# Chapter 8: Moments

Organised by Mr Omar Faruk

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

**4.**

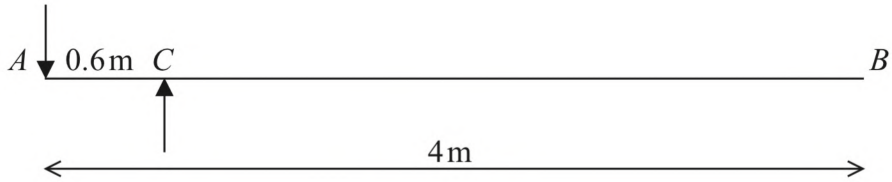


**Figure 1**

A diving board *AB* consists of a wooden plank of length 4 m and mass 30 kg. The plank is held at rest in a horizontal position by two supports at the points *A* and *C*, where $AC = 0.6$ m, as shown in Figure 1. The force on the plank at *A* acts vertically downwards and the force on the plank at *C* acts vertically upwards.

A diver of mass 50 kg is standing on the board at the end *B*. The diver is modelled as a particle and the plank is modelled as a uniform rod. The plank is in equilibrium.

(a) Find

(i) the magnitude of the force acting on the plank at *A*,

(ii) the magnitude of the force acting on the plank at *C*.

**(6)**

The support at *A* will break if subjected to a force whose magnitude is greater than 5000 N.

(b) Find, in kg, the greatest integer mass of a diver who can stand on the board at *B* without breaking the support at *A*.

**(3)**

(c) Explain how you have used the fact that the diver is modelled as a particle.

**(1)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**4.**

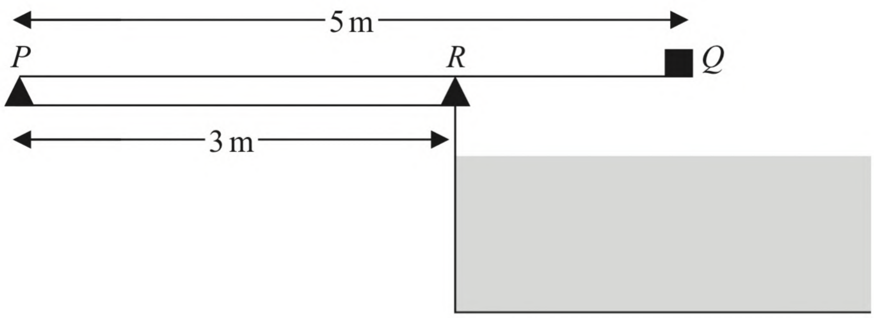


**Figure 2**

A boy sees a box on the end $Q$ of a plank $PQ$ which overhangs a swimming pool. The plank has mass 30 kg, is 5 m long and rests in a horizontal position on two bricks. The bricks are modelled as smooth supports, one acting on the rod at $P$ and one acting on the rod at $R$, where $PR = 3$ m. The support at $R$ is on the edge of the swimming pool, as shown in Figure 2. The boy has mass 40 kg and the box has mass 2.5 kg. The plank is modelled as a uniform rod and the boy and the box are modelled as particles.

The boy steps on to the plank at $P$ and begins to walk slowly along the plank towards the box.

(a) Find the distance he can walk along the plank from $P$ before the plank starts to tilt.
**(4)**

(b) State how you have used, in your working, the fact that the box is modelled as a particle.
**(1)**

A rock of mass $M$ kg is placed on the plank at $P$. The boy is then able to walk slowly along the plank to the box at the end $Q$ without the plank tilting. The rock is modelled as a particle.

(c) Find the smallest possible value of $M$.
**(4)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**3.**

