P2 Chapter 7 Differentiation

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

4.

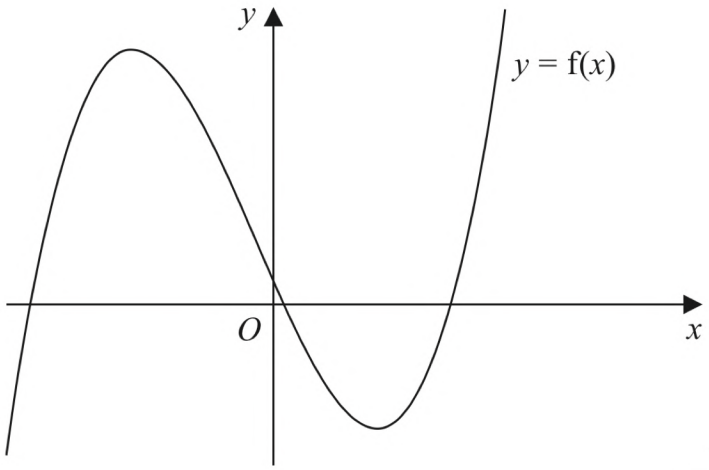

Figure 2

In this question you must show all steps of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = 2x^3 + \frac{3}{2}x^2 - 18x + 3$$

(a) Find the set of values of x for which $f(x)$ is decreasing.

(4)

Given that the equation $f(x) = k$, where k is a constant, has one real root,

(b) find the set of values for k.

(3)

5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

10.

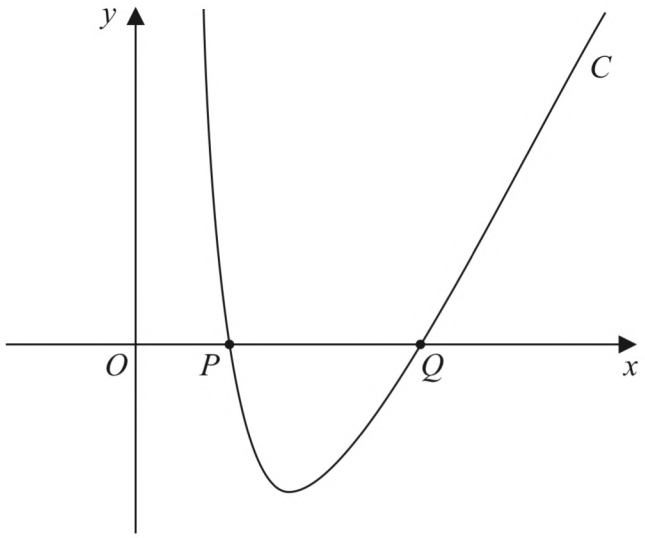

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \qquad x > 0$$

Using calculus,

(a) find the range of values of x for which $f(x)$ is increasing,

(4)

1. A curve C has equation $y = 2x^2(x - 5)$

(a) Find, using calculus, the x coordinates of the stationary points of C.

(4)

(b) Hence find the values of x for which y is increasing.

(2)

10. A curve C has equation

$$y = 4x^3 - 9x + \frac{k}{x} \qquad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C.

Given that P is a stationary point of C,

(a) show that $k = -\frac{3}{2}$

(4)

(b) Determine the nature of the stationary point at P, justifying your answer.

(2)

The curve C has a second stationary point.

(c) Using algebra, find the x coordinate of this second stationary point.

(4)

3. $f(x) = ax^3 - x^2 + bx + 4$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b.

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

(a) Using calculus, find the x coordinates of the stationary points of the curve.

(4)

(b) Justify, by further calculus, the nature of all of the stationary points of the curve.

(3)

9.

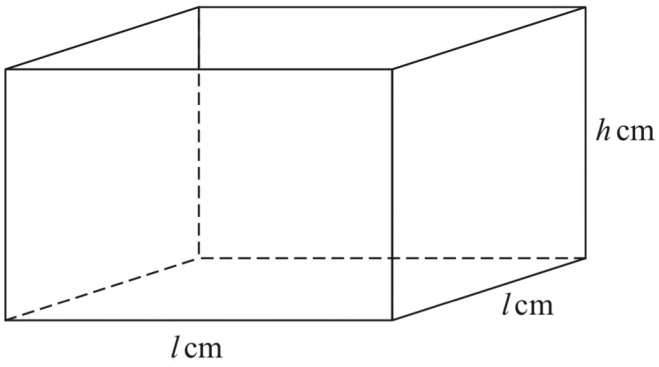

Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is 250 000 cm^3

(a) show that the external surface area, S cm^2, of the box is given by

$$S = \frac{250000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

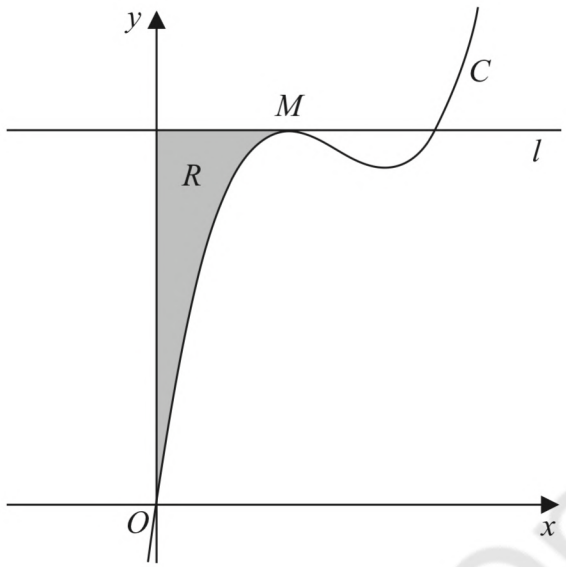

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \qquad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

