Chapter 6: Integration

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P4 Chapter 6 Solutions

3. $$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

5.

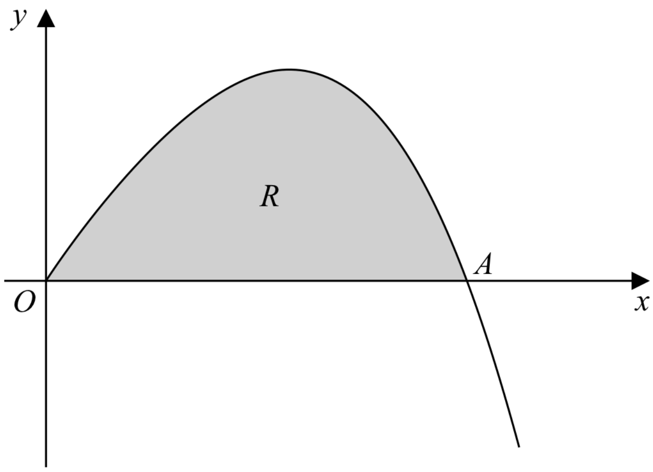

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\,dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

(3)

8. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

3.

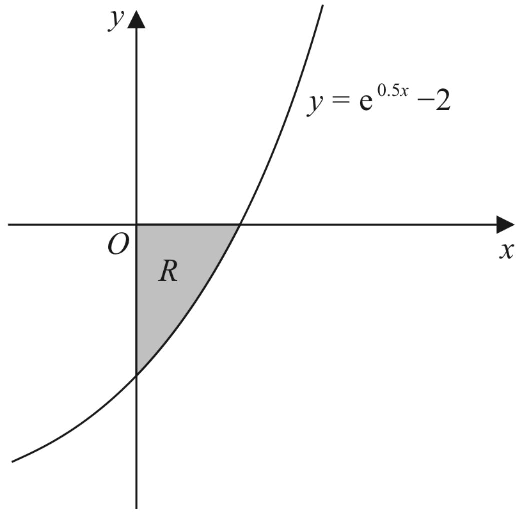

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{0.5x} - 2$

The region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The region R is rotated 360° about the x-axis to form a solid of revolution.

Show that the volume of this solid can be written in the form $a \ln 2 + b$, where a and b are constants to be found.

(6)

5.

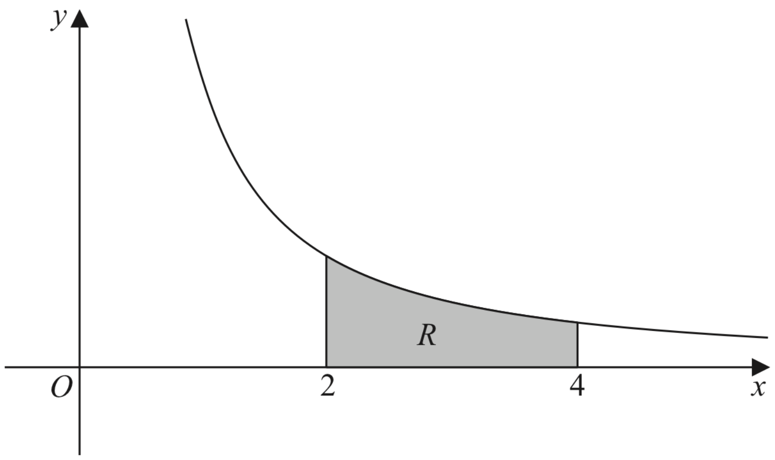

Figure 3

(a) Find $\int \frac{\ln x}{x^2}\,dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \qquad x > 0.5$$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

(b) Use the answer to part (a) to find the exact area of R, writing your answer in simplest form.

(4)

7. (i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(6)

(ii) Find

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,\mathrm{d}x$$

(6)

9. Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, A cm^2, covered by the bacteria, t days after the bacteria start to grow, is modelled by the differential equation

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r}\right)^2$$

where p, q and r are integers to be found.

(7)

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

(2)

5. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_{1}^{13} \frac{4}{3 + \sqrt{2x - 1}}\, dx$$

giving your answer in the form $p + q\ln 2$, where p and q are integers to be found.

(8)

7.

