Chapter 5: Impulses and Collisions

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1. A ball of mass 0.5 kg is moving with velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1} when it receives an impulse $(4\mathbf{i} + 5\mathbf{j})$ N s. Find the gain in kinetic energy of the ball due to the impulse.

(6)

4. A particle P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal plane. The particle P collides directly with a particle Q, of mass m, which is moving on the plane along the same straight line as P but in the opposite direction to P. Immediately before the collision the speed of Q is $3u$. The coefficient of restitution between P and Q is e, where $e > \frac{1}{8}$

(a) Find, in terms of u and e,

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision.

(7)

(b) Show that, for all possible values of e, the direction of motion of P is reversed by the collision.

(2)

After the collision, Q strikes a smooth fixed vertical wall, which is perpendicular to the direction of motion of Q, and rebounds. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{3}{4}$ and that there is a second collision between Q and P,

(c) find the range of possible values of f.

(4)

June 2018

1. A particle P of mass 0.7 kg is moving with velocity $(\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse. Immediately after receiving the impulse, P is moving with velocity $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$.

(a) Find the impulse.

(3)

(b) Find, in degrees, the size of the angle between the direction of the impulse and the direction of motion of P immediately before receiving the impulse.

(3)

7. Three particles A, B and C have masses $2m$, $3m$ and $4m$ respectively. The particles lie at rest in a straight line on a smooth horizontal surface, with B between A and C. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between A and B is e. The kinetic energy of A immediately after the collision is one ninth of the kinetic energy of A immediately before the collision.

Given that the direction of motion of A is unchanged by the collision,

(a) find the value of e.

(7)

After the collision between A and B there is a direct collision between B and C. The coefficient of restitution between B and C is f, where $f < \frac{3}{4}$. The speed of B immediately after the collision with C is V.

(b) (i) Express V in terms of f and u.

(ii) Hence show that there will be a second collision between A and B.

(7)

October 2018

1.

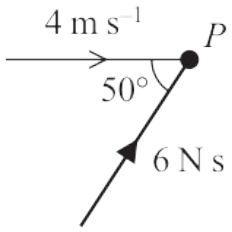

Figure 1

A particle, P, of mass 0.8 kg, moving with speed 4 m s^{-1} along a straight line on a smooth horizontal plane, receives a horizontal impulse of magnitude 6 N s. The angle between the initial direction of motion of P and the direction of the impulse is 50°, as shown in Figure 1.

Find the speed of P immediately after receiving the impulse.

(5)

7. A particle, P, of mass km is moving in a straight line with speed $3u$ on a smooth horizontal surface. Particle P collides directly with another particle, Q, of mass $2m$ which is moving with speed u in the same direction along the same straight line. The coefficient of restitution between P and Q is e.

Given that immediately after the collision P and Q are moving in opposite directions and the speed of Q is $\frac{3}{2}u$,

(a) find the range of possible values of e.

(4)

It is now also given that $e = \frac{7}{8}$.

(b) Show that the kinetic energy lost by P in the collision with Q is $\frac{11}{8}mu^2$.

(6)

The collision between P and Q takes place at the point A. After the collision, Q hits a fixed vertical wall that is perpendicular to the direction of motion of Q. The distance from A to the wall is d. The coefficient of restitution between Q and the wall is $\frac{1}{3}$. Particle Q rebounds from the wall and moves so that P and Q collide directly at the point B.

(c) Find, in terms of d and u, the time interval between the collision at A and the collision at B.

(6)

2. A particle of mass 0.75 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})$ m s^{-1} when it receives an impulse $(-6\mathbf{i} + 4\mathbf{j})$ N s.

Find

(a) the velocity of the particle immediately after receiving the impulse,

(3)

(b) the size of the angle through which the path of the particle is deflected as a result of the impulse.

(3)

7. A particle P of mass $3m$ is moving in a straight line with speed u on a smooth horizontal table. A second particle Q of mass $2m$ is moving with speed $2u$ in the opposite direction to P along the same straight line. Particle P collides directly with Q. The coefficient of restitution between P and Q is e.