(a) show that $k = 28$

(3)

(b) Determine the range of values of x for which y is increasing.

(2)

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the y-axis.

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

(2)

(b) Hence find the coordinates of the stationary point of C.

(4)

(c) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point of C.

(2)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

7. The curve C has equation

$$y = \frac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}} \qquad x > 0$$

(a) Write the equation of C in the form

$$y = ax^{\frac{7}{2}} + bx^{\frac{5}{2}} + cx^{\frac{3}{2}} + dx^{\frac{1}{2}}$$

where a, b, c and d are fully simplified constants.

(3)

The curve C has three turning points.

Using calculus,

(b) show that the x coordinates of the three turning points satisfy the equation

$$2x^3 - 10x^2 + 13x - 5 = 0$$

(3)

Given that the x coordinate of one of the turning points is 1

(c) find, using algebra, the exact x coordinates of the other two turning points.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

2. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

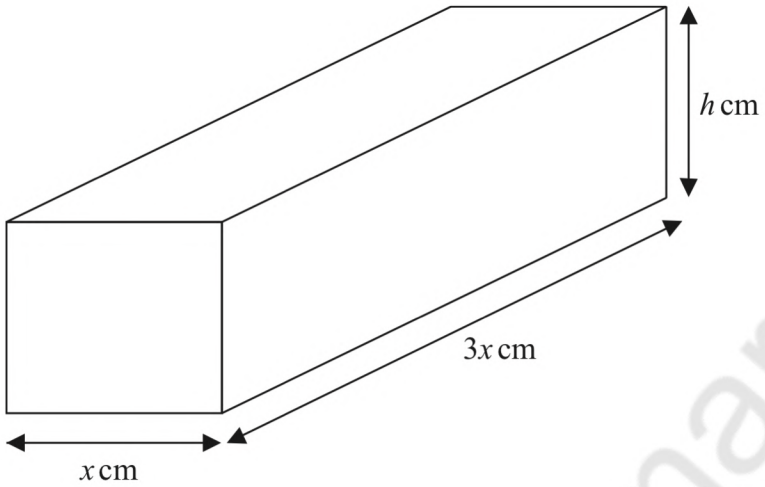

Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is $972\,\text{cm}^3$

(a) Show that the surface area of the brick, $S\,\text{cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x}$$

(3)

(b) Find $\dfrac{\mathrm{d}S}{\mathrm{d}x}$

(1)

(c) Hence find the value of x for which S is stationary.

(2)

(d) Find $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}$ and hence show that the value of x found in part (c) gives the minimum value of S.

(2)

(e) Hence find the minimum surface area of the brick.

(1)

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

9.

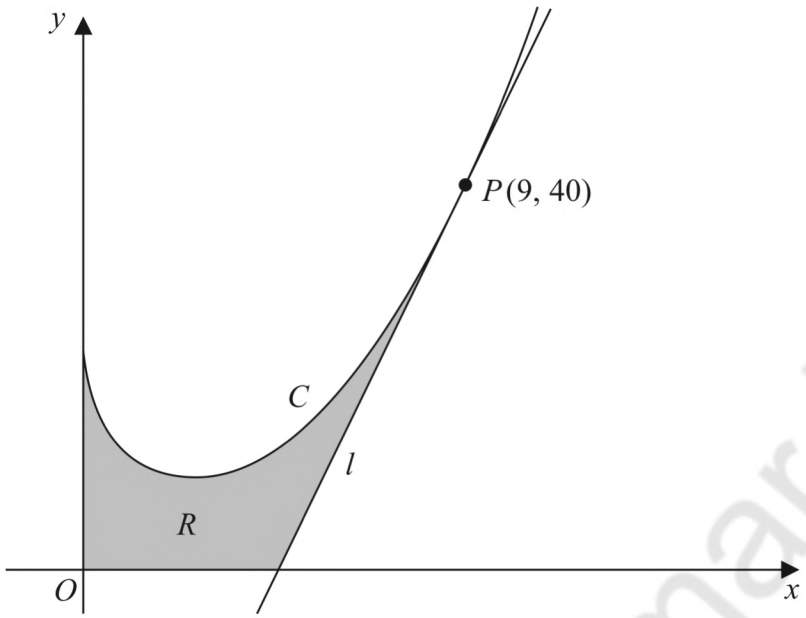

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates (9, 40).