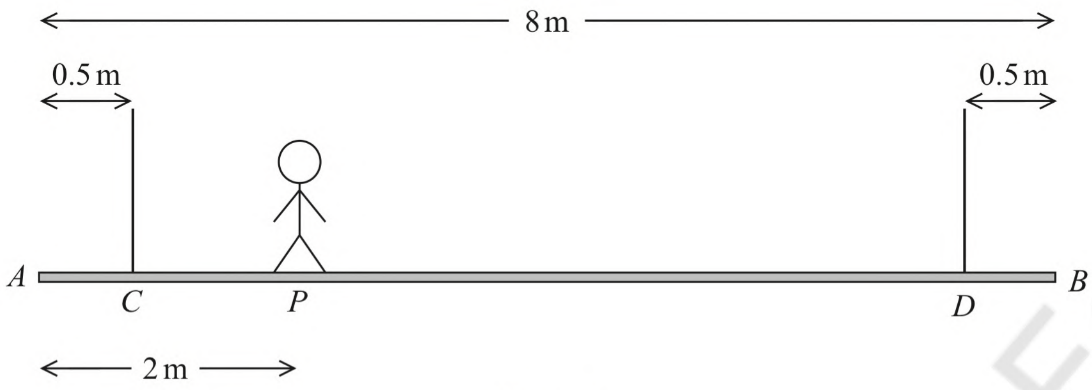


**Figure 2**

A beam $AB$ has mass 100 kg and length 8 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam at $C$ and $D$, where $AC = 0.5$ m and $BD = 0.5$ m. A gymnast of mass $M$ kg stands on the beam at the point $P$, where $AP = 2$ m, as shown in Figure 2. The beam remains horizontal and in equilibrium. The tension in the rope attached to the beam at $D$ is 637 N. The gymnast is modelled as a particle, the beam as a uniform rod and the ropes as light inextensible strings.

(a) Find

(i) the value of $M$,

(ii) the tension in the rope attached to the beam at $C$.

**(6)**

(b) State how you have used the fact that the beam is modelled as a rod.

**(1)**

The gymnast at $P$ now gets off the beam and is replaced by two gymnasts. One gymnast, of mass 60 kg, stands on the beam at $P$ and the other gymnast, of mass 48 kg, stands on the beam at $X$, where $AX = x$ metres. The beam remains horizontal and in equilibrium but the tensions in the two ropes are now equal. The two gymnasts are modelled as particles.

(c) Find the value of $x$.

**(6)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**5.**

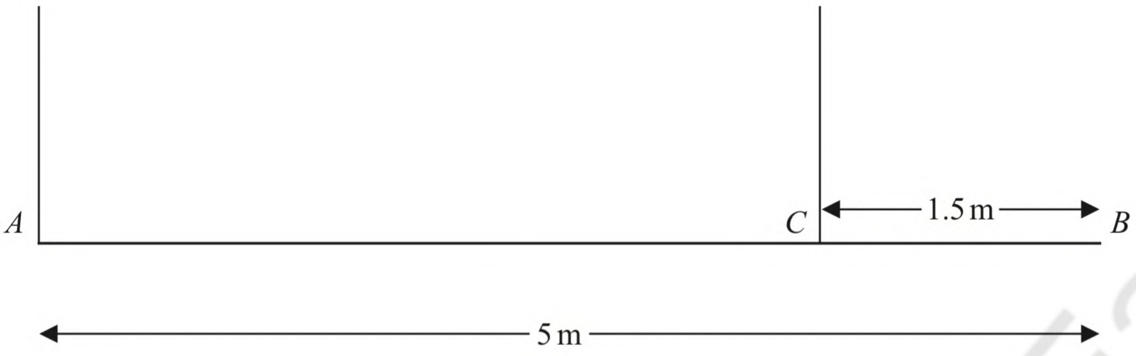


**Figure 2**

A non-uniform beam, $AB$, has length 5 m and mass 12 kg. The beam is suspended in a horizontal position by two vertical ropes. One rope is attached to the beam at $A$. The other rope is attached to the beam at $C$, where $CB = 1.5$ m, as shown in Figure 2. The distance of the centre of mass of the beam from $A$ is 1.75 m. The beam is modelled as a non-uniform rod and the ropes are modelled as light inextensible strings.

A particle of mass $M$ kg is now placed on the beam at $B$ and the beam remains in equilibrium in a horizontal position.

(a) Find the largest possible value of $M$.

**(3)**

The particle at $B$ is now removed and a particle of mass 15 kg is now placed on the beam at the point $D$, where $AD = x$ metres. The beam remains in equilibrium in a horizontal position.

Given that the tension in the rope attached to the beam at $C$ is now twice the tension in the rope attached to the beam at $A$,

(b) find the value of $x$.

