Chapter 3: Equations and Inequalities

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2. **In this question you must show all steps of your working.**
Solutions relying on calculator technology are not acceptable.

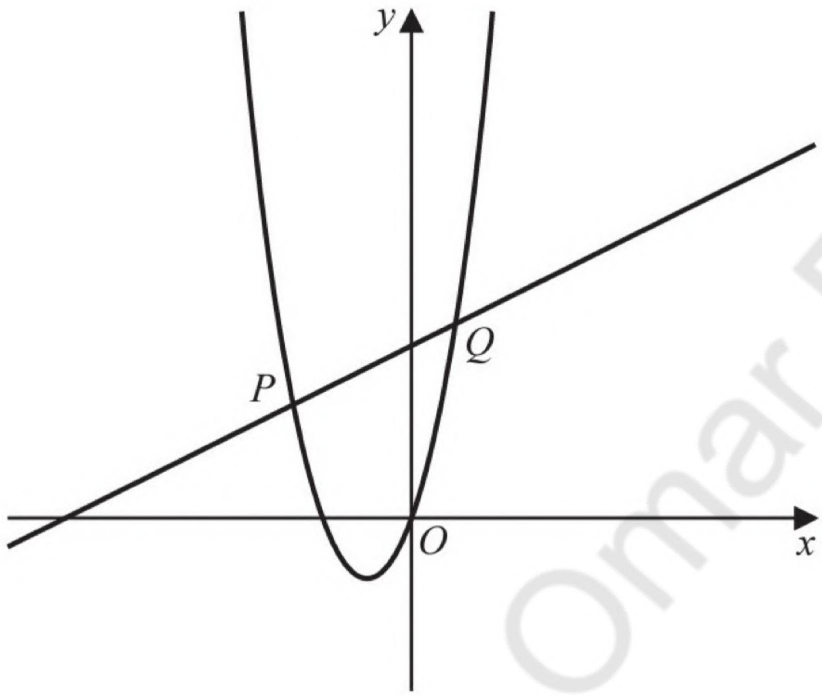

Figure 1

Figure 1 shows a sketch of the curve with equation $y = 2x^2 + 3x$ and the straight line with equation $y = \frac{1}{2}x + 3$

The line meets the curve at the points P and Q, as shown in Figure 1.

(a) Using algebra, find the coordinates of P and the coordinates of Q.

(5)

(b) Hence write down the range of values of x for which $2x^2 + 3x \geqslant \frac{1}{2}x + 3$

(2)

3.2

3. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

4. The straight line with equation $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = 2x^2 + 8x + 3$

Calculate the value of c

(5)

5. (a) On the same axes, sketch the graphs of $y = x + 2$ and $y = x^2 - x - 6$ showing the coordinates of all points at which each graph crosses the coordinate axes.

(4)

(b) On your sketch, show, by shading, the region R defined by the inequalities

$$y < x + 2 \quad \textbf{and} \quad y > x^2 - x - 6$$

(1)

(c) Hence, or otherwise, find the set of values of x for which $x^2 - 2x - 8 < 0$

(3)

4.

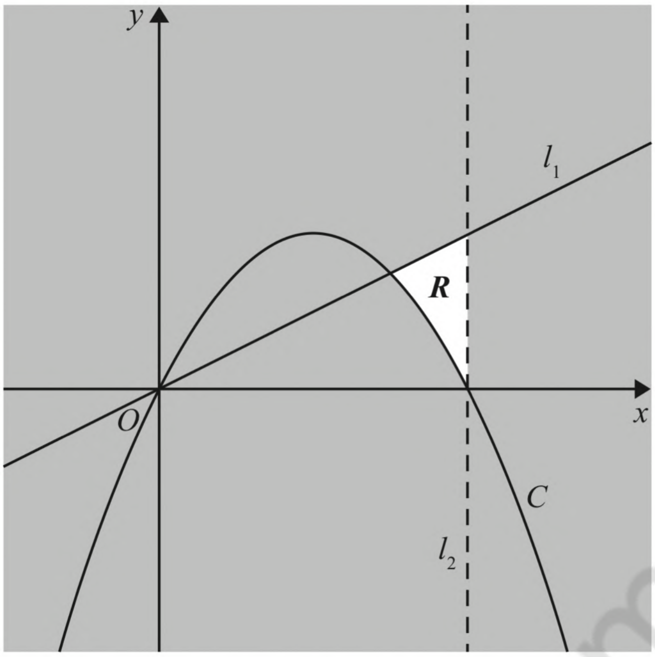

Figure 1

Figure 1 shows a line l_1 with equation $2y = x$ and a curve C with equation $y = 2x - \frac{1}{8}x^2$

The region R, shown unshaded in Figure 1, is bounded by the line l_1, the curve C and a line l_2

Given that l_2 is parallel to the y-axis and passes through the intercept of C with the positive x-axis, identify the inequalities that define R.

