Chapter 4: Dynamic of a Moving Particle in a Straight Line

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5. Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle A.

$\mathbf{F}_1 = (2\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants.

Given that the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is parallel to $(\mathbf{i} + 2\mathbf{j})$,

(a) show that $2p - q + 7 = 0$

(5)

Given that $q = 11$ and that the mass of A is 2 kg, and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on A,

(b) find the magnitude of the acceleration of A.

(5)

3.

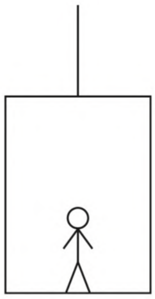

Figure 1

A lift of mass M kg is being raised by a vertical cable attached to the top of the lift. A person of mass m kg stands on the floor inside the lift, as shown in Figure 1. The lift ascends vertically with constant acceleration $1.4\,\text{m s}^{-2}$. The tension in the cable is 2800 N and the person experiences a constant normal reaction of magnitude 560 N from the floor of the lift. The cable is modelled as being light and inextensible, the person is modelled as a particle and air resistance is negligible.

(a) Write down an equation of motion for the person only.

(2)

(b) Write down an equation of motion for the lift only.

(2)

(c) Hence, or otherwise, find

(i) the value of m,

(ii) the value of M.

(3)

5. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle. The resultant of the two forces is $\mathbf{R}$.

Given that $\mathbf{R}$ has the same direction as the vector $(3\mathbf{i} - 2\mathbf{j})$,

(a) find the size of the angle between $\mathbf{R}$ and the vector $\mathbf{j}$.

(2)

Given that $\mathbf{F}_1 = (a\mathbf{i} + 3\mathbf{j})\,\text{N}$ and $\mathbf{F}_2 = (-4\mathbf{i} + b\mathbf{j})\,\text{N}$,

(b) show that $2a + 3b + 1 = 0$

(4)

8.

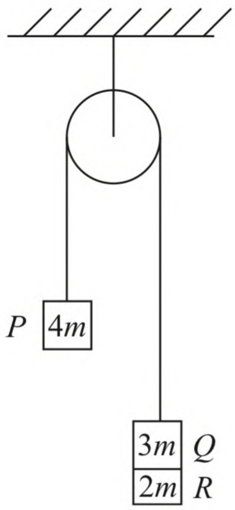

Figure 4

Three particles, P, Q and R, have masses $4m$, $3m$ and $2m$ respectively. Particles P and Q are connected by a light inextensible string that passes over a smooth light fixed pulley. Particle R is attached to particle Q. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 4. The system is released from rest.

(a) Find

(i) the acceleration of particle P,

(ii) the tension in the string.

(7)

(b) State how you have used the fact that the string is inextensible.

(1)

At the instant when particle P has moved a distance d upwards from its initial position, particle R separates from particle Q and falls away. In the subsequent motion, particles P and Q continue to move and particle P does not reach the pulley.

At the instant when particles R and Q separate, particle Q is at the point A, and it continues to move downwards. Particle Q then comes to instantaneous rest at the point B.

(c) Find, in terms of d, the distance AB.

(8)

7. Two forces, **F** and **G**, act on a particle. The force **F** has magnitude 4 N and acts in a direction with a bearing of 120° and the force **G** has magnitude 6 N and acts due north.

Given that $\mathbf{P} = 2\mathbf{F} + \mathbf{G}$, find

(i) the magnitude of **P**

(ii) the direction of **P**, giving your answer as a bearing to the nearest degree.

(7)

6. A force $\mathbf{F}$ is given by $\mathbf{F} = (10\mathbf{i} + \mathbf{j})$ N.

(a) Find the exact value of the magnitude of $\mathbf{F}$.

(2)

(b) Find, in degrees, the size of the angle between the direction of $\mathbf{F}$ and the direction of the vector $(\mathbf{i} + \mathbf{j})$.

(4)

The resultant of the force $\mathbf{F}$ and the force $(-15\mathbf{i} + a\mathbf{j})$ N, where a is a constant, is parallel to, but in the opposite direction to, the vector $(2\mathbf{i} - 3\mathbf{j})$.

(c) Find the value of a.

(5)

7.

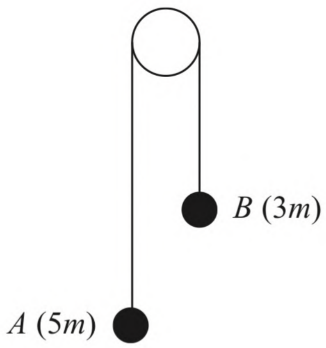

Figure 4

One end of a light inextensible string is attached to a particle A of mass $5m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light fixed pulley. Particle A is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 4.