(a) Show that the direction of motion of P is reversed as a result of the collision with Q.

(3)

(b) Find the range of values of e for which the direction of motion of Q is also reversed as a result of the collision.

(5)

Given that $e = \frac{1}{2}$

(c) find, in terms of m and u, the kinetic energy lost in the collision between P and Q.

(5)

3.

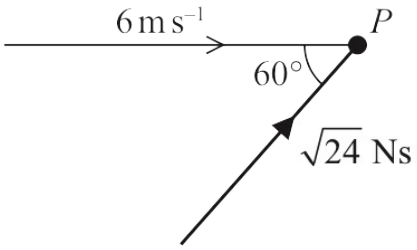

Figure 2

A particle P of mass 0.75 kg is moving along a straight line on a horizontal surface. At the instant when the speed of P is $6\,\text{m s}^{-1}$, it receives an impulse of magnitude $\sqrt{24}$ Ns. The impulse acts in the plane of the horizontal surface. At the instant when P receives the impulse, the line of action of the impulse makes an angle of 60° with the direction of motion of P, as shown in Figure 2.

Find

(i) the speed of P immediately after receiving the impulse,

(ii) the size of the angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse.

(7)

7. Particle A of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal surface. Particle A collides directly with particle B of mass m, which is moving along the same straight line and in the same direction as A.

Immediately before the collision, the speed of B is u.

As a result of the collision, the direction of motion of B is unchanged and the kinetic energy gained by B is $\frac{48}{25}mu^2$

(a) Find the coefficient of restitution between A and B.

(8)

After the collision, B hits a smooth fixed vertical wall that is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is f.

Given that the speed of B immediately after first hitting the wall is equal to the speed of A immediately after its first collision with B,

(b) find the value of f.

(2)

May 2020

1. A particle of mass 2 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}\,\text{N s}$, such that $\mathbf{I} = a\mathbf{i} + b\mathbf{j}$

Immediately after receiving the impulse, the particle is moving with velocity $\lambda(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$, where λ is a constant.

Given that the magnitude of $\mathbf{I}$ is $\sqrt{40}$, find the two possible impulses.

(5)

7. Particle A of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal surface. Particle A collides directly with particle B of mass m, which is moving along the same straight line and in the same direction as A.

Immediately before the collision, the speed of B is u.

As a result of the collision, the direction of motion of B is unchanged and the kinetic energy gained by B is $\frac{48}{25}mu^2$

(a) Find the coefficient of restitution between A and B.

(8)

After the collision, B hits a smooth fixed vertical wall that is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is f.

Given that the speed of B immediately after first hitting the wall is equal to the speed of A immediately after its first collision with B,

(b) find the value of f.

(2)

1. A particle P of mass 1.5 kg is moving with velocity $(4\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse of magnitude 15 Ns. Immediately after P receives the impulse, the velocity of P is $v\mathbf{i}\,\text{m s}^{-1}$.

Find the two possible values of v.

(7)

8. Two particles, A and B, have masses $3m$ and $4m$ respectively. The particles are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately after the collision, A and B are moving in the same direction with speeds $\frac{u}{3}$ and u respectively. In the collision, A receives an impulse of magnitude $8mu$.

(a) Find the coefficient of restitution between A and B.

(6)

When A and B collide they are at a distance d from a smooth vertical wall, which is perpendicular to their direction of motion. After the collision with A, particle B collides directly with the wall and rebounds so that there is a second collision between A and B. This second collision takes place at distance x from the wall.

Given that the coefficient of restitution between B and the wall is $\frac{1}{4}$

(b) find x in terms of d.

(6)

4.