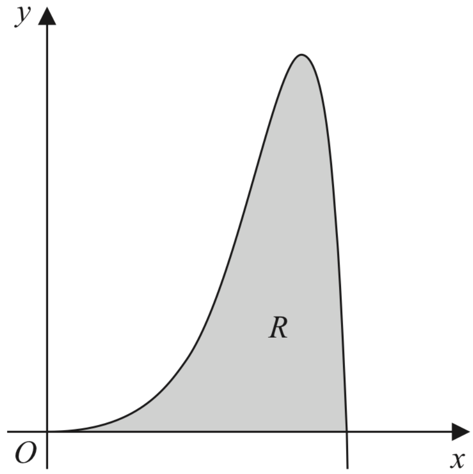

Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \qquad x \geqslant 0$$

The finite region R is bounded by the curve and the x-axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\dfrac{e^{2\pi} + 1}{5}$

(*Solutions relying on calculator technology are not acceptable*.)

(2)

9.

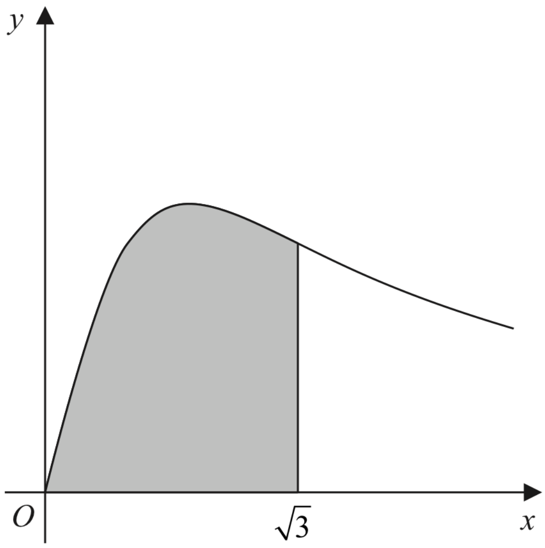

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan\theta \qquad y = 2\sin 2\theta \qquad \theta \geqslant 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta)\,d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

10. (a) Write $\frac{1}{(H-5)(H+3)}$ in partial fraction form.

(3)

The depth of water in a storage tank is being monitored.

The depth of water in the tank, H metres, is modelled by the differential equation

$$\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$$

where t is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}}$$

(7)

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(3)

According to the model, the depth of water in the tank will eventually fall to k metres.

(d) State the value of the constant k.

(1)

2.

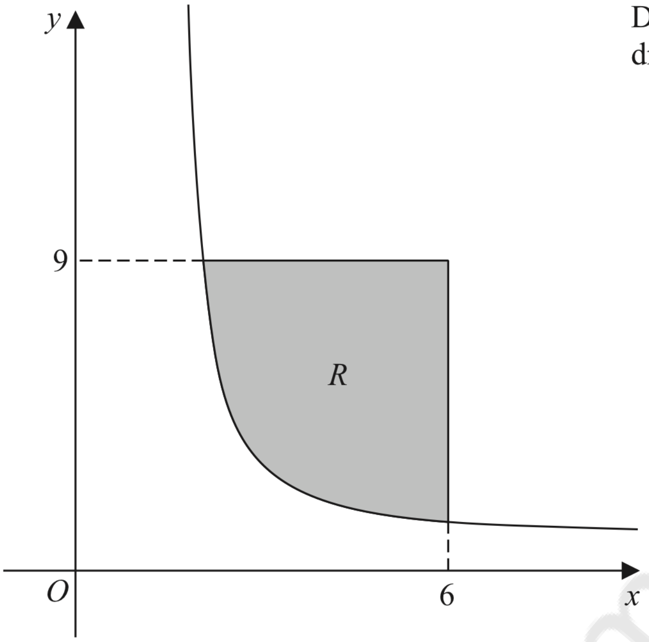

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x-3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

(7)

4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,\mathrm{d}x$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where a and b are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(8)

6.