(3)

9. The equation

$$\frac{3}{x} + 5 = -2x + c$$

where c is a constant, has no real roots.

Find the range of possible values of c.

(7)

3.

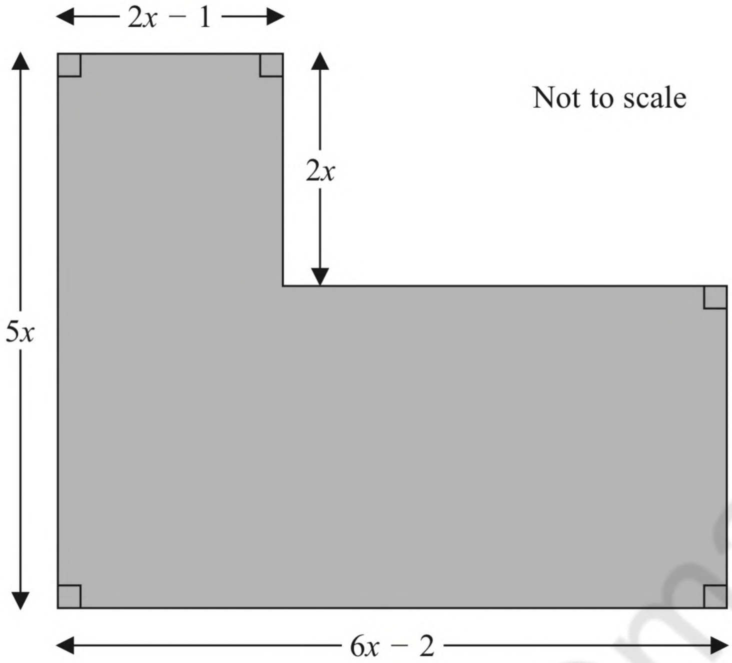

Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 29 m,

(a) show that $x > 1.5$ m

(3)

Given also that the area of the garden is less than 72 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of possible values of x.

(1)

5. (a) Find, using algebra, all real solutions of

$$2x^3 + 3x^2 - 35x = 0$$

(3)

(b) Hence find all real solutions of

$$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$$

(4)

6. The line with equation $y = 4x + c$, where c is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(a) Find the value of c.

(4)

(b) Hence find the coordinates of the point of intersection.

(3)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

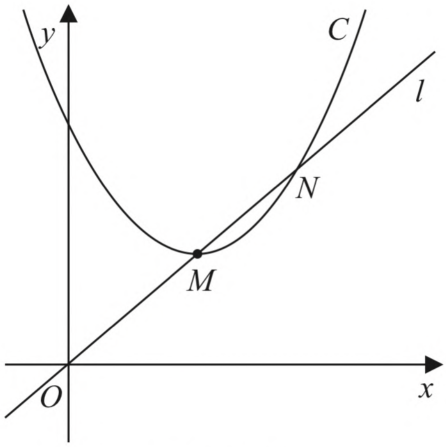

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = x^2 - 5x + 13$

The point M is the minimum point of C.

The straight line l passes through the origin O and intersects C at the points M and N as shown.

Find, showing your working,

(a) the coordinates of M,

(3)

(b) the coordinates of N.

(5)

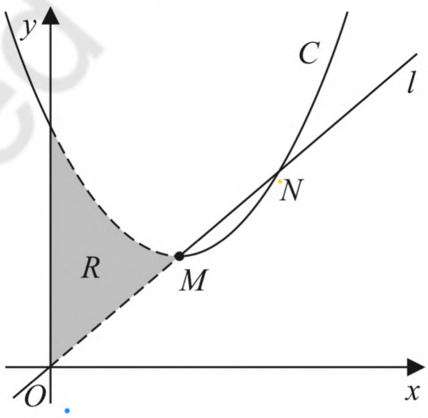

Figure 3

Figure 3 shows the curve C and the line l. The finite region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Use inequalities to define the region R.

