P2 Chapter 3 Exponential and Logarithms

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7. (i) Find, in terms of $\log_2 3$, an exact solution of the equation

$$3 \times 2^{x-2} = 8^x$$

(4)

(ii) Solve, using algebra, the equation

$$2\log_5(2y + 1) - \log_5(2 - y) = 1$$

explaining clearly why there is only one solution.

(4)

6. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a. Give your answer in its simplest form.

(3)

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P.

(b) Show that the x coordinate of P is $\dfrac{\log_{10} 2}{\log_{10} 24}$

(5)

9. (i) Find the exact value of x for which

$$\log_3(x + 5) - 4 = \log_3(2x - 1)$$

(4)

(ii) Given that

$$3^{y+3} \times 2^{1-2y} = 108$$

(a) show that

$$0.75^y = 2$$

(4)

(b) Hence find the value of y, giving your answer to 3 decimal places.

(2)

3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

7. (a) Given that

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

4. **In this question you must show all stages of your working.**

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$a - b = 8$$
$$\log_4 a + \log_4 b = 3$$

(6)

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

4. The weight of a baby mammal is monitored over a 16-month period.

The weight of the mammal, w kg, is given by

$$w = \log_a(t + 5) - \log_a 4 \qquad 2 \leqslant t \leqslant 18$$

where t is the age of the mammal in months and a is a constant.

Given that the weight of the mammal was 10 kg when $t = 3$

(a) show that $a = 1.072$ correct to 3 decimal places.

(3)

Using $a = 1.072$

(b) find an equation for t in terms of w

(3)

(c) find the value of t when $w = 15$, giving your answer to 3 significant figures.

(2)

10. Given $a = \log_2 3$

(i) write, in simplest form, in terms of a,

(a) $\log_2 9$

(b) $\log_2\left(\frac{\sqrt{3}}{16}\right)$

(3)

(ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a.

(4)

4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1, a > 1, y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x + 1) = 3 + \log_2(x + 6)$$

(5)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve

$$3^a = 70$$

giving the answer to 3 decimal places.

(2)

(ii) Find the exact value of b such that

$$4 + 3\log_3 b = \log_3 5b$$

(4)

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that –1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve

$$2\log_2(2-x) = 4 + \log_2(x+10)$$

(5)

(ii) Find the value of

$$\log_{\sqrt{a}} a^6$$

where a is a positive constant greater than 1

(1)

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)