

Chapter 7: Vectors

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9. With respect to a fixed origin O , the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

- (a) Find the coordinates of A .

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line l_2 passes through the point P and is parallel to the line l_1

- (b) Write down a vector equation for the line l_2

(2)

- (c) Find the exact value of the distance AP .

Give your answer in the form $k\sqrt{2}$, where k is a constant to be found.

(2)

The acute angle between AP and l_2 is θ

- (d) Find the value of $\cos \theta$

(3)

A point E lies on the line l_2

Given that $AP = PE$,

- (e) find the area of triangle APE ,

(2)

- (f) find the coordinates of the two possible positions of E .

(5)

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8. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \text{where } \mu \text{ is a scalar parameter}$$

Given that l_1 and l_2 meet at the point X ,

(a) find the position vector of X .

(5)

The point $P(10, -7, 0)$ lies on l_1

The point Q lies on l_2

Given that \vec{PQ} is perpendicular to l_2

(b) calculate the coordinates of Q .

(5)

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7. Relative to a fixed origin O , the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that \vec{OA} is a unit vector parallel to l ,

(a) find \vec{OA} (2)

The point X lies on l .

Given that X is the point on l that is closest to the origin,

(b) find the coordinates of X . (5)

The points O , X and A form the triangle OXA .

(c) Find the exact area of triangle OXA . (3)

9. (i) Relative to a fixed origin O , the points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively.

Points A , B and C lie in a straight line, with B lying between A and C .

Given $AB:AC = 1:3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

(3)

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8. With respect to a fixed origin O the points A and B have position vectors

$$\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix}$$

respectively.

The line l_1 passes through the points A and B .

- (a) Write down an equation for l_1

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where λ is a scalar parameter.

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

where μ is a scalar parameter.

- (b) Show that l_1 and l_2 do **not** meet.

(4)

The point C is on l_2 where $\mu = -1$

- (c) Find the acute angle between AC and l_2

Give your answer in degrees to one decimal place.

(5)

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3.

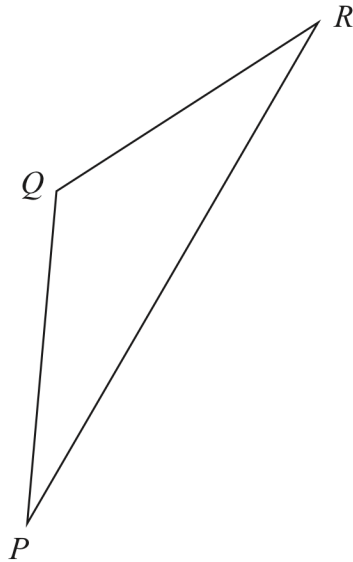


Figure 1

Figure 1 shows a sketch of triangle PQR .

Given that

- $\vec{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\vec{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find \vec{RQ}

(2)

(b) Find the size of angle PQR , in degrees, to three significant figures.

(3)

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4. Relative to a fixed origin O ,
- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
 - the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
 - the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B .

- (a) Find a vector equation for l .

(2)

The point C lies on l so that PC is perpendicular to l .

- (b) Find the coordinates of C .

(4)

The point P' is the reflection of P in the line l .

- (c) Find the coordinates of P'

(2)

- (d) Hence find $|\overrightarrow{PP'}|$, giving your answer as a simplified surd.

(2)

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6. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

(a) find the value of p , (4)

(b) find the position vector of the point of intersection. (2)

(c) Find the acute angle between l_1 and l_2
 Give your answer in degrees to one decimal place. (3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with \vec{AB} perpendicular to l_2

(d) Find the coordinates of B (5)

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