Chapter 5: Forces and Friction

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

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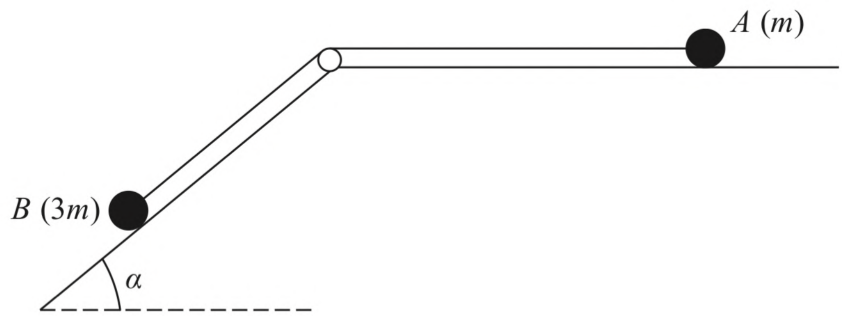

Figure 4

Two particles A and B have masses m and $3m$ respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a rough horizontal table. The coefficient of friction between particle A and the table is $\frac{1}{5}$. The string lies along the table and passes over a small smooth light pulley that is fixed at the edge of the table. Particle B is at rest on a rough plane that is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 4. The coefficient of friction between particle B and the inclined plane is $\frac{1}{3}$. The string lies in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane. The system is released from rest with the string taut and B slides down the inclined plane. Given that A does not reach the pulley,

(a) find the tension in the string,

(11)

(b) state where in your working you have used the fact that the string is modelled as being light,

(1)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

2.

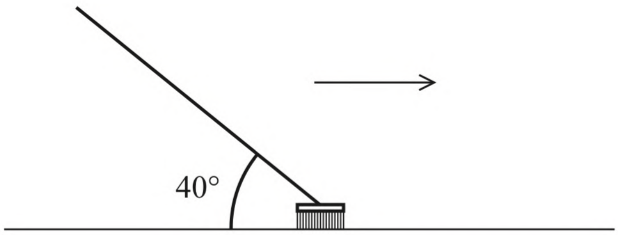

Figure 1

A broom is being used to sweep a rough horizontal floor. The handle of the broom makes a constant angle of 40° with the horizontal, as shown in Figure 1. The broom head is modelled as a particle of mass 0.5 kg and the handle of the broom is modelled as a light rod. The coefficient of friction between the broom head and the floor is $\frac{1}{4}$. The broom head is pushed along the floor in a straight line at constant speed. Find the magnitude of the force that is being applied along the handle of the broom to the broom head.

(6)

4.

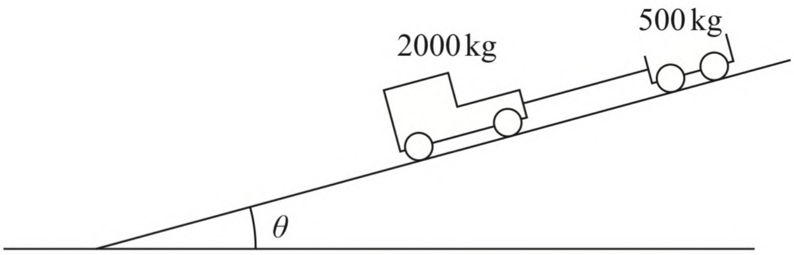

Figure 3

A railway truck of mass 500 kg is pushed up a straight track by a railway engine of mass 2000 kg. The track is inclined to the horizontal at an angle θ, where $\sin\theta = \frac{1}{14}$, as shown in Figure 3. The engine produces a constant driving force of magnitude 3050 N. The truck experiences a constant resistance to motion of magnitude 100 N and the engine experiences a constant resistance to motion of magnitude 200 N. The engine and the truck are connected by a coupling which is modelled as a light rod that is parallel to the track.

Find

(i) the acceleration of the system,

(ii) the magnitude of the force exerted on the truck by the coupling.