(2)

8. Solve, using algebra, the equation

$$x - 6x^{\frac{1}{2}} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

5. (a) Find, using algebra, all solutions of

$$20x^3 - 50x^2 - 30x = 0$$

(3)

(b) Hence find all real solutions of

$$20(y+3)^{\frac{3}{2}} - 50(y+3) - 30(y+3)^{\frac{1}{2}} = 0$$

(4)

4. Use algebra to solve the simultaneous equations

$$y - 3x = 4$$
$$x^2 + y^2 + 6x - 4y = 4$$

You must show all stages of your working.

(7)

2. A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

where p and q are constants,

(a) find, to 3 significant figures where necessary, the value of p and the value of q.
(4)

Exactly T years after the tree was planted, its height was 5 m.

(b) Find the value of T according to the model, giving your answer to one decimal place.
(2)

4.

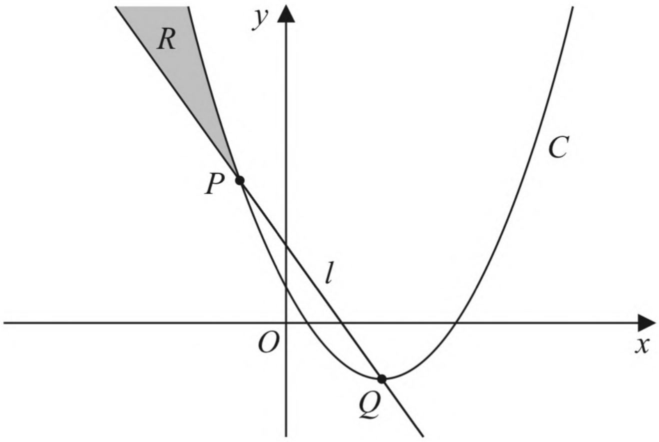

Figure 2

The points P and Q, as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q.

(a) Find an equation for l, writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(3)

The quadratic curve C passes through P and has a minimum point at Q.

(b) Find an equation for C.

(3)

The region R, shown shaded in Figure 2, lies in the second quadrant and is bounded by C and l only.

(c) Use inequalities to define region R.

(2)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3(9^{3z}) + 26(9^{2z}) - 9(9^{z}) = 0$

(5)

5.

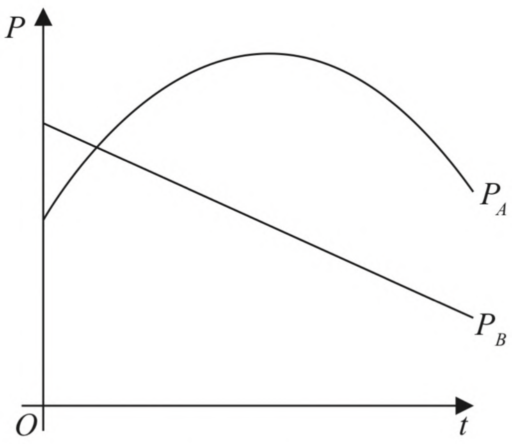

Figure 2

The share value of two companies, company A and company B, has been monitored over a 15-year period.

The share value P_A of **company *A***, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

The share value P_B of **company *B***, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

(a) Find the difference between the share value of **company *A*** and the share value of **company *B*** at the point monitoring began.
(2)

(b) State the maximum share value of **company *A*** during the 15-year period.
(1)

(c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company *A*** was greater than the share value of **company *B***.
(4)

(d) Explain why the model for **company *A*** should not be used to predict its share value when $t = 20$
(1)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$2xy - 3x^2 = 50$$

and

$$y - x^3 + 6x = 0$$

show that

$$2x^4 - 15x^2 - 50 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$2xy - 3x^2 = 50$$
$$y - x^3 + 6x = 0$$

Give your answers in fully simplified surd form.