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

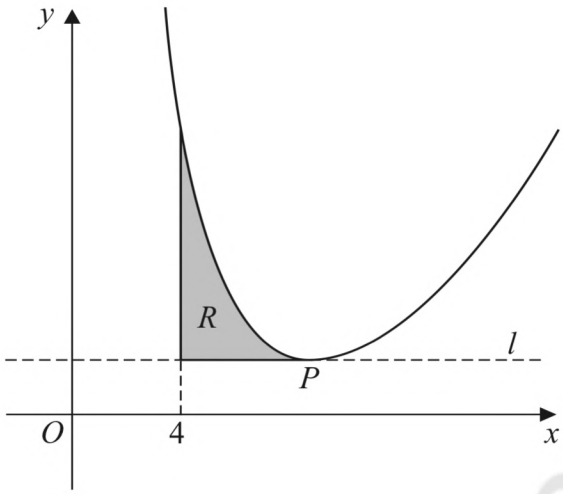

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

9.

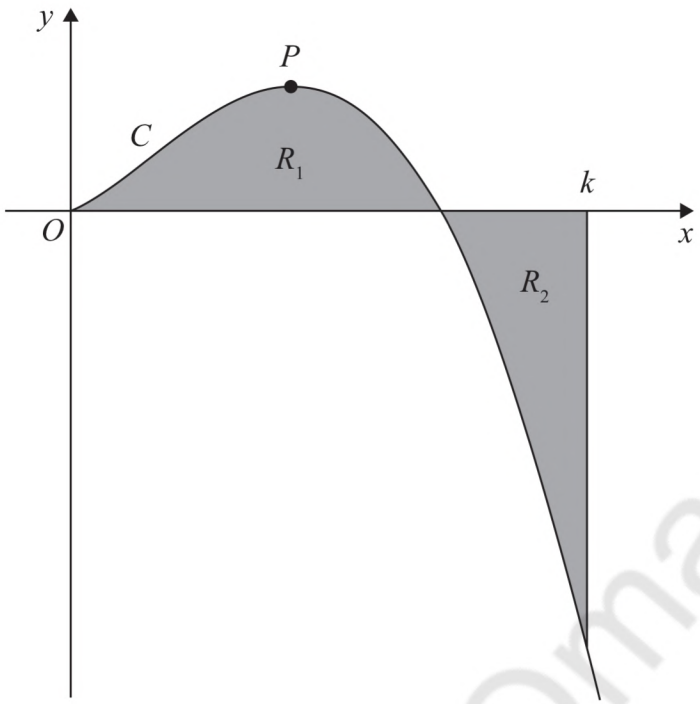

Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

10.

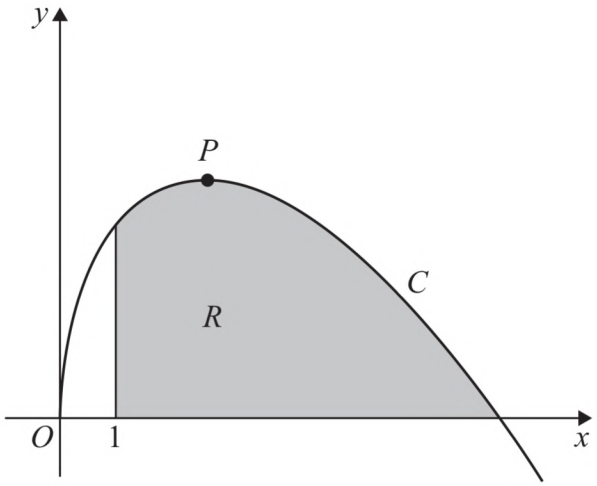

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ giving each term in simplest form.

(2)

(b) Hence find the x coordinate of the stationary point of C.

(2)

(c) (i) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ giving each term in simplest form.

(ii) Hence determine the nature of the stationary point of C, giving a reason for your answer.

(2)

(d) State the range of values of x for which y is decreasing.

(1)

3.

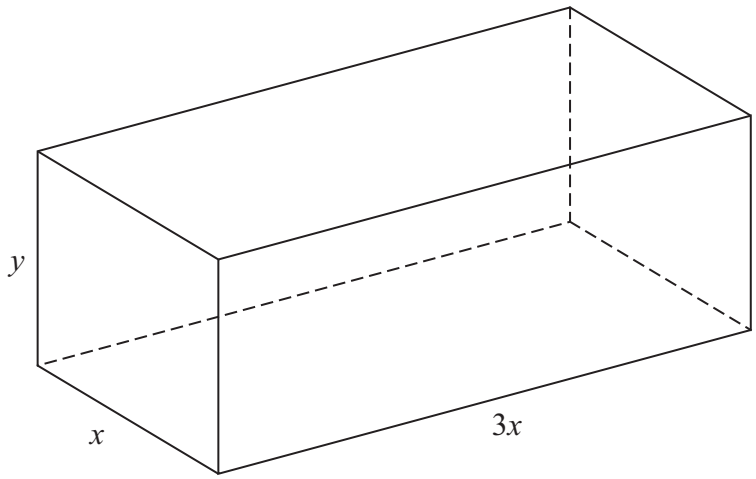

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)