(8)

3. A car of mass 800 kg is towing a trailer of mass 400 kg up a straight road using a towbar. The towbar is parallel to the road and parallel to the direction of motion of the car. The road is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{7}$. The engine of the car produces a constant driving force of magnitude D newtons. The resistance to the motion of the car from non-gravitational forces is modelled as a single force of magnitude 420 N. The resistance to the motion of the trailer from non-gravitational forces is modelled as a single force of magnitude 300 N. The car and trailer are modelled as particles and the towbar is modelled as a light rod.

Given that the tension in the towbar is 2060 N, find the value of D.

(7)

7.

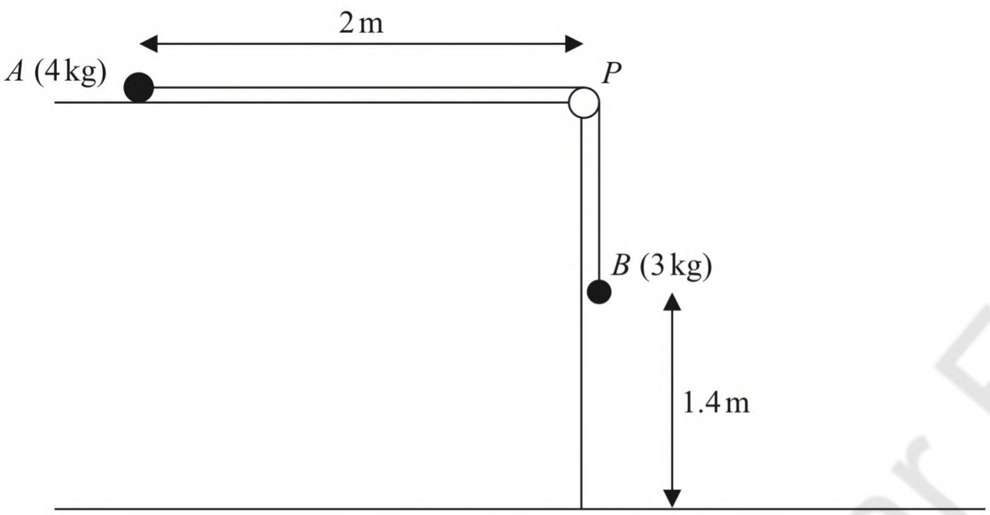

Figure 3

A particle A of mass 4 kg is held at rest on a rough horizontal table. Particle A is attached to one end of a string that passes over a pulley P. The pulley is fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below P.

The part of the string from A to P is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane.

The string is modelled as being light and inextensible and the pulley is modelled as being small, smooth and light.

The system is released from rest with the string taut. At the instant of release, A is 2 m from the edge of the table and B is 1.4 m above a horizontal floor, as shown in Figure 3.

After descending with constant acceleration for 2 seconds, B hits the floor and does not rebound.

(a) Show that the acceleration of A before B hits the floor is $0.7\,\text{m}\,\text{s}^{-2}$

(2)

(b) State which of the modelling assumptions you have used in order to answer part (a).

(1)

(c) Find the magnitude of the resultant force exerted on the pulley by the string.

(4)

The coefficient of friction between A and the table is μ.

(d) Find the value of μ.

(6)

(e) Determine, by calculation, whether or not A reaches the pulley.

(5)

6.

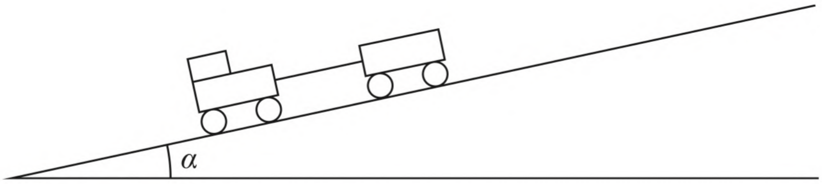

Figure 3

A railway engine of mass 1500 kg is attached to a railway truck of mass 500 kg by a straight rigid coupling. The engine pushes the truck up a straight track, which is inclined to the horizontal at an angle α, where $\sin\alpha = \dfrac{7}{25}$. The coupling is parallel to the track and parallel to the direction of motion, as shown in Figure 3.