**(5)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**2.**

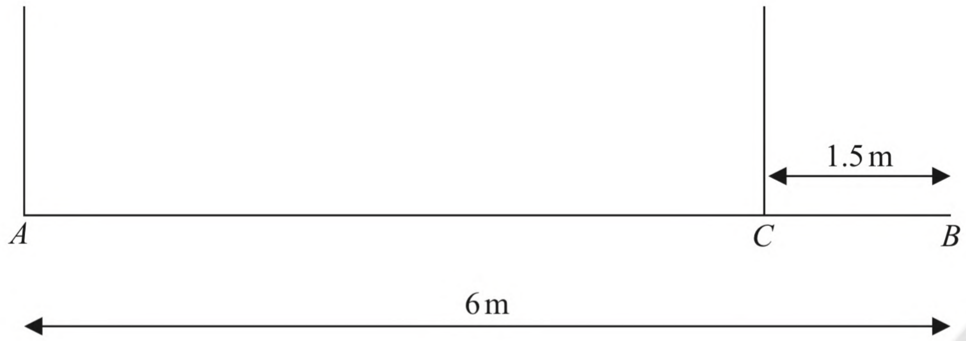


**Figure 1**

A non-uniform beam $AB$ has length 6 m and weight $W$ newtons. The beam is supported in equilibrium in a horizontal position by two vertical ropes, one attached to the beam at $A$ and the other attached to the beam at $C$, where $CB = 1.5$ m, as shown in Figure 1.

The centre of mass of the beam is 2.625 m from $A$.

The ropes are modelled as light strings. The beam is modelled as a non-uniform rod.

Given that the tension in the rope attached at $C$ is 20 N greater than the tension in the rope attached at $A$,

(a) find the value of $W$.

**(6)**

(b) State how you have used the fact that the beam is modelled as a rod.

**(1)**

Organised by Mr Omar Faruk

**4.**

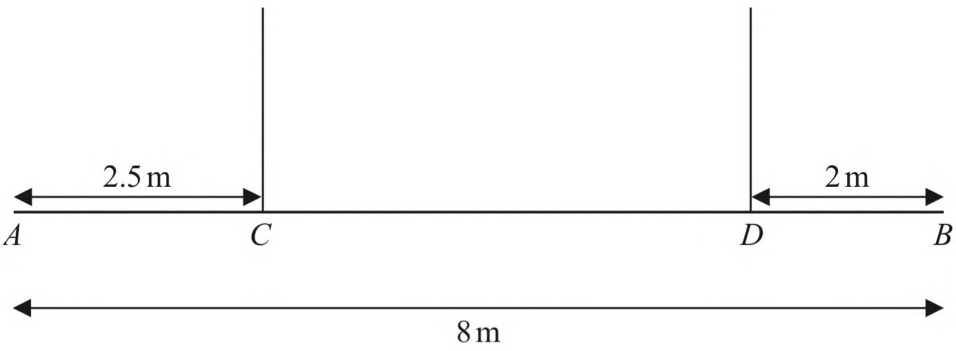


**Figure 2**

A non-uniform beam $AB$ has length 8 m and mass $M$ kg.

The centre of mass of the beam is $d$ metres from $A$.

The beam is supported in equilibrium in a horizontal position by two vertical light ropes. One rope is attached to the beam at $C$, where $AC = 2.5$ m and the other rope is attached to the beam at $D$, where $DB = 2$ m, as shown in Figure 2.

A gymnast, of mass 64 kg, stands on the beam at the point $X$, where $AX = 1.875$ m, and the beam remains in equilibrium in a horizontal position but is now on the point of tilting about $C$.

The gymnast then dismounts from the beam.

A second gymnast, of mass 48 kg, now stands on the beam at the point $Y$, where $YB = 0.5$ m, and the beam remains in equilibrium in a horizontal position but is now on the point of tilting about $D$.

The beam is modelled as a non-uniform rod and the gymnasts are modelled as particles.

Find the value of $M$.

**(8)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**4.**

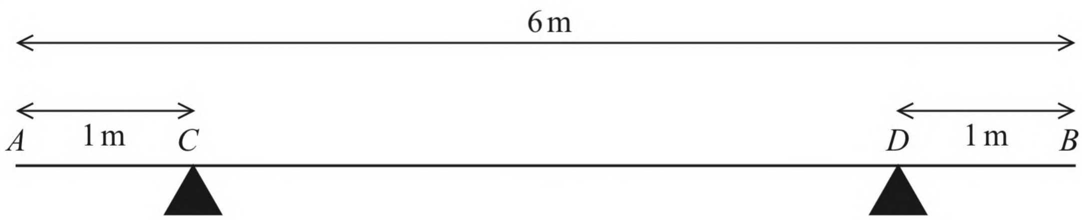


**Figure 2**

A metal girder $AB$ has weight $W$ newtons and length 6 m. The girder rests in a horizontal position on two supports $C$ and $D$ where $AC = DB = 1$ m, as shown in Figure 2.