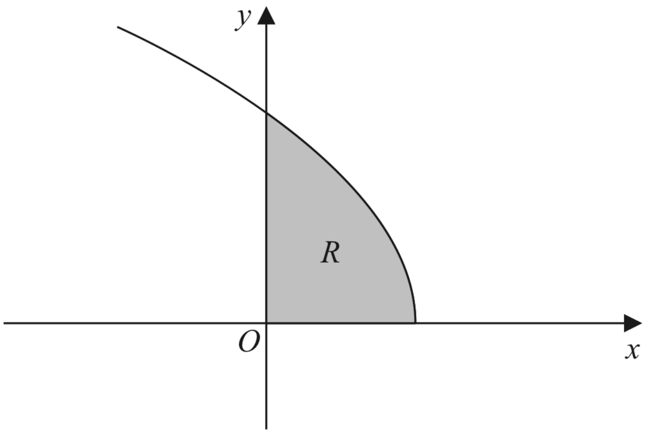

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t \, dt$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax+b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \sqrt{ax+b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

8. (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = g(x)$.

(7)

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

(2)

2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

(6)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

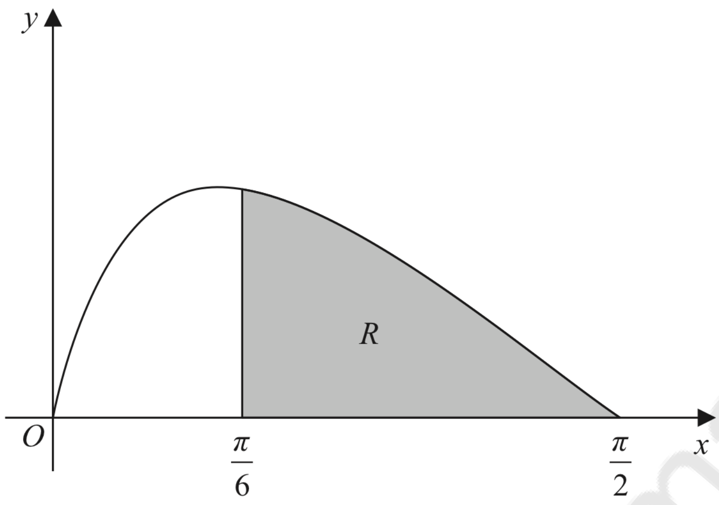

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

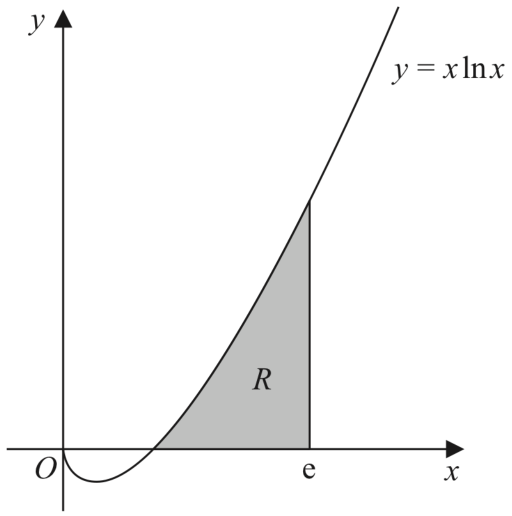

Figure 3

(a) Find $\int x^2 \ln x \, dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \qquad x > 0$$

The region R, shown shaded in Figure 3, lies entirely above the x-axis and is bounded by the curve, the x-axis and the line with equation $x = \mathrm{e}$.

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

(4)

9.

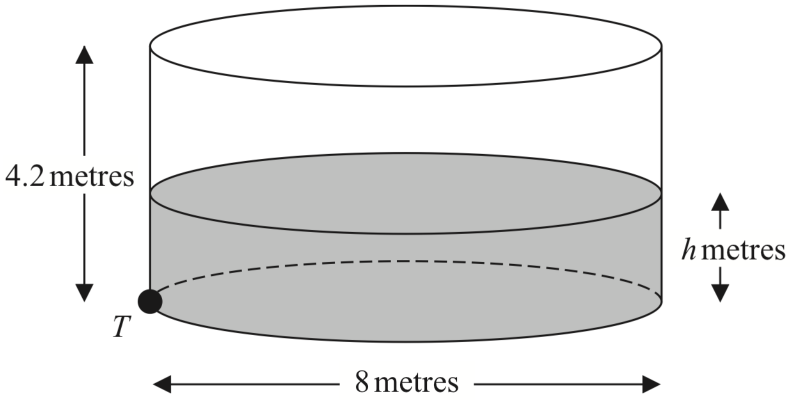

Figure 4

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\,\text{m}^3$ per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)\,\text{m}^3$ per minute

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

5.