The engine produces a constant driving force of magnitude D newtons. The engine and the truck experience constant resistances to motion, from non-gravitational forces, of magnitude 1200 N and 500 N respectively.

The thrust in the coupling is 2000 N.

The coupling is modelled as a light rod.

(a) Find the acceleration of the engine and the truck.

(4)

(b) Find the value of D.

(4)

8.

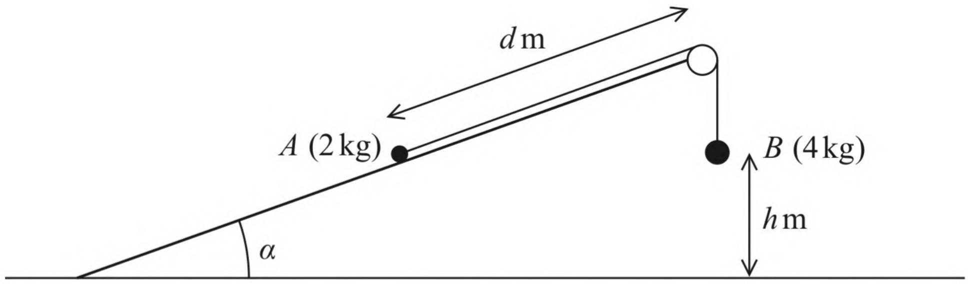

Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

6. A fixed rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \dfrac{5}{12}$

A particle of mass 6 kg is projected with speed 5 $\mathrm{m\,s^{-1}}$ from a point A on the plane, up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is $\dfrac{1}{4}$

(a) Find the magnitude of the frictional force acting on the particle as it moves up the plane.

(3)

The particle comes to instantaneous rest at the point B.

(b) Find the distance AB.

(5)

The particle now slides down the plane from B. At the instant when the particle passes through the point C on the plane, the speed of the particle is again 5 $\mathrm{m\,s^{-1}}$

(c) Find the distance BC.

(5)

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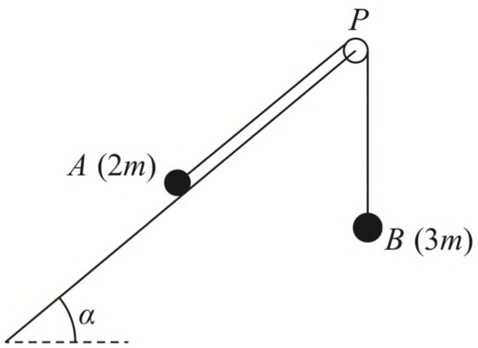

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light pulley P which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

Particle A is held at rest on the plane with the string taut and B hanging freely below P, as shown in Figure 4. The section of the string AP is parallel to a line of greatest slope of the plane.

The coefficient of friction between A and the plane is $\frac{1}{2}$

Particle A is released and begins to move up the plane.

For the motion before A reaches the pulley,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B,

(4)

(b) find, in terms of g, the acceleration of A,

(5)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State how you have used the information that P is a smooth pulley.

(1)

7.

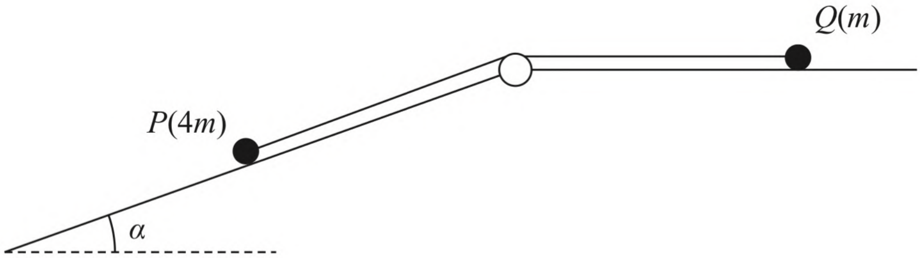

Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

4.