When a force of magnitude 900 N is applied vertically upwards to the girder at $A$, the girder is about to tilt about $D$.

When a force of magnitude 1500 N is applied vertically upwards to the girder at $B$, the girder is about to tilt about $C$.

The girder is modelled as a non-uniform rod whose centre of mass is a distance $x$ metres from $A$.

Find the value of $x$.

**(6)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**7.**

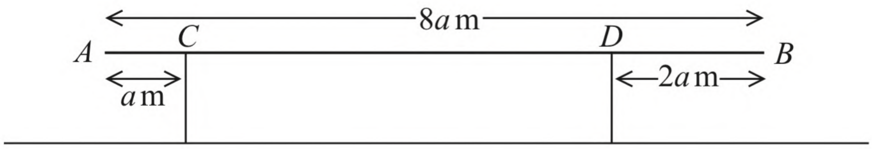


**Figure 2**

A non-uniform beam $AB$, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at $C$, where $AC = a$ metres and the other support is at $D$, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at $D$ is three times the magnitude of the normal reaction between the beam and the support at $C$.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance $x$ metres from $A$,

(a) find an expression for $x$ in terms of $a$.

**(5)**

A box of mass $M$ kg is placed on the beam at $E$, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at $C$ is now equal to the magnitude of the normal reaction between the beam and the support at $D$.

By modelling the box as a particle,

(b) find the value of $M$.

**(5)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**1.**

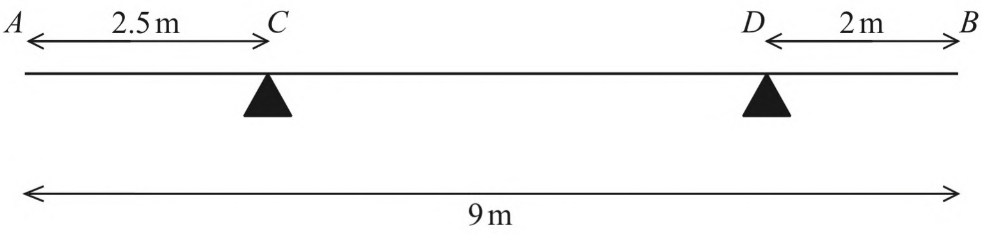


**Figure 1**

A non-uniform rod $AB$ has length 9 m and mass $M$ kg.

The rod rests in equilibrium in a horizontal position on two supports, one at $C$ where $AC = 2.5$ m and the other at $D$ where $DB = 2$ m, as shown in Figure 1.

The magnitude of the force acting on the rod at $D$ is twice the magnitude of the force acting on the rod at $C$.

The centre of mass of the rod is $d$ metres from $A$.

Find the value of $d$.

**(6)**

Organised by Mr Omar Faruk

**3.**

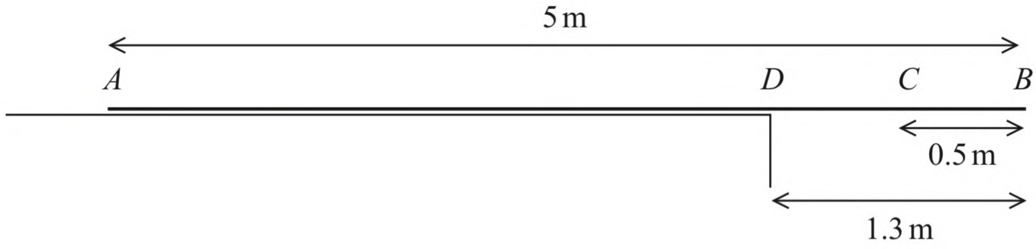


**Figure 2**

A beam *ADCB* has length 5 m. The beam lies on a horizontal step with the end *A* on the step and the end *B* projecting over the edge of the step. The edge of the step is at the point *D* where *DB* = 1.3 m, as shown in Figure 2.

When a small boy of mass 30 kg stands on the beam at *C*, where *CB* = 0.5 m, the beam is on the point of tilting.

The boy is modelled as a particle and the beam is modelled as a uniform rod.

(a) Find the mass of the beam.

**(3)**

A block of mass *X* kg is now placed on the beam at *A*.

The block is modelled as a particle.

(b) Find the smallest value of *X* that will enable the boy to stand on the beam at *B* without the beam tilting.

