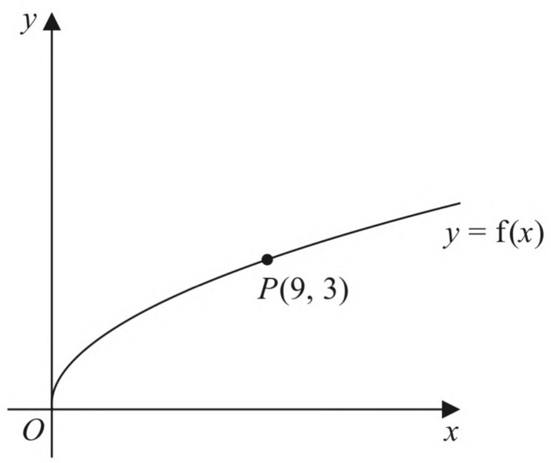


Diagram 1

Turn over for a copy of Diagram 1 if you need to redraw your graphs.

7.

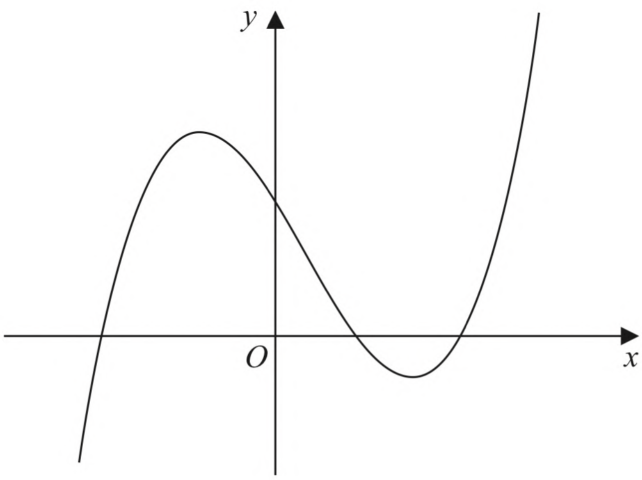


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 4)(x - 2)(2x - 9)$$

Given that the curve with equation $y = \mathrm{f}(x) - p$ passes through the point with coordinates (0, 50)

(a) find the value of the constant p.

(2)

Given that the curve with equation $y = \mathrm{f}(x + q)$ passes through the origin,

(b) write down the possible values of the constant q.

(2)

10. The curve C has equation

$$y = \frac{1}{x^2} - 9$$

(a) Sketch the graph of C.

On your sketch

- show the coordinates of any points of intersection with the coordinate axes
- state clearly the equations of any asymptotes

(4)

The curve D has equation $y = kx^2$ where k is a constant.

Given that C meets D at 4 distinct points,

(b) find the range of possible values for k.

(5)

4.

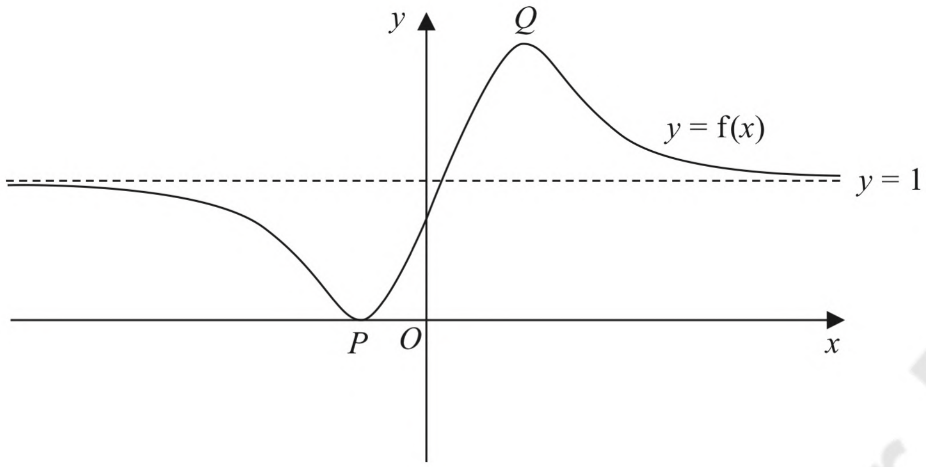


Figure 1

Figure 1 shows a sketch of a curve with equation $y = \mathrm{f}(x)$

The curve has a minimum at $P(-1, 0)$ and a maximum at $Q\left(\frac{3}{2}, 2\right)$

The line with equation $y = 1$ is the only asymptote to the curve.

On separate diagrams sketch the curves with equation

(i) $y = \mathrm{f}(x) - 2$

(3)

(ii) $y = \mathrm{f}(-x)$

(3)

On each sketch you must clearly state

- the coordinates of the maximum and minimum points
- the equation of the asymptote

6. (a) Given that k is a positive constant such that $0 < k < 4$ sketch, on **separate axes**, the graphs of

(i) $y = (2x - k)(x + 4)^2$

(ii) $y = \dfrac{k}{x^2}$

showing the coordinates of any points where the graphs cross or meet the coordinate axes, leaving coordinates in terms of k, where appropriate.

