P2 Chapter 6 Trigonometric Identities and Equations

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

8. The depth of water, D metres, in a harbour on a particular day is modelled by the equation

$$D = 3 - 1.2\sin\left(\frac{\pi t}{6}\right), \qquad 0 \leqslant t < 19$$

where t is the number of hours after 05:00, and $\frac{\pi t}{6}$ is measured in radians.

(a) Show that the depth of the water at 06:00 is 2.4 metres.

(1)

(b) State the greatest value of D.

(1)

(c) Find the first two times of the day, after 05:00, when the depth of the water in the harbour is 3.5 metres. Show all steps in your working and give your answers to the nearest minute.

(6)

9. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k

(3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

9. (a) Show that the equation

$$\cos\theta - 1 = 4\sin\theta\tan\theta$$

can be written in the form

$$5\cos^2\theta - \cos\theta - 4 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < \frac{\pi}{2}$

$$\cos 2x - 1 = 4\sin 2x\tan 2x$$

giving your answers, where appropriate, to 2 decimal places.

(4)

9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

7. (a) Show that the equation

$$8\tan\theta = 3\cos\theta$$

may be rewritten in the form

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $0 \leqslant x \leqslant 90°$, the equation

$$8\tan 2x = 3\cos 2x$$

giving your answers to 2 decimal places.

(4)

7. (i) Show that

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta} \qquad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(ii) Solve, for $0 \leqslant x < 90°$, the equation

$$3\cos^2(2x + 10°) = 1$$

giving your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < 360°$, the equation

$$3\sin(\theta + 30°) = 7\cos(\theta + 30°)$$

giving your answers to one decimal place.

(4)

(ii) (a) Show that the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

can be written in the form

$$\sin x(a\cos^2 x + b\cos x + c) = 0$$

where a, b and c are constants to be found.

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

(6)

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\tan^2\left(2x + \frac{\pi}{4}\right) = 3$$

(5)

(ii) Solve, for $0 < \theta < 360°$

$$(2\sin\theta - \cos\theta)^2 = 1$$

giving your answers, as appropriate, to one decimal place.

(5)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leqslant 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(6)

5. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$(3\cos\theta - \tan\theta)\cos\theta = 2$$

can be written as

$$3\sin^2\theta + \sin\theta - 1 = 0$$

(3)

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

$$(3\cos 2x - \tan 2x)\cos 2x = 2$$

(5)

8. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \pi$, the equation

$$5\sin(3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

(ii) Solve, for $0 < \theta < 360°$, the equation

$$2\tan\theta\sin\theta = 5 + \cos\theta$$

giving your answers, **in degrees**, to one decimal place.

(5)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$3\cos\theta(\tan\theta\sin\theta + 3) = 11 - 5\cos\theta$$

may be written as

$$3\cos^2\theta - 14\cos\theta + 8 = 0$$

(3)

(b) Hence solve, for $0 < x < 360°$

$$3\cos 2x(\tan 2x \sin 2x + 3) = 11 - 5\cos 2x$$

giving your answers to one decimal place.

(4)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Solve, for $0 < \theta \leqslant 360°$ the equation

$$2\tan\theta + 3\sin\theta = 0$$

giving your answers, as appropriate, to one decimal place.

(5)

(b) Hence, or otherwise, find the smallest positive solution of

$$2\tan(2x + 40°) + 3\sin(2x + 40°) = 0$$

giving your answer to one decimal place.

(2)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

(ii)

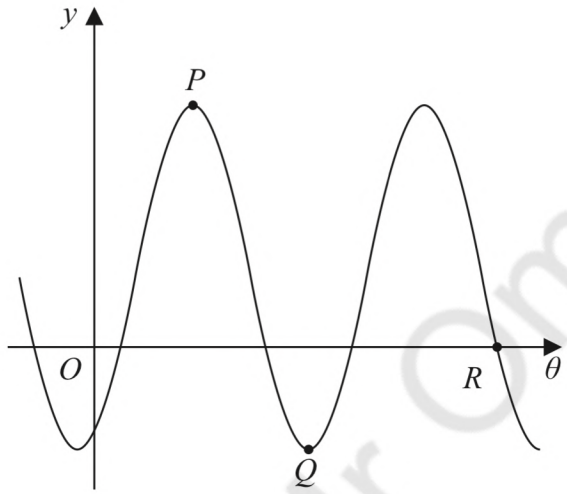

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, $H°$C, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$, the equation

$$3\sin x \tan x = 11 + \cos x$$

giving the answers in radians to 3 decimal places.

(5)

(ii) Given that

- $0 < \theta < 90°$
- $\cos\theta = \frac{1}{3}$

find, in simplest form, the exact value of $\tan\theta$

(2)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}\qquad\theta\neq\frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x+\frac{2}{\tan x}\right)=4\sin x-5$$

giving your answers to 3 significant figures.

(4)