**(3)**

(c) State how you have used the modelling assumption that the block is a particle in your calculations.

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**5.**

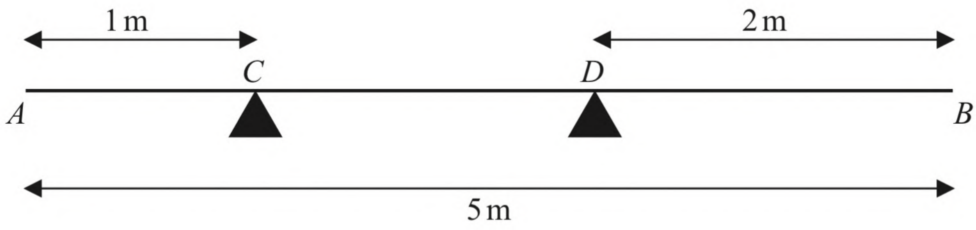


**Figure 2**

A uniform rod $AB$ has length 5 m and mass 5 kg. The rod rests in equilibrium in a horizontal position on two supports $C$ and $D$, where $AC = 1\text{ m}$ and $DB = 2\text{ m}$, as shown in Figure 2.

A particle of mass 10 kg is placed on the rod at $A$ and a particle of mass $M$ kg is placed on the rod at $B$. The rod remains horizontal and in equilibrium.

(a) Find, in terms of $M$, the magnitude of the reaction on the rod at $C$.
**(3)**

(b) Find, in terms of $M$, the magnitude of the reaction on the rod at $D$.
**(3)**

(c) Hence, or otherwise, find the range of possible values of $M$.
**(3)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**2.**


**Figure 1**

A uniform rod $AB$ has length $2a$ and mass $M$. The rod is held in equilibrium in a horizontal position by two vertical light strings which are attached to the rod at $C$ and $D$, where $AC = \frac{2}{5}a$ and $DB = \frac{3}{5}a$, as shown in Figure 1.

A particle $P$ is placed on the rod at $B$.

The rod remains horizontal and in equilibrium.

(a) Find, in terms of $M$, the largest possible mass of the particle $P$

**(3)**

Given that the mass of $P$ is $\frac{1}{2}M$

(b) find, in terms of $M$ and $g$, the tension in the string that is attached to the rod at $C$.

**(3)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**4.**

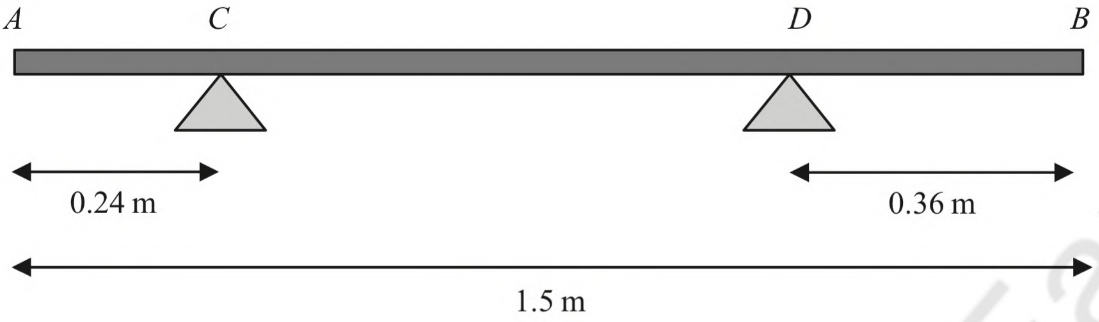


**Figure 1**

A branch $AB$, of length 1.5 m, rests horizontally in equilibrium on two supports.

The two supports are at the points $C$ and $D$, where $AC = 0.24\text{ m}$ and $DB = 0.36\text{ m}$, as shown in Figure 1.

When a force of 150 N is applied vertically upwards at $B$, the branch is on the point of tilting about $C$.

When a force of 225 N is applied vertically downwards at $B$, the branch is on the point of tilting about $D$.

The branch is modelled as a non-uniform rod $AB$ of weight $W$ newtons.

The distance from the point $C$ to the centre of mass of the rod is $x$ metres.