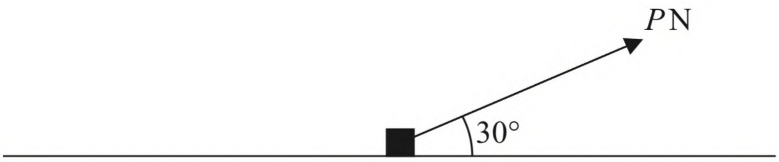

Figure 1

A small block of mass 5 kg lies at rest on a rough horizontal plane.

The coefficient of friction between the block and the plane is $\frac{3}{7}$

A force of magnitude P newtons is applied to the block in a direction which makes an angle of 30° with the plane, as shown in Figure 1.

The block is modelled as a particle.

Given that $P = 14$

(a) find the magnitude of the frictional force exerted on the block by the plane and describe what happens to the block, justifying your answer.

(6)

The value of P is now changed so that the block is on the point of slipping along the plane.

(b) Find the value of P

(6)

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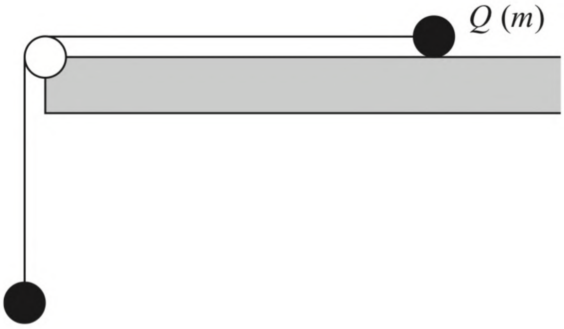

Figure 4

A particle P of mass m is attached to one end of a light inextensible string. Another particle Q, also of mass m, is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P hangs vertically below the pulley with the string taut, as shown in Figure 4.

The pulley, P and Q all lie in the same vertical plane.

The coefficient of friction between Q and the table is μ, where $\mu < 1$

Particle Q is released from rest.

The tension in the string before Q hits the pulley is kmg, where k is a constant.

(a) Find k in terms of μ.

(7)

Given that Q is initially a distance d from the pulley,

(b) find, in terms of d, g and μ, the time taken by Q, after release, to reach the pulley.

(4)

(c) Describe what would happen if $\mu \geqslant 1$, giving a reason for your answer.

(2)

6.

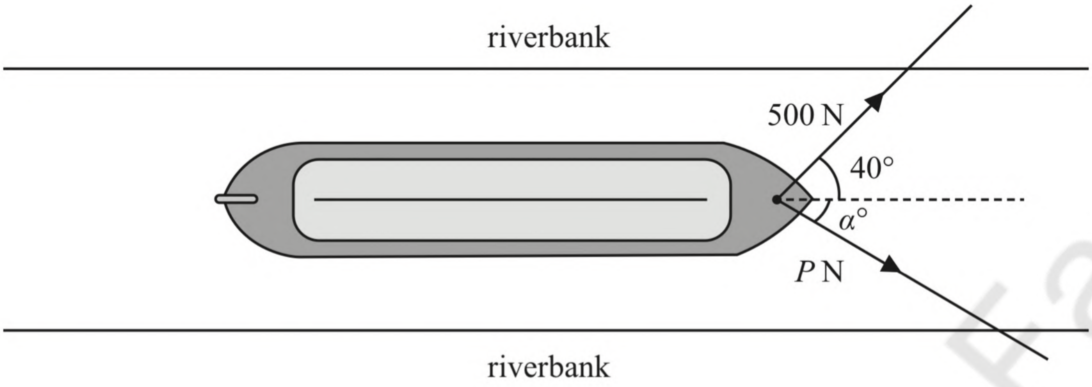

Figure 3

A boat is pulled along a river at a constant speed by two ropes.

The banks of the river are parallel and the boat travels horizontally in a straight line, parallel to the riverbanks.