Particle A is released.

(a) Find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while A is falling and before B hits the pulley.

(8)

(b) State how, in your solution to part (a), you have used the fact that the pulley is smooth.

(1)

5. A particle is acted upon by two forces **F** and **G**. The force **F** has magnitude 8 N and acts in a direction with a bearing of 240°. The force **G** has magnitude 10 N and acts due South.

Given that **R** = **F** + **G**, find

(i) the magnitude of **R**,

(ii) the direction of **R**, giving your answer as a bearing to the nearest degree.

(7)

3. [*In this question* **i** *and* **j** *are perpendicular horizontal unit vectors.*]

Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, are given by

$$\mathbf{F}_1 = (5\mathbf{i} + 2\mathbf{j})\,\text{N} \qquad \mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})\,\text{N} \qquad \mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})\,\text{N}$$

where a and b are constants.

The forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle P of mass 4 kg.

Given that P rests in equilibrium on a smooth horizontal surface under the action of these three forces,

(a) find the size of the angle between the direction of $\mathbf{F}_3$ and the direction of $-\mathbf{j}$.

(4)

The force $\mathbf{F}_3$ is now removed and replaced by the force $\mathbf{F}_4$ given by $\mathbf{F}_4 = \lambda(\mathbf{i} + 3\mathbf{j})\,\text{N}$, where λ is a positive constant.

When the three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ act on P, the acceleration of P has magnitude $3.25\ \text{m s}^{-2}$

(b) Find the value of λ.

(5)

4.

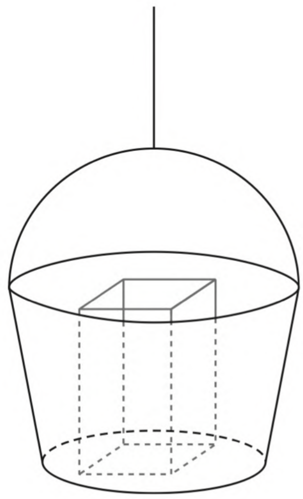

Figure 1

Figure 1 shows a large bucket used by a crane on a building site to move materials between the ground and the top of the building. The mass of the bucket is 15 kg.

The bucket is attached to a vertical cable with the bottom of the bucket horizontal. The cable is modelled as light and inextensible.

When the bucket is on the ground, a bag of cement of mass 25 kg is placed in the bucket.

The bucket with the bag of cement moves vertically upwards with constant acceleration $0.2\,\text{m}\,\text{s}^{-2}$. Air resistance is modelled as being negligible.

(a) Find the tension in the cable.

(3)

At the top of the building, the bag of cement is removed. A box of tools of mass 12 kg is now placed in the bucket.

Later on the bucket with the box of tools is moving vertically downwards with constant deceleration $0.1\,\text{m}\,\text{s}^{-2}$. Air resistance is again modelled as being negligible.

(b) Find the magnitude of the normal reaction between the bucket and the box of tools.

(3)

3. A tractor of mass 6 tonnes is dragging a large block of mass 2 tonnes along rough horizontal ground. The cable connecting the tractor to the block is horizontal and parallel to the direction of motion.

The cable is modelled as being light and inextensible.

The driving force of the tractor is 7400 N and the resistance to the motion of the tractor is 200 N. The resistance to the motion of the block is R newtons, where R is a constant.

Given that the tension in the cable is 6000 N and the tractor is accelerating,

(a) find the value of R.

(6)

(b) State how you have used the fact that the cable is modelled as being inextensible.

(1)

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle A of mass 0.5 kg is at rest on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants, are applied to A.

Given that A moves in the direction of the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) show that $2p + q - 4 = 0$

(4)

Given that $p = 5$

(b) find the speed of A at time $t = 4$ seconds.

(5)

4.

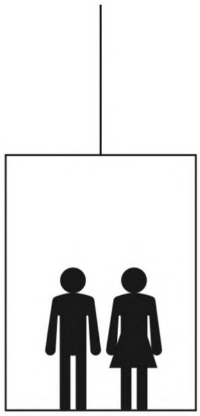

Figure 3

Two children, Alan and Bhavana, are standing on the horizontal floor of a lift, as shown in Figure 3.