(5)

3. (i) Solve

$$\frac{3}{x} > 4$$

(3)

(ii)

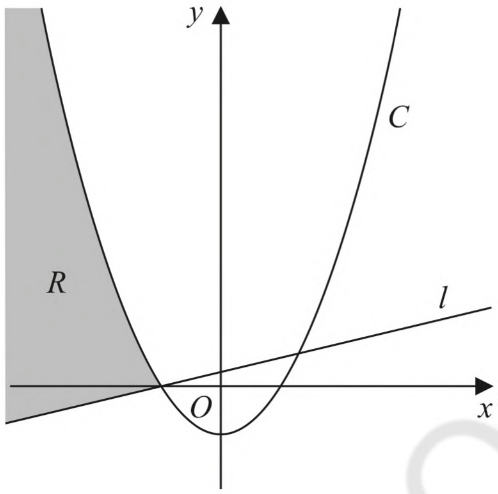

Figure 1

Figure 1 shows a sketch of the curve C and the straight line l.

The infinite region R, shown shaded in Figure 1, lies in quadrants 2 and 3 and is bounded by C and l only.

Given that

- l has a gradient of 3
- C has equation $y = 2x^2 - 50$
- C and l intersect on the negative x-axis

use inequalities to define the region R.

(3)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express f(x) in the form $ax^2 + bx\sqrt{2} + c$ where a, b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

4.

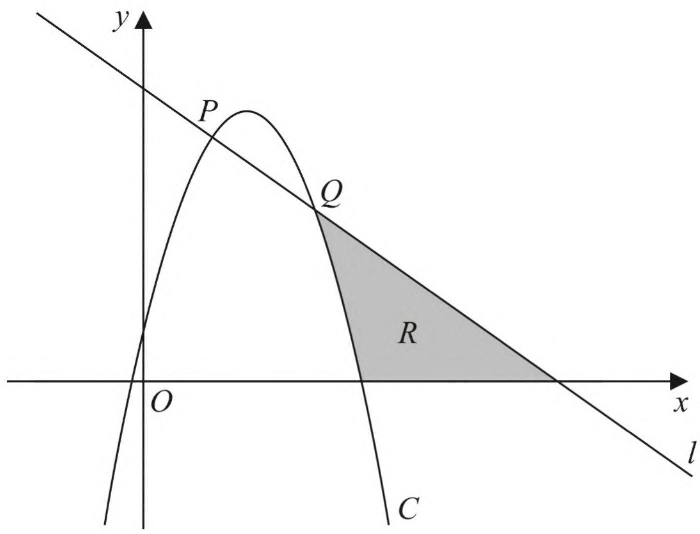

Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows a line l with equation $x + y = 6$ and a curve C with equation $y = 6x - 2x^2 + 1$

The line l intersects the curve C at the points P and Q as shown in Figure 1.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 1, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

5. The curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}(x)$ is a quadratic expression
- the maximum turning point on C has coordinates $(-2, 12)$
- C cuts the negative x-axis at -5

(a) find $\mathrm{f}(x)$

(4)

The line l_1 has equation $y = \frac{4}{5}x$

Given that the line l_2 is perpendicular to l_1 and passes through $(-5, 0)$

(b) find an equation for l_2, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

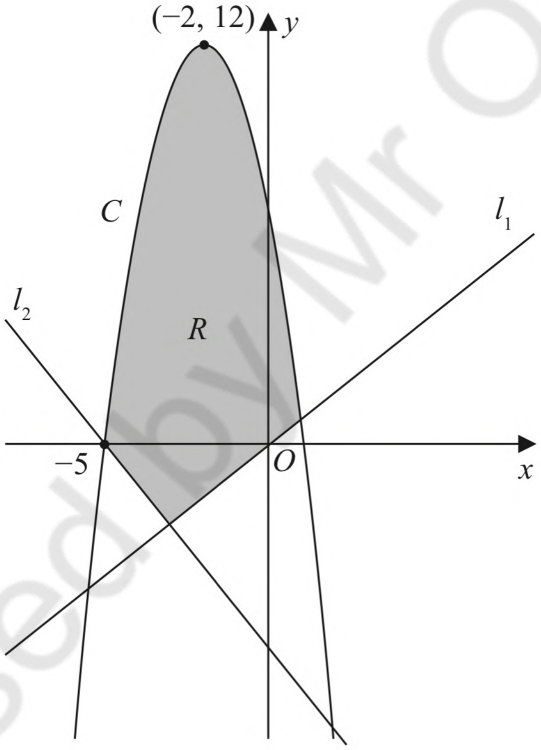

Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(c) Define the region R, shown shaded in Figure 2, using inequalities.