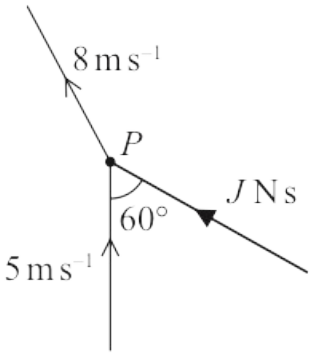

Figure 2

A particle P of mass 0.3 kg is moving with speed $5\,\text{m s}^{-1}$ along a straight line on a smooth horizontal plane. The particle receives a horizontal impulse of magnitude J N s. The speed of P immediately after receiving the impulse is $8\,\text{m s}^{-1}$. The angle between the direction of motion of P before it receives the impulse and the direction of the impulse is 60°, as shown in Figure 2.

Find the value of J.

(6)

8. Particles A, B and C, of masses $2m$, m and $3m$ respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particle A is projected towards particle B with speed $2u$ and collides directly with B.

The coefficient of restitution between each pair of particles is e.

(a) (i) Show that the speed of B immediately after the collision with A is $\frac{4}{3}u(1 + e)$

(ii) Find the speed of A immediately after the collision with B.

(7)

At the instant when A collides with B, particle C is projected with speed u towards B so that B and C collide directly.

(b) Show that there will be a second collision between A and B.

(6)

4. A particle P of mass 0.75 kg is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{J}$ Ns. Immediately after P receives the impulse, the speed of P is $8\,\text{m s}^{-1}$

Given that $\mathbf{J} = c(-\mathbf{i} + 2\mathbf{j})$, where c is a constant, find the two possible values of c.

(6)

6. Two particles, A and B, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of A is $2m$ and the mass of B is $3m$.

Immediately **after** the collision, A and B are moving in opposite directions with the same speed v.
In the collision, A receives an impulse of magnitude $5mv$.

(a) Find the coefficient of restitution between A and B.

(6)

After the collision with A, particle B strikes a smooth fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of the particles.
The coefficient of restitution between B and the wall is f.

As a result of its collision with A and with the wall, the total kinetic energy lost by B is E.
As a result of its collision with B, the kinetic energy lost by A is $2E$.

(b) Find the value of f.

(4)

1. A particle of mass 0.5 kg is moving with velocity $(2\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse of $(-4\mathbf{i} + 6\mathbf{j})\,\text{N s}$.

(a) Find the speed of the particle immediately after it receives the impulse.

(5)

(b) Find the size of the angle between the direction of motion of the particle immediately before it receives the impulse and the direction of motion of the particle immediately after it receives the impulse.

(3)

4. Two small balls, A and B, are moving in opposite directions along the same straight line on smooth horizontal ground. The mass of A is $2m$ and the mass of B is $3m$.
The balls collide directly. Immediately before the collision, the speed of A is $2u$ and the speed of B is u. The coefficient of restitution between A and B is e, where $e > 0$

By modelling the balls as particles,

(a) show that the speed of B immediately after the collision is $\frac{1}{5}u(1 + 6e)$.

(6)

After the collision with ball A, ball B hits a smooth fixed vertical wall which is perpendicular to the direction of motion of B.

The coefficient of restitution between B and the wall is $\frac{5}{7}$

Ball B rebounds from the wall and there is a second direct collision between A and B.

(b) Find the range of possible values of e.

(4)

3. A particle P of mass 0.5 kg is moving with velocity $\lambda(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when P receives an impulse of magnitude $\sqrt{\frac{5}{2}}\,\text{N s}$

Immediately after P receives the impulse, the velocity of P is $4\mathbf{i}\,\text{m s}^{-1}$
Given that λ is a constant, find the two possible values of λ

(6)

6. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface so that the particles collide directly.
The mass of P is km and the mass of Q is m.
Immediately before the collision, the speed of P is x and the speed of Q is y.
Immediately after the collision, P and Q are moving in the same direction, the speed of P is v and the speed of Q is $2v$.

The coefficient of restitution between P and Q is $\frac{1}{5}$

The magnitude of the impulse received by Q in the collision is $5mv$

(a) Find (i) y in terms of v
(ii) x in terms of v
(iii) the value of k

(9)

(b) Find, in terms of m and v, the total kinetic energy lost in the collision between P and Q.