Use the model to find

(i) the value of $W$

(ii) the value of $x$

**(8)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**4.**

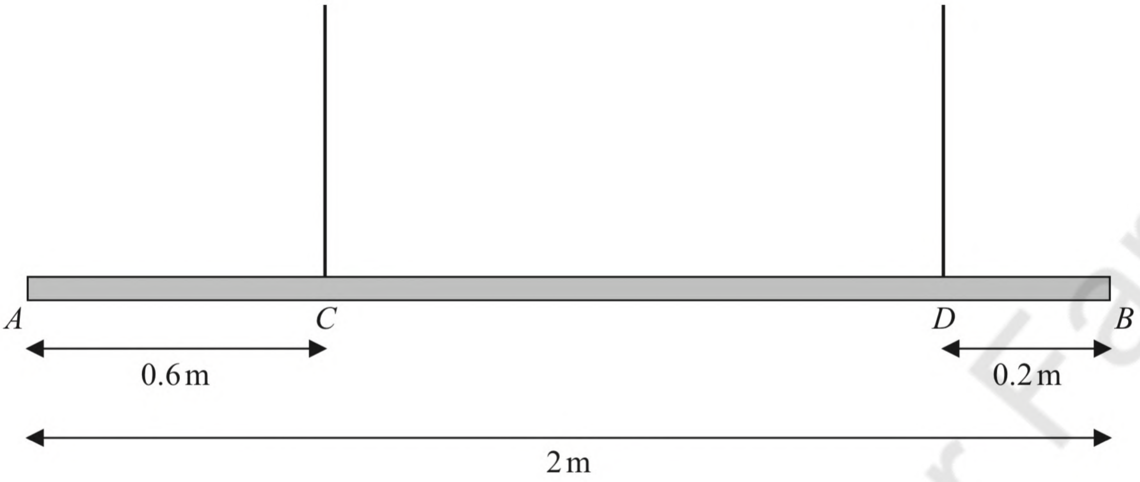


**Figure 1**

Figure 1 shows a beam $AB$, of mass $m$ kg and length 2 m, suspended by two light vertical ropes.
The ropes are attached to the points $C$ and $D$ on the beam, where $AC = 0.6$ m and $DB = 0.2$ m
The beam is in equilibrium in a horizontal position.

A particle of mass $pm$ kg is attached to the beam at $A$ and the beam remains in equilibrium in a horizontal position.

The beam is modelled as a uniform rod.

(a) Given that the tension in the rope attached at $C$ is four times the tension in the rope attached at $D$, use the model to find the exact value of $p$.

**(7)**

The particle of mass $pm$ kg at $A$ is removed and replaced by a particle of mass $qm$ kg at $A$.
The beam remains in equilibrium in a horizontal position but is now on the point of tilting.

(b) Using the model, find the exact value of $q$

**(4)**

(c) State how you have used the modelling assumption that the beam is uniform.

**(1)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**1.**

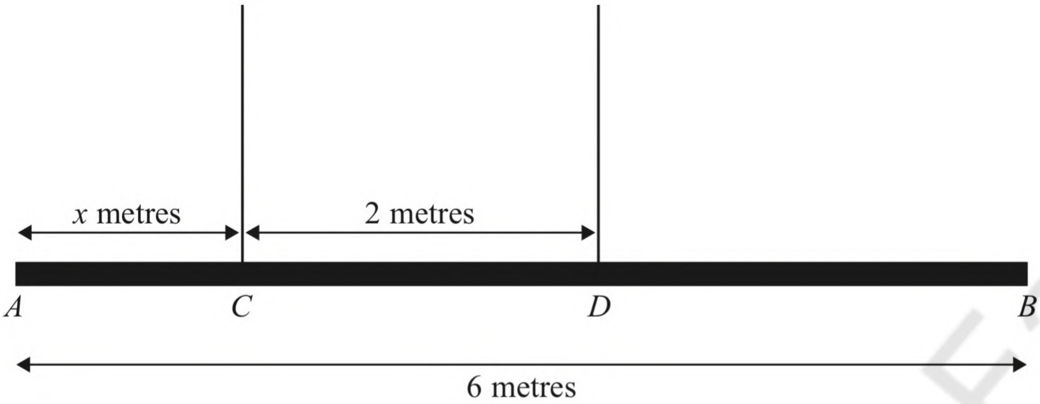


**Figure 1**

Figure 1 shows a beam $AB$ with weight 24 N and length 6 m.

The beam is suspended by two light vertical ropes. The ropes are attached to the points $C$ and $D$ on the beam where $AC = x$ metres and $CD = 2$ m.

The tension in the rope attached to the beam at $C$ is double the tension in the rope attached to the beam at $D$.

The beam is modelled as a uniform rod, resting horizontally in equilibrium.

