# P2 Chapter 5 Sequences and Series

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**3.** In an arithmetic series

- the sixth term is 11
- the sum of the first three terms is –15

For this series,

(a) show that the first term is –9 and calculate the common difference.

**(4)**

Given that $S_n > 888$

(b) find the least value of $n$.

**(3)**

**9.** The first three terms of a geometric series are

$$\sin\theta,\ \cos\theta \text{ and } 0.5 \qquad \text{where } \theta \text{ is a constant}$$

(a) Show that

$$2\sin^2\theta + \sin\theta - 2 = 0$$

**(3)**

Given that $\theta$ lies in the interval $90° < \theta < 180°$

(b) find the value of $\theta$ giving your answer to one decimal place.

**(4)**

(c) Hence prove that this series is convergent.

**(3)**

(d) Find, to two significant figures, $S_\infty$

**(2)**

**2.** The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is $S_\infty$

(a) Find the value of $S_\infty$

**(2)**

The sum to $N$ terms of the series is $S_N$

(b) Find, to 1 decimal place, the value of $S_{12}$

**(2)**

(c) Find the smallest value of $N$, for which $S_\infty - S_N < 0.5$

**(4)**

5. An arithmetic series has first term $a$ and common difference $d$.

(a) Prove that the sum of the first $n$ terms of the series is

$$\frac{1}{2}n[2a + (n - 1)d]$$

**(4)**

A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week $N$.

(b) Find the value of $N$

**(2)**

The company then plans to continue to make 600 mobile phones each week.

(c) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

**(5)**

**Answer all questions. Write your answers in the spaces provided.**

**1.** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4 - a_n$$
$$a_1 = 3$$

Find the value of

(a) (i) $a_2$

(ii) $a_{107}$

**(2)**

(b) $\sum_{n=1}^{200}(2a_n - 1)$

**(2)**

**7.** Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model $A$ assumes that her annual salary will increase by the same amount each year.

(a) According to model $A$, determine Kim's annual salary in year 2.

**(3)**

Model $B$ assumes that her annual salary will increase by the same percentage each year.

(b) According to model $B$, determine Kim's annual salary in year 2. Give your answer to the nearest £10

**(3)**

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

**(3)**

**2.** The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

**(3)**

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

**(3)**

**5.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of $p$, giving your answer to 2 decimal places.

**(3)**

According to the model, at the end of $N$ years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of $N$.

**(5)**

**8.** (i) An arithmetic series has first term $a$ and common difference $d$.

Prove that the sum to $n$ terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\}$$

**(3)**

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) $u_5$

**(1)**

(b) $\sum_{n=1}^{59} u_n$

**(3)**

**5.** Ben is saving for the deposit for a house over a period of 60 months.

Ben saves £100 in the first month and in each subsequent month, he saves £5 more than the previous month, so that he saves £105 in the second month, £110 in the third month, and so on, forming an arithmetic sequence.

(a) Find the amount Ben saves in the 40th month.

**(2)**

(b) Find the total amount Ben saves over the 60-month period.

**(3)**

Lina is also saving for a deposit for a house.

Lina saves £600 in the first month and in each subsequent month, she saves £10 less than the previous month, so that she saves £590 in the second month, £580 in the third month, and so on, forming an arithmetic sequence.

Given that, after $n$ months, Lina will have saved exactly £18 200 for her deposit,

(c) form an equation in $n$ and show that it can be written as

$$n^2 - 121n + 3640 = 0$$

**(3)**

(d) Solve the equation in part (c).

**(2)**

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible value for $n$.

**(1)**

**8.** A geometric series has first term $a$ and common ratio $r$.

(a) Prove that the sum of the first $n$ terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

**(3)**

The second term of a geometric series is –320 and the fifth term is $\frac{512}{25}$

(b) Find the value of the common ratio.

**(2)**

(c) Hence find the sum of the first 13 terms of the series, giving your answer to 2 decimal places.

**(3)**

**8.** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where $p$ is a constant.

(a) Find an expression for $a_2$ in terms of $p$, giving your answer in simplest form.

**(1)**

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of $a_2$

**(6)**

**10. In this question you must show detailed reasoning.**

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan $A$, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan $A$.

**(3)**

In training plan $B$, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan $B$.
Give your answer in km to one decimal place.

**(3)**

Owen will also run a fixed distance, $x$ km, each Wednesday over the 12-week period.

Given that

- $x$ is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of $x$, if he uses training plan $A$,

(ii) find the maximum value of $x$, if he uses training plan $B$.

**(5)**

**1.** Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of $n$, for the amount, in pounds (£), saved in week $n$.

**(3)**

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

**(4)**

**2.** A sequence is defined by

$$u_1 = 6$$
$$u_{n+1} = ku_n + 3$$

where $k$ is a positive constant.

(a) Find, in terms of $k$, an expression for $u_3$

**(2)**

Given that $\sum_{n=1}^{3} u_n = 117$

(b) find the value of $k$.

**(3)**

5. A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

(a) Find the mass of silver extracted in year 14

**(2)**

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year $N$,

(b) show that

$$N^2 - 65N + 1036 = 0$$

**(3)**

(c) Hence, state the value of $N$.

**(1)**

**7.** (i) A geometric sequence has first term 4 and common ratio 6
Given that the $n^{\text{th}}$ term is greater than $10^{100}$, find the minimum possible value of $n$.
**(3)**

(ii) A different geometric sequence has first term $a$ and common ratio $r$.