(2)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$2xy - 3x^2 = 50$$

and

$$y - x^3 + 6x = 0$$

show that

$$2x^4 - 15x^2 - 50 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$2xy - 3x^2 = 50$$
$$y - x^3 + 6x = 0$$

Give your answers in fully simplified surd form.

(5)

2. Given that

$$(x - 5)(2x + 1)(x + 3) \equiv ax^3 + bx^2 - 32x - 15$$

where a and b are constants,

(a) find the value of a and the value of b.

(2)

3. The share price of a company is monitored.

Exactly 3 years after monitoring began, the share price was £1.05

Exactly 5 years after monitoring began, the share price was £1.65

The share price, £V, of the company is modelled by the equation

$$V = pt + q$$

where t is the number of years after monitoring began and p and q are constants.

(a) Find the value of p and the value of q.

(3)

Exactly T years after monitoring began, the share price was £2.50

(b) Find the value of T, according to the model, giving your answer to one decimal place.

(2)

4. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^2(2x + 1) - 15x$$

(a) Solve

$$f(x) = 0$$

(4)

(b) Hence solve

$$y^{\frac{4}{3}}\left(2y^{\frac{2}{3}} + 1\right) - 15y^{\frac{2}{3}} = 0 \qquad y > 0$$

giving your answer in simplified surd form.

(2)

9.

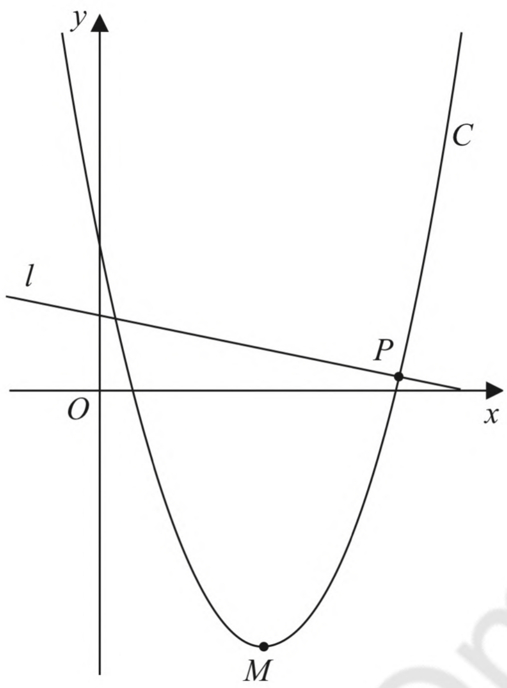

Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = \frac{1}{2}x^2 - 10x + 22$$

(3)

The point M is the minimum turning point of C, as shown in Figure 3.

(b) Deduce the coordinates of M

(2)

The line l is the normal to C at the point P, as shown in Figure 3.

Given that l has equation $y = k - \frac{1}{8}x$, where k is a constant,

(c) (i) find the coordinates of P

(ii) find the value of k

(6)

Question 9 continued

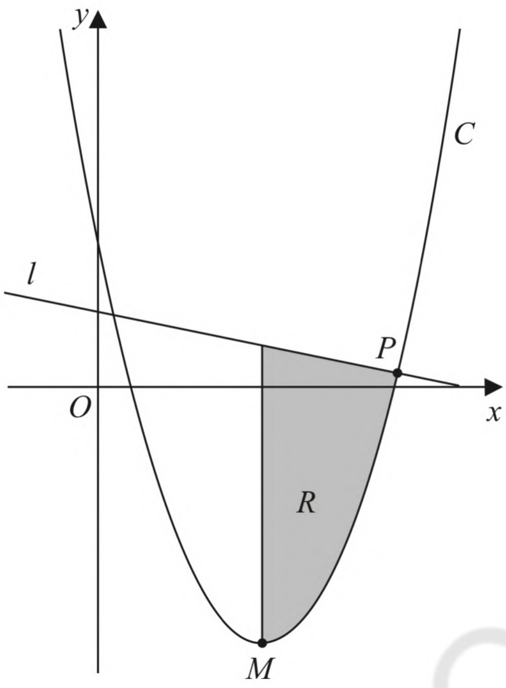

Figure 4

Figure 4 is a copy of Figure 3. The finite region R, shown shaded in Figure 4, is bounded by l, C and the line through M parallel to the y-axis.