(5)

(b) State, with a reason, the number of roots of the equation

$$(2x - k)(x + 4)^2 = \frac{k}{x^2}$$

(1)

7.

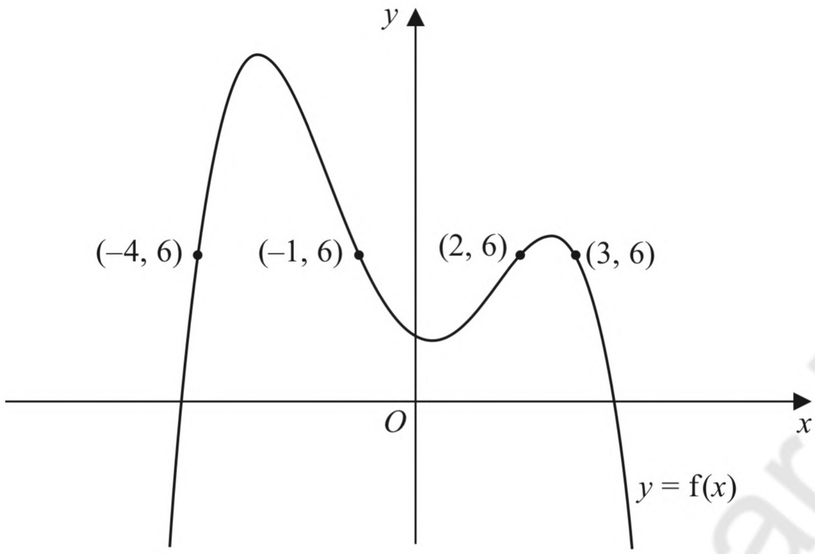


Figure 1

Figure 1 shows the curve with equation $y = \mathrm{f}(x)$.

The points $P(-4, 6)$, $Q(-1, 6)$, $R(2, 6)$ and $S(3, 6)$ lie on the curve.

(a) Using Figure 1, find the range of values of x for which

$$\mathrm{f}(x) < 6$$

(3)

(b) State the largest solution of the equation

$$\mathrm{f}(2x) = 6$$

(1)

(c) (i) Sketch the curve with equation $y = \mathrm{f}(-x)$.

On your sketch, state the coordinates of the points to which P, Q, R and S are transformed.

(ii) Hence find the set of values of x for which

$$\mathrm{f}(-x) \geqslant 6 \text{ and } x < 0$$

(4)

7. (a) On Diagram 1, sketch a graph of the curve C with equation

$$y = \frac{6}{x} \qquad x \neq 0$$

(2)

The curve C is transformed onto the curve with equation $y = \dfrac{6}{x-2} \qquad x \neq 2$

(b) Fully describe this transformation.

(2)

The curve with equation

$$y = \frac{6}{x-2} \qquad x \neq 2$$

and the line with equation

$$y = kx + 7 \qquad \text{where } k \text{ is a constant}$$

intersect at exactly two points, P and Q.

Given that the x coordinate of point P is -4

(c) find the value of k,

(2)

(d) find, using algebra, the coordinates of point Q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

4.

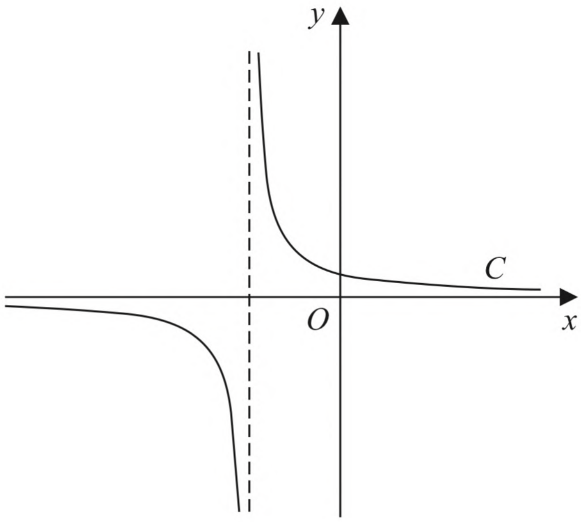


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \dfrac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y-axis.

(1)

(b) Factorise fully $x^3 + 4x^2 + 4x$

(2)

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

(3)

(d) Hence state the number of real solutions of the equation

$$(x + 2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

(1)