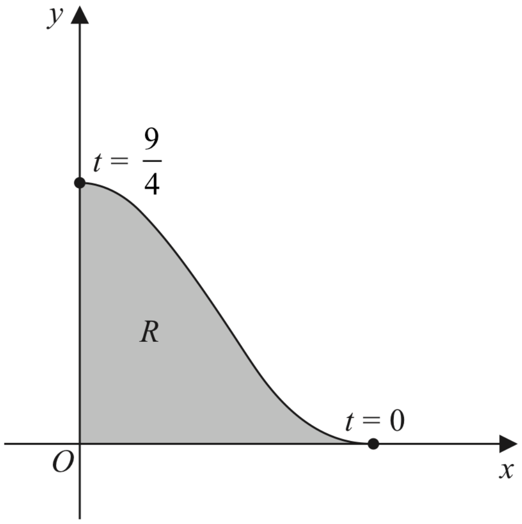

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K\int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}}\,\mathrm{d}t$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

7.

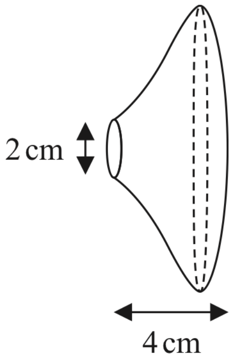

Figure 3

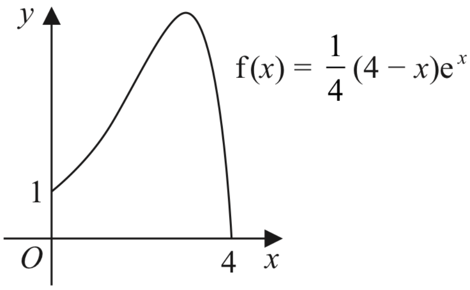

Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x-axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4 - x)e^{x} \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, V cm³, of the doorknob is given by

$$V = K\int_{0}^{4}\left(x^{2} - 8x + 16\right)e^{2x}\,dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^{q} + r)$ cm³ where p, q and r are simplified rational numbers to be found.

(5)

9. (a) Find the derivative with respect to y of

$$\frac{1}{(1+2\ln y)^2}$$

(2)

(b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3 \qquad y > 0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = e^{A\sec x - \frac{1}{2}}$$

where A is an irrational number to be found.

(5)

2. (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \frac{1}{2}$ when $y = \frac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2\sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx = \int_0^p \left(\frac{3}{2}\sec u \tan u + \frac{1}{2}\sec^2 u\right) du$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx$$

(4)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

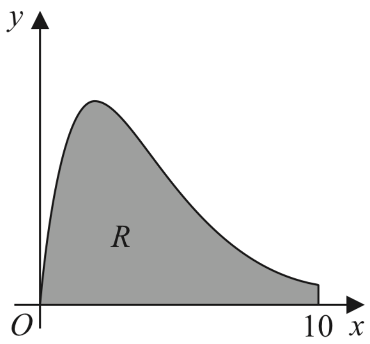

Figure 2

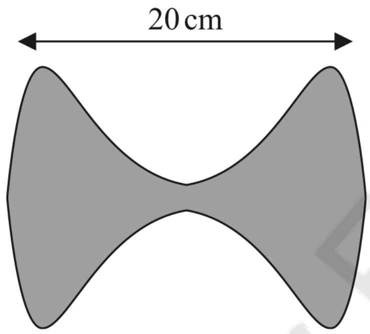

Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2 e^{-x}\,dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2 e^{-x}\,dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