- The tension in the first rope is 500 N acting at an angle of 40° to the direction of motion, as shown in Figure 3.
- The tension in the second rope is P newtons, acting at an angle of $\alpha°$ to the direction of motion, also shown in Figure 3.
- The resistance to motion of the boat as it moves through the water is a constant force of magnitude 900 N

The boat is modelled as a particle. The ropes are modelled as being light and lying in a horizontal plane.

Use the model to find

(i) the value of α

(ii) the value of P

(8)

8.

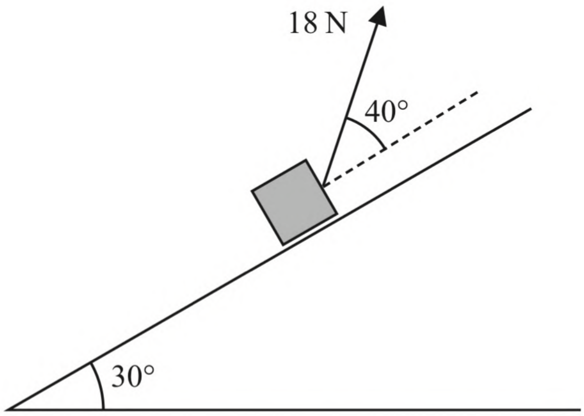

Figure 5

A parcel of mass 2 kg is pulled up a rough inclined plane by the action of a constant force.

The force has magnitude 18 N and acts at an angle of 40° to the plane.

The line of action of the force lies in a vertical plane containing a line of greatest slope of the inclined plane.

The plane is inclined at an angle of 30° to the horizontal, as shown in Figure 5.

The coefficient of friction between the plane and the parcel is 0.3

The parcel is modelled as a particle *P*

(a) Find the acceleration of *P*

(8)

The points *A* and *B* lie on a line of greatest slope of the plane, where $AB = 5\,\text{m}$ and *B* is above *A*. Particle *P* passes through *A* with speed $2\,\text{m}\,\text{s}^{-1}$ in the direction *AB*.

(b) Find the speed of *P* as it passes through *B*.

(3)

The force of 18 N is removed at the instant *P* passes through *B*. As a result, *P* comes to rest at the point *C*.

(c) Determine whether *P* will remain at rest at *C*. You must show all stages of your working clearly.

(4)

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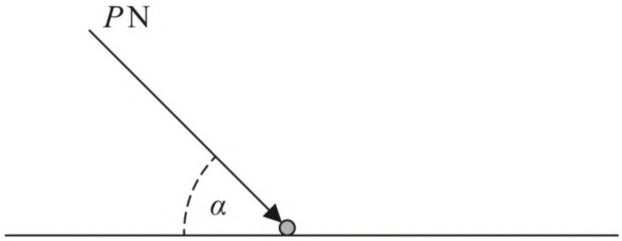

Figure 3

A particle of weight W newtons lies at rest on a rough horizontal surface, as shown in Figure 3.
A force of magnitude P newtons is applied to the particle.

The force acts at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$

The coefficient of friction between the particle and the surface is $\frac{1}{4}$

Given that the particle does not move, show that

$$P \leqslant \frac{5W}{8}$$

(7)

7.

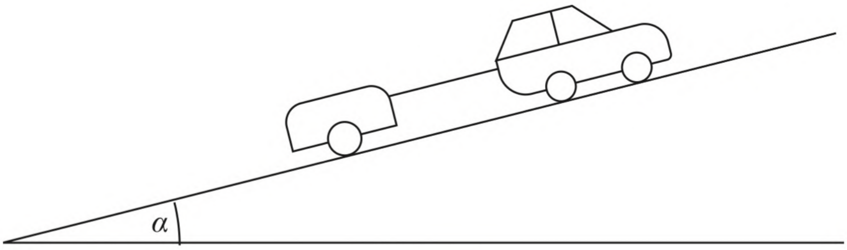

Figure 4

A car of mass 1200 kg is towing a trailer of mass 600 kg up a straight road, as shown in Figure 4.

The road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$

The driving force produced by the engine of the car is 3000 N.