Find

(i) the tension in the rope attached to the beam at $D$.

(ii) the value of $x$.

**(5)**

Organised by Mr Omar Faruk

**5.**

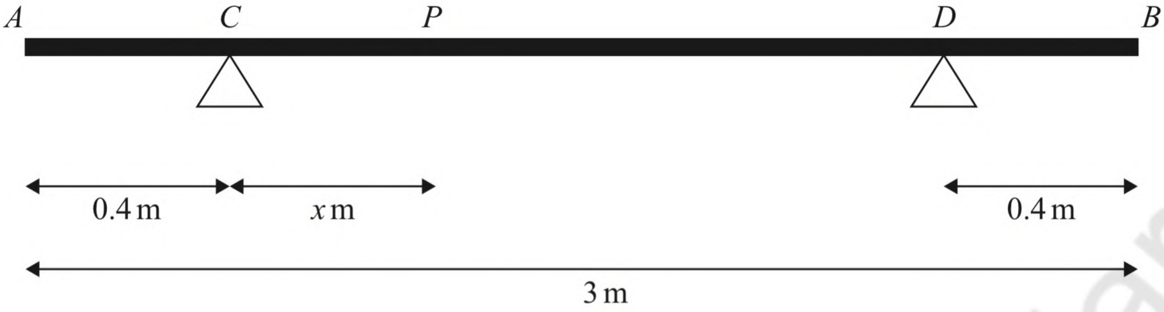


**Figure 4**

A beam $AB$ has mass 30 kg and length 3 m.

The beam rests on supports at $C$ and $D$ where $AC = 0.4$ m and $DB = 0.4$ m, as shown in Figure 4.

A person of mass 55 kg stands on the beam between $C$ and $D$.

The person is modelled as a particle at the point $P$, where $CP = x$ metres and $0 < x < 2.2$

The beam is modelled as a uniform rod resting in equilibrium in a horizontal position.

Using the model,

(a) show that the magnitude of the reaction at $C$ is $(686 - 245x)$ N.

**(3)**

The magnitude of the reaction at $C$ is **four** times the magnitude of the reaction at $D$.

Using the model,

(b) find the value of $x$

**(4)**

The person steps off the beam and places a package of mass $M$ kg at $A$.

The package is modelled as a particle at the point $A$.

The beam is now on the point of tilting about $C$.

Using the model,

(c) find the value of $M$

**(3)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**4.**

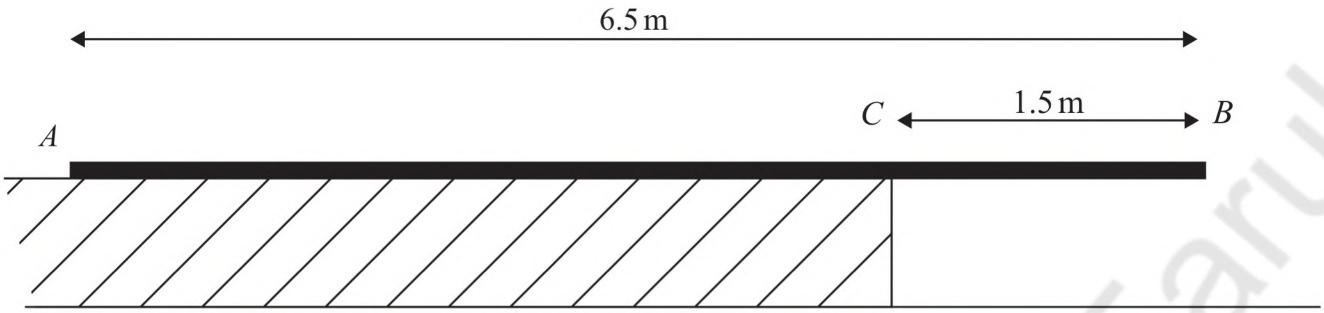


**Figure 2**

A non-uniform rod $AB$ has length 6.5 m and mass 1.2 kg. The centre of mass of the rod is 3 m from $A$. The rod rests on a horizontal step and overhangs the end of the step $C$ by 1.5 m, as shown in Figure 2.

The rod is perpendicular to the edge of the step.

A particle of mass 4 kg is placed on the rod at $B$ and another particle, whose mass is $M$ kg, is placed on the rod at $D$, where $AD = 0.5$ m.

The rod remains in equilibrium in a horizontal position.