(d) Identify the inequalities that define R.

(3)

4. Given that the equation

$$kx^2 + 6kx + 5 = 0 \qquad \text{where } k \text{ is a non zero constant}$$

has no real roots, find the range of possible values for k.

(4)

8.

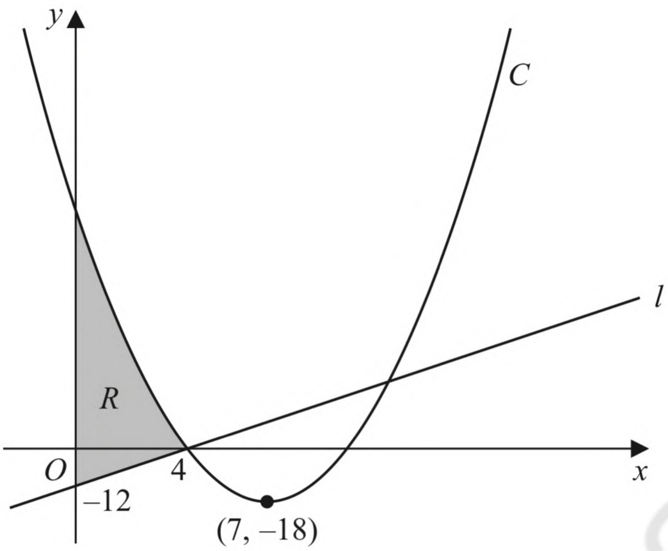

Figure 2

Figure 2 shows a sketch of the straight line l and the curve C.

Given that l cuts the y-axis at –12 and cuts the x-axis at 4, as shown in Figure 2,

(a) find an equation for l, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

Given that C

- has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression
- has a minimum point at (7, –18)
- cuts the x-axis at 4 and at k, where k is a constant

(b) deduce the value of k,

(1)

(c) find $\mathrm{f}(x)$.

(3)

The region R is shown shaded in Figure 2.

(d) Use inequalities to define R.

(2)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A rectangular sports pitch has length x metres and width y metres, where $x > y$

Given that the perimeter of the pitch is 350 m,

(a) write down an equation linking x and y

(1)

Given also that the area of the pitch is $7350\,\text{m}^2$

(b) write down a second equation linking x and y

(1)

(c) hence find the value of x and the value of y

(4)

7.

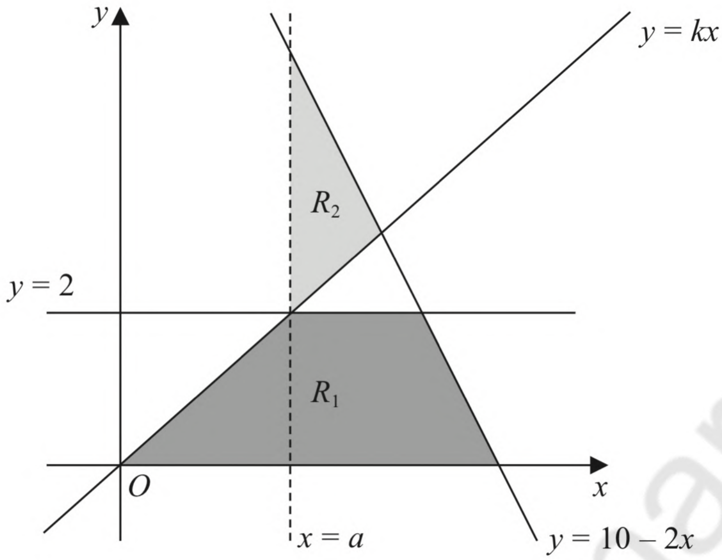

Figure 2

The region R_1, shown shaded in Figure 2, is defined by the inequalities

$$0 \leqslant y \leqslant 2 \qquad y \leqslant 10 - 2x \qquad y \leqslant kx$$

where k is a constant.

The line $x = a$, where a is a constant, passes through the intersection of the lines $y = 2$ and $y = kx$

Given that the area of R_1 is $\frac{27}{4}$ square units,

(a) find

(i) the value of a

(ii) the value of k

(4)

(b) Define the region R_2, also shown shaded in Figure 2, using inequalities.