(3)

3. A particle P of mass 0.2 kg is moving with velocity $(4\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$

The particle receives an impulse $\lambda(\mathbf{i} + \mathbf{j})\,\text{N s}$, where λ is a constant.

Immediately after receiving the impulse, the speed of P is $7\,\text{m s}^{-1}$

Find the possible values of λ

(6)

7. Particle A has mass m and particle B has mass $2m$.
The particles are moving in the same direction along the same straight line on a smooth horizontal surface.
Particle A collides directly with particle B.
Immediately before the collision, the speed of A is $3u$ and the speed of B is u.
The coefficient of restitution between A and B is e.

(a) (i) Show that the speed of B immediately after the collision is $\frac{5 + 2e}{3}u$

(ii) Find the speed of A immediately after the collision.

(7)

After the collision, B hits a smooth fixed vertical wall that is perpendicular to the direction of motion of B.

The coefficient of restitution between B and the wall is $\frac{1}{3}$

Particle B rebounds and there is a second collision between A and B.
The first collision between A and B occurs at a distance d from the wall.
The time between the two collisions is T.

Given that $e = \frac{1}{2}$

(b) find T in terms of d and u.

(6)

2. A particle P of mass 0.5 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$
The particle receives an impulse $(-2\mathbf{i} + \lambda\mathbf{j})\,\text{N s}$, where λ is a constant.
Immediately after receiving the impulse, the velocity of P is $(x\mathbf{i} + y\mathbf{j})\,\text{m s}^{-1}$
The kinetic energy gained by P as a result of receiving the impulse is 22 J.

Find the possible values of λ.

(7)

7. Particle P has mass $3m$ and particle Q has mass km. The particles are moving towards each other on the same straight line on a smooth horizontal surface.
The particles collide directly.
Immediately **before** the collision, the speed of P is $2u$ and the speed of Q is $3u$.
Immediately **after** the collision, the speed of P is u and the speed of Q is v.

The direction of motion of P is unchanged by the collision.

(a) Show that $v = \dfrac{(3 - 3k)}{k} u$

(3)

(b) Find, in terms of m and u, the magnitude of the impulse received by Q in the collision.

(2)

The coefficient of restitution between P and Q is e.

Given that $v \neq u$

(c) find the range of possible values of k.

(5)

May 2023

1. A particle P of mass 0.3 kg is moving with velocity $5\mathbf{i}\ \text{m s}^{-1}$

The particle receives an impulse $\mathbf{I}$ Ns.

Immediately after receiving the impulse, the velocity of P is $(7\mathbf{i} + 7\mathbf{j})\ \text{m s}^{-1}$

(a) Find the magnitude of $\mathbf{I}$

(4)

(b) Find the angle between the direction of $\mathbf{I}$ and the direction of motion of P immediately before receiving the impulse.

(3)

4. A particle P of mass $3m$ and a particle Q of mass $5m$ are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide directly.

Immediately **before** the collision, the speed of P is u and the speed of Q is ku.

Immediately **after** the collision, the speed of P is $2v$ and the speed of Q is v.

The direction of motion of each particle is reversed by the collision.

In the collision, P receives an impulse of magnitude $15mv$.

(a) Show that $u = 3v$.

(3)

(b) Find the value of k.

(3)

The coefficient of restitution between P and Q is e.

(c) Find the value of e.

(3)

The total kinetic energy lost in the collision is λmv^2

(d) Find the value of λ.

(3)

3.

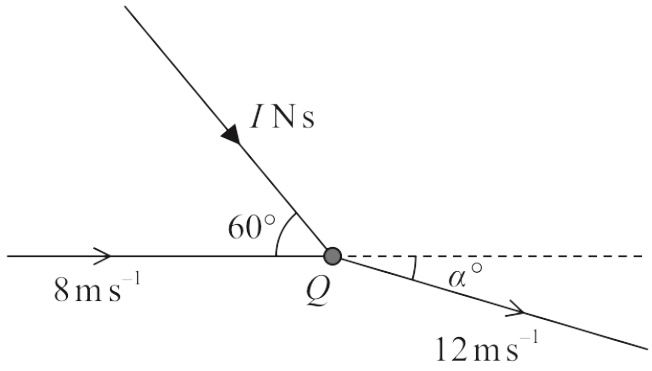

Figure 2

A particle Q of mass 0.25 kg is moving in a straight line on a smooth horizontal surface with speed $8\,\text{m s}^{-1}$ when it receives an impulse of magnitude I N s.