Given that

- the second term of the sequence is −6
- the sum to infinity of the series is 25

(a) show that

$$25r^2 - 25r - 6 = 0$$

**(3)**

(b) Write down the solutions of

$$25r^2 - 25r - 6 = 0$$

**(1)**

Hence,

(c) state the value of $r$, giving a reason for your answer,
**(1)**

(d) find the sum of the first 4 terms of the series.
**(2)**

**8.** A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.
**(1)**

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.
**(2)**

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio $r$
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of $r$, giving your answer to 3 decimal places.
**(2)**

After a total of $N$ hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of $N$.
**(4)**

**6.** In a geometric sequence $u_1,\ u_2,\ u_3,\ \ldots$

- the common ratio is $r$
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that $r$ satisfies

$$3r^2 - 4r - 4 = 0$$

**(3)**

Given that the geometric sequence has a sum to infinity,

(b) find $u_1$

**(3)**

(c) find $S_\infty$

**(2)**

**3.** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

(a) (i) $a_1$

(ii) $a_2$

(iii) $a_3$

**(3)**

(b) Hence find the exact value of

$$\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\}$$

You must make your method clear.

**(4)**

**8.** A geometric sequence has first term $a$ and common ratio $r$

Given that $S_\infty = 3a$

(a) show that $r = \frac{2}{3}$

**(2)**

Given also that

$$u_2 - u_4 = 16$$

where $u_k$ is the $k^{\text{th}}$ term of this sequence,

(b) find the value of $S_{10}$ giving your answer to one decimal place.

**(5)**

**7.** A geometric sequence has first term $a$ and common ratio $r$, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

**(1)**

(b) Hence find the value of $a$.

**(2)**

Given that the sum of the first $n$ terms of this sequence is greater than 156

(c) find the smallest possible value of $n$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(4)**

**6.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where $k$ is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

**(2)**

(b) predict the number of downloads in the 4th month.

**(4)**

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the $N$th month.

(c) Calculate the value of $N$ according to the model.

**(3)**

**11.** A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = b - au_n$$

$$u_1 = 3$$

where $a$ and $b$ are constants.

(a) Find, in terms of $a$ and $b$,

(i) $u_2$

(ii) $u_3$

**(2)**

Given

- $\sum_{n=1}^{3} u_n = 153$
- $b = a + 9$

(b) show that

$$a^2 - 5a - 66 = 0$$

**(3)**

(c) Hence find the larger possible value of $u_2$

**(3)**

**2.** A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}$$

(a) Find the value of $u_2$, the value of $u_3$ and the value of $u_4$

**(3)**

(b) Find the value of

$$\sum_{r=1}^{100} u_r$$

**(2)**

**8.** In a large theatre there are $n$ rows of seats, where $n$ is a constant.

The number of seats in the first row is $a$, where $a$ is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of $a$.

**(2)**

Given also that the total number of seats in the $n$ rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

**(2)**

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

**(3)**

**5.** (i) Find the value of

$$\sum_{r=1}^{\infty} 6 \times (0.25)^{r}$$

**(3)**

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$
$$u_{n+1} = \frac{u_n - 3}{u_n - 2} \qquad n \in \mathbb{N}$$

(a) Show that this sequence is periodic.

**(2)**

(b) State the order of this sequence.

**(1)**

(c) Hence find

$$\sum_{n=1}^{70} u_n$$

**(2)**

**7.** Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4000 tonnes of wheat.

Model $A$ assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model $A$, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.
**(3)**

Model $B$ assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model $B$, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.
**(3)**

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.
**(3)**

**2.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is –80

For this series,

(a) find the value of the first term and the value of the common difference.

**(4)**

(b) Hence find the smallest value of $n$ for which

$$S_n > 8000$$

**(3)**

**10.** **In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.**

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

**(2)**

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where $N$ is the number of voles, $t$ years after monitoring began and $a$ and $b$ are constants. Give the value of $a$ and the value of $b$ to 2 significant figures.

**(3)**

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of $T$, giving your answer to one decimal place.

**(3)**

**1.** The sequence $u_1, u_2, u_3, \dots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

(a) find the value of $u_2$

**(2)**

(b) find $\sum_{r=1}^{4}(u_r + 2r)$

**(3)**

**8.** (i) (a) In an **arithmetic** series the first term is $a$ and the common difference is $d$.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

**(3)**

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

**(3)**

(ii) Given that the first three terms of a **geometric** series are

$$k + 4 \qquad k - 2 \qquad 11 - k$$

where $k$ is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

**(3)**

Given also that this series is convergent,

(b) find the value of $S_\infty$

**(4)**

**2.** A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7$$

$$u_{n+1} = (-1)^n u_n + k$$

where $k$ is a constant.

(a) Show that $u_5 = 7$

**(3)**

Given that $\sum_{r=1}^{4} u_r = 30$

(b) find the value of $k$.

**(2)**

(c) Hence find the value of $\sum_{r=1}^{150} u_r$

**(2)**

**7.** Jem pays money into a savings scheme, $A$, over a period of 300 months.

Jem pays £20 into scheme $A$ in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme $A$ in month 100

**(1)**

(b) Find the **total** amount that Jem pays into scheme $A$ over the period of 300 months.

**(2)**

Kim pays money into a different savings scheme, $B$, over the same period of 300 months.

In a model, the amounts Kim pays into scheme $B$ increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme $B$ in month 1
- £250 into scheme $B$ in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme $A$ and the total amount paid into scheme $B$ over the period of 300 months.

**(3)**

**1.** The arithmetic series $S$ is given by

$$S = 2 + 5 + 8 + 11 + \dots\dots\dots + 254$$

Find

(a) the number of terms in the series,

**(2)**

(b) the sum of the series.

**(2)**

**4.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

**(4)**

(b) the first term.

**(2)**