The car moves with acceleration $0.75\,\text{m}\,\text{s}^{-2}$

The non-gravitational resistance to motion of

- the **car** is modelled as a constant force of magnitude $2R$ newtons
- the **trailer** is modelled as a constant force of magnitude R newtons

The car and the trailer are modelled as particles.
The tow bar between the car and trailer is modelled as a light rod that is parallel to the direction of motion.

Using the model,

(a) show that the value of R is 60

(4)

(b) find the tension in the tow bar.

(3)

When the car and trailer are moving at a speed of $12\,\text{m}\,\text{s}^{-1}$, the tow bar breaks.

Given that the non-gravitational resistance to motion of the trailer remains unchanged,

(c) use the model to find the further distance moved by the trailer before it first comes to rest.

(4)

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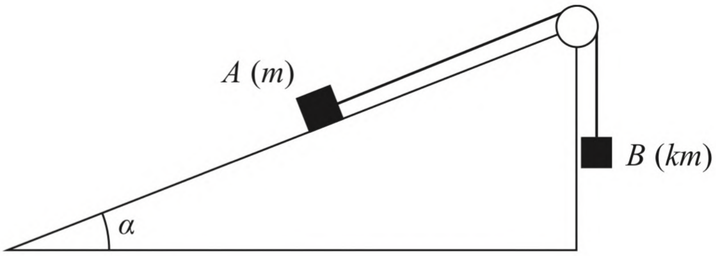

Figure 4

Figure 4 shows a block A of mass m held at rest on a rough plane.
The plane is inclined at an angle α to the horizontal and the coefficient of friction between the block and the plane is μ.

One end of a light inextensible string is now attached to A. The string passes over a small smooth pulley which is fixed at the top of the plane.
The other end of the string is attached to a block B of mass km.
Block B hangs vertically below the pulley, with the string taut.

The string from A to the pulley lies along a line of greatest slope of the plane.

Both A and B are modelled as particles.

When the system is released from rest, A moves up the plane and the tension in the string is $\dfrac{4mg}{3}$

Given that $\mu = \dfrac{1}{3}$ and $\tan\alpha = \dfrac{7}{24}$

(a) (i) find the magnitude of the acceleration of A, giving your answer in terms of g,

(ii) find the value of k.

(9)

(b) Find the magnitude of the resultant force exerted on the pulley by the string, giving your answer in terms of m and g.

(4)

8.

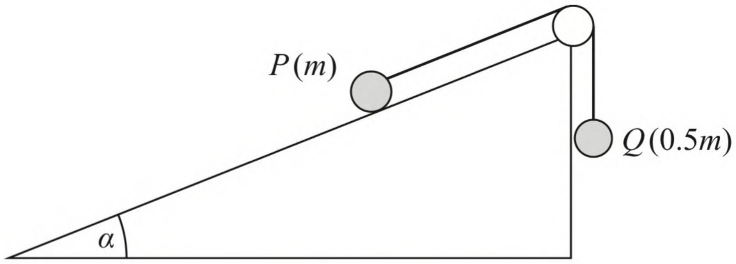

Figure 5

A fixed rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = \dfrac{5}{12}$

A small smooth pulley is fixed at the top of the plane.

One end of a light inextensible string is attached to a particle P which is at rest on the plane. The string passes over the pulley and the other end of the string is attached to a particle Q which hangs vertically below the pulley, as shown in Figure 5.

Particle P has mass m and particle Q has mass $0.5m$

The string from P to the pulley lies along a line of greatest slope of the plane.

The coefficient of friction between P and the plane is μ.

The system is in **limiting equilibrium** with the string taut and P is on the point of slipping **up** the plane.

(a) Find the value of μ.

(8)

The string breaks and P begins to move down the plane.

When particle P has travelled a distance of 0.8 m down the plane, the speed of P is $V\,\mathrm{m\,s^{-1}}$

(b) Find the value of V.