The impulse acts parallel to the horizontal surface and at 60° to the original direction of motion of Q.

Immediately after receiving the impulse, the speed of Q is $12\,\text{m s}^{-1}$

As a result of receiving the impulse, the direction of motion of Q is turned through $\alpha°$, as shown in Figure 2.

Find the value of I

(6)

7. Particle P has mass $4m$ and particle Q has mass $2m$.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface.

Particle P collides directly with particle Q.

Immediately **before** the collision, the speed of P is $2u$ and the speed of Q is $3u$.

Immediately **after** the collision, the speed of P is x and the speed of Q is y.

The direction of motion of each particle is reversed as a result of the collision.

The total kinetic energy of P and Q after the collision is half of the total kinetic energy of P and Q before the collision.

(a) Show that $y = \frac{8}{3}u$

(6)

The coefficient of restitution between P and Q is e.

(b) Find the value of e.

(3)

After the collision, Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q.

Particle Q rebounds.

The coefficient of restitution between Q and the wall is f.

Given that there is no second collision between P and Q,

(c) find the range of possible values of f.

(3)

Given that $f = \frac{1}{4}$

(d) find, in terms of m and u, the magnitude of the impulse received by Q as a result of its impact with the wall.

(2)

2. [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle Q of mass 0.5 kg is moving on a smooth horizontal surface. Particle Q is moving with velocity $(3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse of $(2\mathbf{i} + 5\mathbf{j})\,\text{N s}$.

(a) Find the speed of Q immediately after receiving the impulse.

(4)

As a result of receiving the impulse, the direction of motion of Q is turned through an angle $\theta°$

(b) Find the value of θ

(2)

7. Particle P has mass m and particle Q has mass $5m$.

The particles are moving in the same direction along the same straight line on a smooth horizontal surface.

Particle P collides directly with particle Q.

Immediately **before** the collision, the speed of P is $6u$ and the speed of Q is u.

Immediately **after** the collision, the speed of P is x and the speed of Q is y.

The direction of motion of P is reversed as a result of the collision.

The coefficient of restitution between P and Q is e.

(a) Find the complete range of possible values of e.

(7)

Given that $e = \frac{3}{5}$

(b) find the total kinetic energy lost in the collision between P and Q.

(4)

After the collision, Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q.

Particle Q rebounds.

The coefficient of restitution between Q and the wall is f.

Given that there is a second collision between P and Q,

(c) find the complete range of possible values of f.

(3)

1. [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors*.]

A particle A has mass 2 kg and a particle B has mass 3 kg. The particles are moving on a smooth horizontal plane when they collide.

Immediately before the collision, the velocity of A is $5\mathbf{j}\,\text{m s}^{-1}$ and the velocity of B is $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Immediately after the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the total kinetic energy of the two particles before the collision.

(3)

(b) Find, in terms of **i** and **j**, the impulse received by A in the collision.

(2)

Given that, in the collision, the impulse of A on B is equal and opposite to the impulse of B on A,

(c) find the velocity of B immediately after the collision.

(3)

5. A particle P of mass m and a particle Q of mass $2m$ are at rest on a smooth horizontal plane.

Particle P is projected with speed u along the plane towards Q and the particles collide. The coefficient of restitution between the particles is e.

As a result of the collision, the direction of motion of P is reversed.

(a) Find, in terms of u and e, the speed of P after the collision.

(6)

After the collision, Q goes on to hit a vertical wall which is fixed at right angles to the direction of motion of Q. The coefficient of restitution between Q and the wall is $\frac{1}{3}$

Given that there is a second collision between P and Q

(b) find the full range of possible values of e.

(5)