Question 4 continued

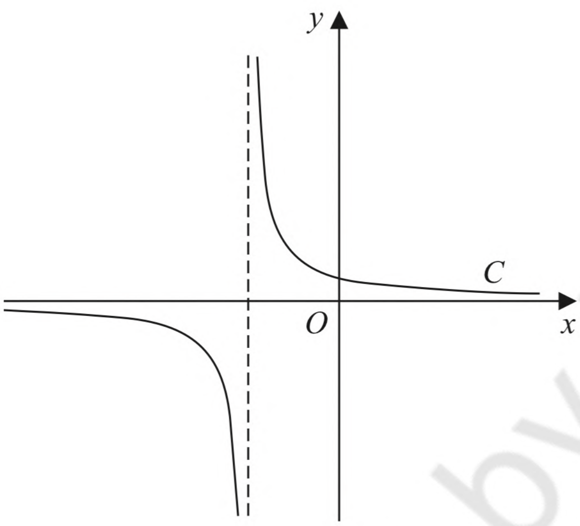
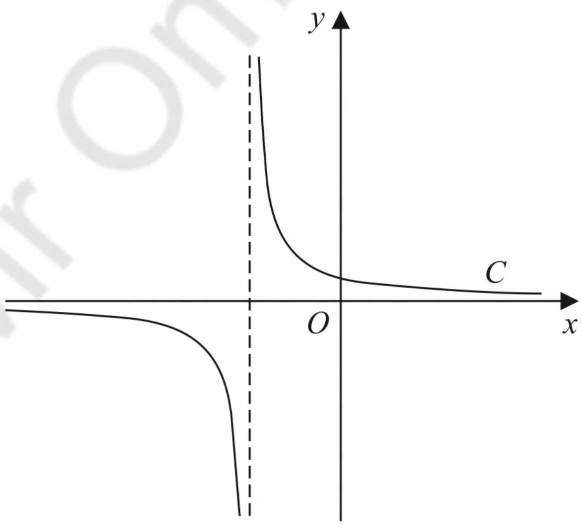


Diagram 1



copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

7. (a) Sketch the graph of the curve C with equation

$$y = \frac{4}{x - k}$$

where k is a positive constant.

Show on your sketch

- the coordinates of any points where C cuts the coordinate axes
- the equation of the vertical asymptote to C

(4)

Given that the straight line with equation $y = 9 - x$ does not cross or touch C

(b) find the range of values of k.

(5)

3.

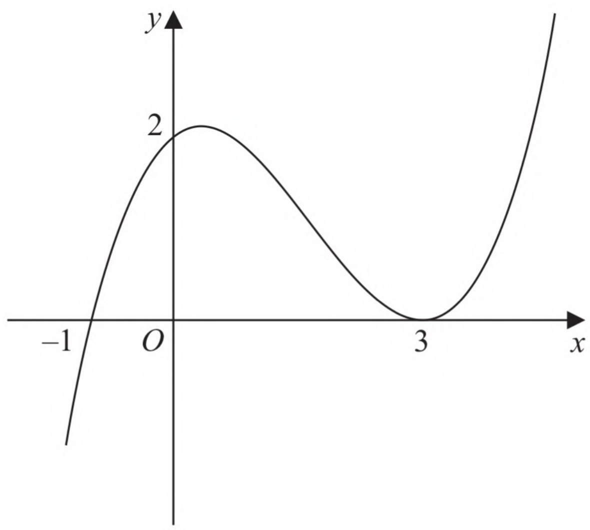


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

The curve passes through the points (–1, 0) and (0, 2) and touches the x-axis at the point (3, 0).

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x + 3)$

(3)

(b) $y = \mathrm{f}(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

Question 3 continued

8. The curve C_1 has equation

$$y = x(4 - x^2)$$

(a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \dfrac{A}{x}$ where A is a constant.

(b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$x^4 - 4x^2 + A = 0$$

(1)

(c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

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4.

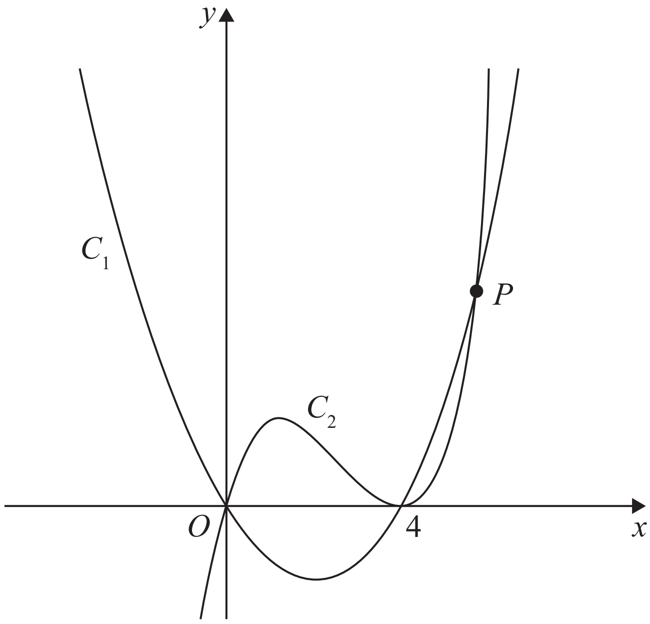

Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a quadratic function
- cuts the x-axis at the origin and at $x = 4$
- has a minimum turning point at $(2, -4.8)$

(a) find $f(x)$

(3)

Given that C_2

- has equation $y = g(x)$ where $g(x)$ is a cubic function
- cuts the x-axis at the origin and meets the x-axis at $x = 4$
- passes through the point $(6, 7.2)$

(b) find $g(x)$

(3)

The curves C_1 and C_2 meet in the first quadrant at the point P, shown in Figure 1.

(c) Use algebra to find the coordinates of P.

(4)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

(a) Sketch the graph of C.

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k.

(3)