The lift has mass 250 kg. The lift is raised vertically upwards with constant acceleration by a vertical cable which is attached to the top of the lift. The cable is modelled as being light and inextensible. While the lift is accelerating upwards, the tension in the cable is 3616 N.

As the lift accelerates upwards, the floor of the lift exerts a force of magnitude 565 N on Alan and a force of magnitude 226 N on Bhavana.

Air resistance is modelled as being negligible and Alan and Bhavana are modelled as particles.

(a) By considering the forces acting on the lift only, find the acceleration of the lift.

(3)

(b) Find the mass of Alan.

(3)

7.

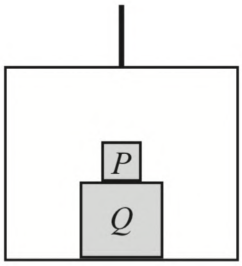

Figure 4

A simple lift operates by means of a vertical cable which is attached to the top of the lift.
The lift has mass m
A box Q is placed on the floor of the lift.
A box P is placed directly on top of box Q, as shown in Figure 4.

The cable is modelled as being light and inextensible and air resistance is modelled as being negligible.

The tension in the cable is $\frac{42mg}{5}$

The lift and its contents move vertically upwards with acceleration $\frac{2g}{5}$

Using the model,

(a) find, in terms of m, the combined mass of boxes P and Q

(4)

During the motion of the lift, the force exerted on box P by box Q is $\frac{14mg}{5}$

Using the model,

(b) find, in terms of m, the mass of box P

(3)

[*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors*.]

2. A particle P rests in equilibrium on a smooth horizontal plane.

A system of **three** forces, $\mathbf{F}_1$ N, $\mathbf{F}_2$ N and $\mathbf{F}_3$ N where

$$\mathbf{F}_1 = (3c\mathbf{i} + 4c\mathbf{j})$$
$$\mathbf{F}_2 = (-14\mathbf{i} + 7\mathbf{j})$$

is applied to P.

Given that P remains in equilibrium,

(a) find $\mathbf{F}_3$ in terms of c, **i** and **j**.

(2)

The force $\mathbf{F}_3$ is **removed** from the system.

Given that $c = 2$

(b) find the size of the angle between the direction of **i** and the direction of the resultant force acting on P.

(4)

The mass of P is m kg.

Given that the magnitude of the acceleration of P is $8.5\,\text{m s}^{-2}$

(c) find the value of m.

(4)

4.

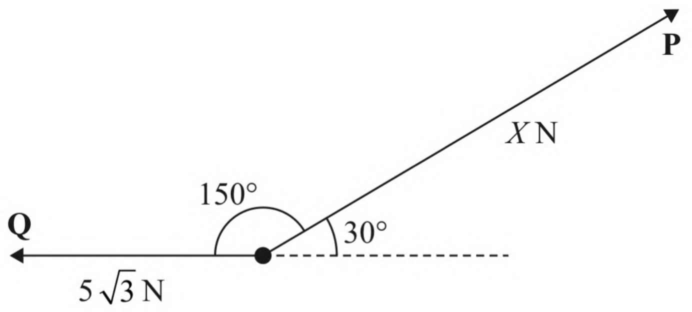

Figure 3

Figure 3 shows two horizontal forces **P** and **Q** acting on a particle.

The angle between the direction of **P** and the direction of **Q** is 150°

Force **P** has magnitude X newtons.

Force **Q** has magnitude $5\sqrt{3}$ N.

The resultant of **P** and **Q** has magnitude $\sqrt{129}$ N.

Find

(i) the value of X.

(ii) the angle between **Q** and the resultant, giving your answer to the nearest degree.

(8)

2. Two forces, **P** and **Q**, act on a particle.

- **P** has magnitude 10 N and acts due west
- **Q** has magnitude 8 N and acts on a bearing of 330°

Given that **F** = **P** + **Q**, find the magnitude of **F**.

(4)

3.

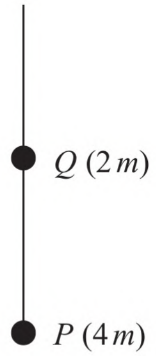

Figure 1

Two particles, P and Q, have masses $4m$ and $2m$ respectively. The particles are connected by a light inextensible string. A second light inextensible string has one end attached to Q. Both strings are taut and vertical, as shown in Figure 1.

The particles are **accelerating** vertically **downwards**.

Given that the tension in the string connecting the two particles is $3mg$, find, in terms of m and g, the tension in the upper string.

(6)

5.