(a) Find the smallest possible value of $M$.

**(3)**

The particle at $B$ **and** the particle at $D$ are now **removed**.

A new particle is placed on the rod at the point $E$, where $EB = 0.9$ m.

The rod remains in equilibrium in a horizontal position but is on the point of tilting about $C$.

(b) Find the magnitude of the force acting on the rod at $C$.

**(3)**

Organised by Mr Omar Faruk

Organised by Mr Omar Faruk

**2.**

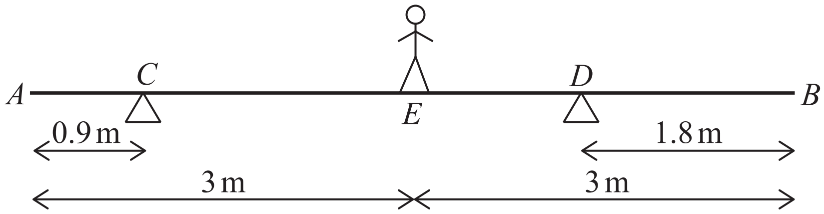


**Figure 1**

A non-uniform beam $AB$ has length 6 m and mass 50 kg. The beam rests horizontally on two supports at $C$ and $D$, where $AC = 0.9$ m and $DB = 1.8$ m.

A child of mass 25 kg stands on the beam at $E$, where $AE = EB = 3$ m, as shown in Figure 1.

The beam is in equilibrium.

The magnitude of the normal reaction between the beam and the support at $C$ is $R_C$ newtons.

The magnitude of the normal reaction between the beam and the support at $D$ is $R_D$ newtons.

The beam is modelled as a rod and the child is modelled as a particle.

The centre of mass of the beam is between $C$ and $D$ and is a distance $x$ metres from $D$.

Given that $2R_D = 3R_C$

(a) show that $x = 1.38$

**(6)**

The child remains at $E$ and a block of mass $M$ kg is placed on the beam at $B$.

The block is modelled as a particle.

Given that the beam is on the point of tilting,

(b) find the value of $M$.

**(3)**

**4.**

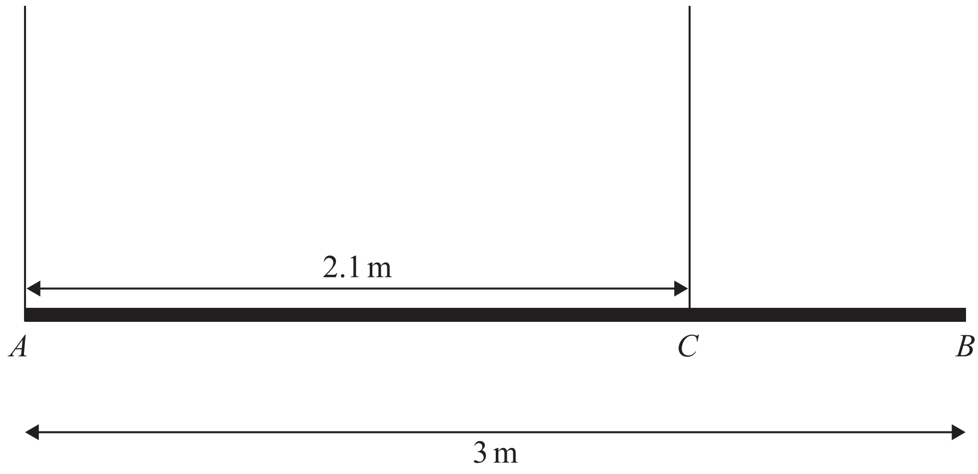


**Figure 2**

A uniform rod *AB* has length 3 m and weight *W* newtons.
The rod is suspended by two light vertical ropes.
One rope is attached to the rod at *A* and the other rope is attached to the rod at *C*, where *AC* = 2.1 m.

The rod is in equilibrium in a horizontal position, as shown in Figure 2.

The tension in the rope at *C* is 350 N.

(a) Show that $W = 490$

**(3)**

A particle *P* of weight 210 N is attached to the rod at a distance *d* metres from *A*.
The tension in the rope at *C* is now 600 N.
The rod remains in equilibrium in a horizontal position.

(b) Find the value of *d*.

**(3)**

Particle *P* is removed from the rod.
A particle *Q* of weight *X* newtons is now attached at *B*.
The rod remains in equilibrium in a horizontal position and is now on the point of tilting.

(c) Find the value of *X*.

**(4)**