(4)

5. A parachute is used to deliver a box of supplies. The parachute is attached to the box.

- the parachute and box are dropped from rest from a helicopter that is hovering at a height of 520 m above the ground
- the parachute and box fall vertically and freely under gravity for 5 seconds, then the parachute opens
- from the instant the parachute opens, it provides a resistance to motion of magnitude 3200 N
- the parachute and box continue to fall vertically downwards after the parachute opens
- the parachute and box are modelled throughout the motion as a particle P of mass 250 kg

(a) Find the distance fallen by P in the first 5 seconds.

(2)

(b) Find the speed with which P lands on the ground.

(7)

(c) Find the total time from the instant when P is dropped from the helicopter to the instant when P lands on the ground.

(3)

(d) Sketch a speed-time graph for the motion of P from the instant when P is dropped from the helicopter to the instant when P lands on the ground.

(2)

6.

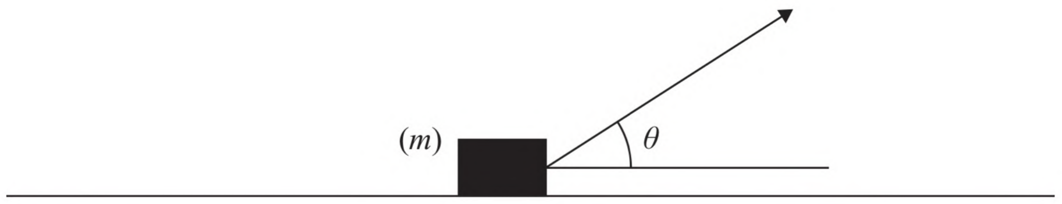

Figure 3

A box of mass m lies on a rough horizontal plane. The box is pulled along the plane in a straight line at **constant speed** by a light rope. The rope is inclined at an angle θ to the plane, as shown in Figure 3.

The coefficient of friction between the box and the plane is $\frac{1}{3}$

The box is modelled as a particle.

Given that $\tan\theta = \frac{3}{4}$

(a) find, in terms of m and g, the tension in the rope.

(7)

The rope is now removed and the box is placed at rest on the plane.
The box is then projected horizontally along the plane with speed u.

The box is again modelled as a particle.

When the box has moved a distance d along the plane, the speed of the box is $\frac{1}{2}u$.

(b) Find d in terms of u and g.

(5)

8.

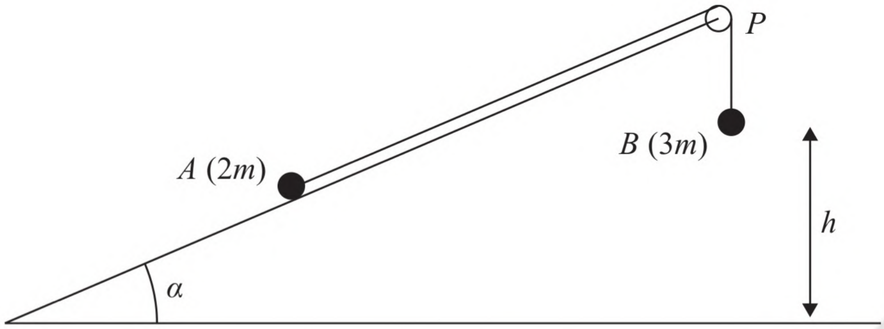

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. Particle A is held at rest on a rough plane which is inclined to horizontal ground at an angle α, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley P which is fixed at the top of the plane. Particle B hangs vertically below P with the string taut, at a height h above the ground, as shown in Figure 4.

The part of the string between A and P lies along a line of greatest slope of the plane. The two particles, the string and the pulley all lie in the same vertical plane.

The coefficient of friction between A and the plane is $\frac{11}{36}$

The particle A is released from rest and begins to move up the plane.

(a) Show that the frictional force acting on A as it moves up the plane is $\frac{22mg}{39}$ **(3)**

(b) Write down an equation of motion for B. **(2)**

(c) Show that the acceleration of A immediately after its release is $\frac{1}{3}g$ **(4)**

In the subsequent motion, A comes to rest before it reaches the pulley.

(d) Find, in terms of h, the total distance travelled by A from when it was released from rest to when it first comes to rest again. **(6)**