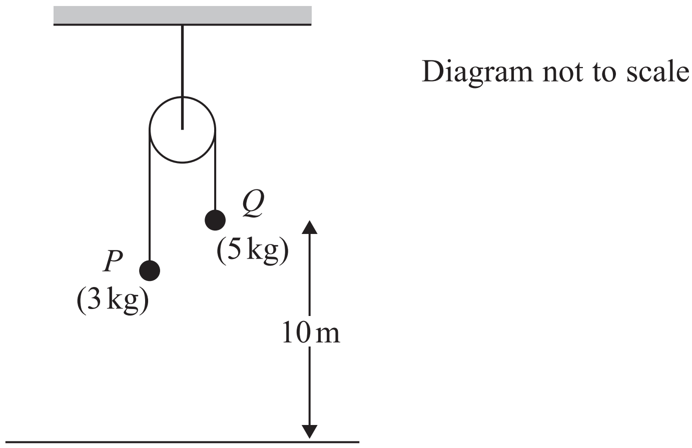

Figure 3

Two particles, P and Q, have masses 3 kg and 5 kg respectively. The particles are connected by a light inextensible string which passes over a small smooth fixed pulley.

The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 3.

Immediately after the particles are released from rest, P moves upwards with acceleration $a\,\text{m s}^{-2}$ and the tension in the string is T newtons.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of T.

(4)

The total force acting on the pulley due to the string has magnitude F newtons.

(c) Find the value of F.

(2)

Initially, Q is 10 m above horizontal ground and P is more than 2 m below the pulley.

At the instant when Q has descended a distance of 2 m, the string breaks and Q falls to the ground.

(d) Find the speed of Q at the instant it hits the ground.

(5)

1. A particle of mass 2.5 kg moves on a smooth horizontal plane under the action of three horizontal forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, where

$$\mathbf{F}_1 = (6\mathbf{i} + 8\mathbf{j})\,\text{N}$$

$$\mathbf{F}_2 = (-16\mathbf{i} + 2\mathbf{j})\,\text{N}$$

$$\mathbf{F}_3 = (-2\mathbf{i} + 8\mathbf{j})\,\text{N}$$

(a) Find the magnitude of the acceleration of the particle.

(4)

A fourth force, $\mathbf{F}_4 = (p\mathbf{i} + p\mathbf{j})\,\text{N}$, where p is a constant, is added.

The resultant of the four forces acts in the direction of the vector $(7\mathbf{i} + 2\mathbf{j})$.

(b) Find the value of p.

(4)

7.

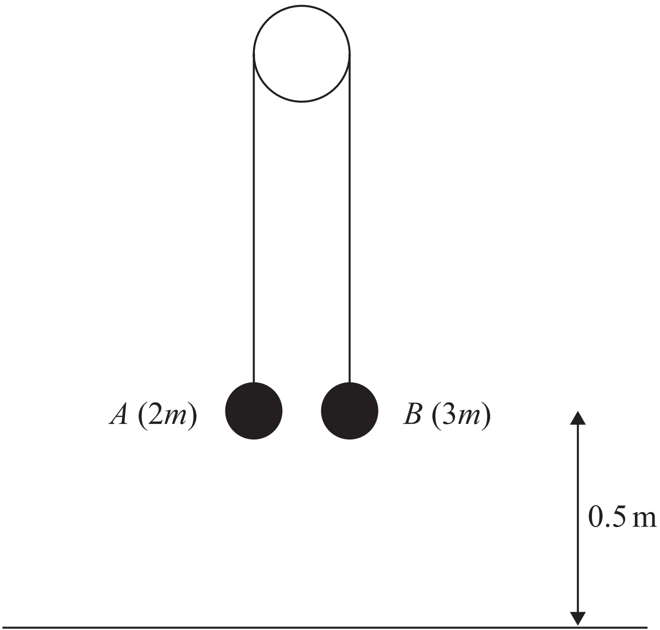

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$.
The other end is attached to a particle B of mass $3m$.
The string passes over a small smooth fixed pulley.
The string is taut and both straight parts of the string are vertical.
Both particles are held at a distance 0.5 m above a horizontal surface, as shown in Figure 4.

The system is released from rest and B moves downwards.

(a) Find the tension in the string in terms of m and g.

(5)

(b) Find the speed of B at the instant it strikes the surface.

(4)

In the subsequent motion, A does not reach the pulley and B does not rebound after it strikes the surface.

(c) Find the time from the instant when the system is released from rest to the instant when A first reaches a height of 1.06 m above the surface.

(6)