(2)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The equation

$$4(p - 2x) = \frac{12 + 15p}{x + p} \qquad x \neq -p$$

where p is a constant, has two distinct real roots.

(a) Show that

$$3p^2 - 10p - 8 > 0$$

(3)

(b) Hence, using algebra, find the range of possible values of p

(3)

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$xy = \frac{15}{2} - 5x \qquad x \neq 0$$

The curve C_2 has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that C_1 and C_2 meet when

$$2x^4 - 7x^2 - 15 = 0$$

(2)

Given that C_1 and C_2 meet at points P and Q

(b) find, using algebra, the exact distance PQ

(5)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 2^x$, show that the equation

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

can be written in the form

$$4p^2 - 33p + 8 = 0$$

(3)

(b) Hence solve

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

4. The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

6.

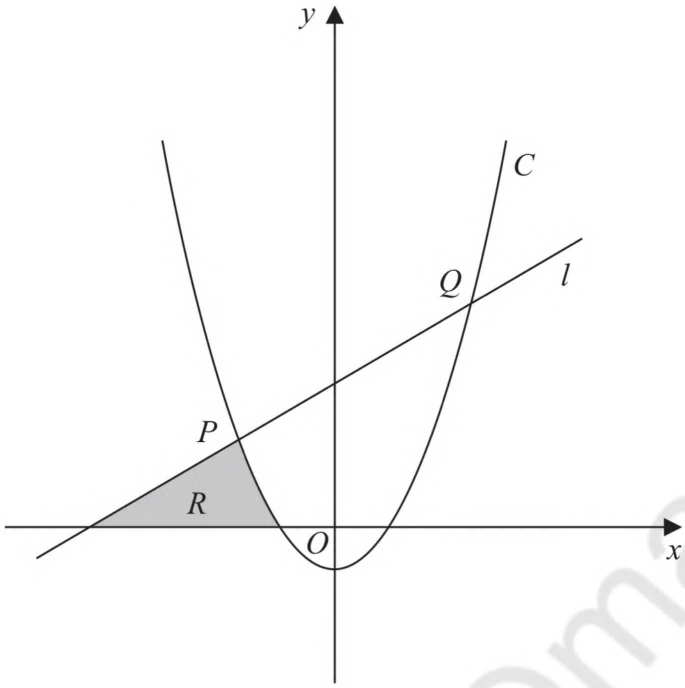

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows

- the line l with equation $y - 5x = 75$
- the curve C with equation $y = 2x^2 + x - 21$

The line l intersects the curve C at the points P and Q, as shown in Figure 3.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Solve

$$5(x+3) > 4(2x-5)$$

(2)

(b) (i) Write

$$x^2 - 6x + 1$$

in the form $(x+a)^2 + b$ where a and b are constants.

(ii) Hence solve

$$x^2 - 6x + 1 \geqslant 0$$

(4)

(c) Hence find the values of x that satisfy both

$$5(x+3) > 4(2x-5) \quad \textbf{and} \quad x^2 - 6x + 1 \geqslant 0$$

(1)

4. The curve C has equation

$$y = \frac{2}{x} + 3x - 4 \qquad x \neq 0$$

The straight line l has equation

$$y = kx + 2$$

where k is a constant.

(a) Show that l meets C when

$$(k-3)x^2 + 6x - 2 = 0$$

(2)

(b) Hence find the value of k for which l is a tangent to C

(3)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Fully factorise

$$9x^3 - 10x^2 + x$$

(2)

(b) Hence solve

$$9 \times 27^y - 10 \times 9^y + 3^y = 0$$

(3)

2.

(ii) Find the exact value of a such that

$$\frac{16}{\sqrt{3}+1} = a\sqrt{27} + 4$$

Write your answer in the form $p\sqrt{3} + q$ where p and q are fully simplified rational constants.

(4)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Curve C with equation

$$y = \frac{1}{2-x} \qquad x \neq 2$$

The straight line l has equation $y = kx - 4$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$k^2 - 5k + 4 \geqslant 0$$

(ii) find the range of possible values for k.

(6)

3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000

Exactly 10 years after monitoring began, the population was 65 000

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(2)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

The line l has equation $y = -kx - 6$

Given that l intersects C at 2 distinct points,

(b) find the range of possible values of k.

(5)