2. (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_{5}^{25} \frac{3x}{(2x-1)(x-2)}\,\mathrm{d}x = \ln k$$

where k is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

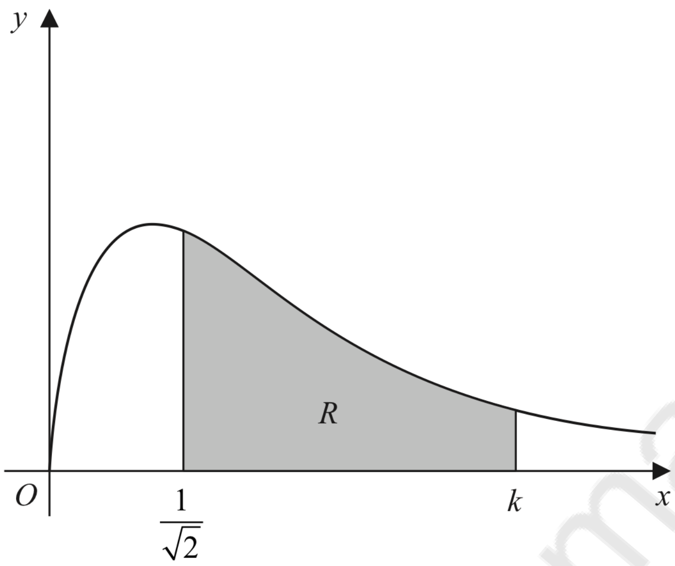

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x-axis and the line with equation $x = k$.

This region is rotated through 360° about the x-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k.

(6)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\, dx = a + b\ln 2$$

where a and b are constants to be found.

(7)

(ii) Show, by integration, that

$$\int 3e^x \cos 2x \, dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where p and q are constants to be found and c is an arbitrary constant.

(5)

3.

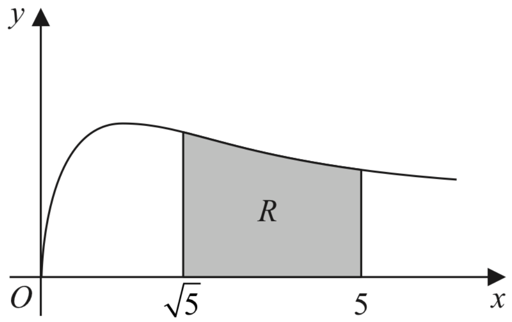

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

4. (a) Using the substitution $u = \sqrt{2x + 1}$, show that

$$\int_4^{12} \sqrt{8x + 4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$$

may be expressed in the form

$$\int_a^b ku^2\mathrm{e}^u\ \mathrm{d}u$$

where a, b and k are constants to be found.

(4)

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x + 4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$$

giving your answer in simplest form.

(5)

7. The volume $V\,\text{cm}^3$ of a spherical balloon with radius r cm is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\frac{\mathrm{d}V}{\mathrm{d}r}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

3. $$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_{k}^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

5. (i) Find

$$\int x^2 e^x \, dx$$

(4)

(ii) Use the substitution $u = \sqrt{1 - 3x}$ to show that

$$\int \frac{27x}{\sqrt{1 - 3x}} \, dx = -2(1 - 3x)^{\frac{1}{2}}(Ax + B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a, b, c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

(2)

8.

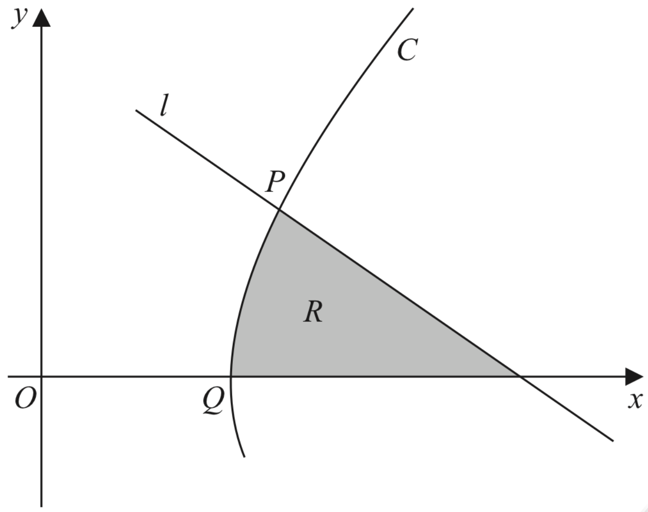

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R, shown shaded in Figure 2 is bounded by the curve C, the line l and the x-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x-axis.

(7)

3. **In this question you must show all stages of your working.**

Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x} \, dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2} \, dx = a + \ln b$$

where a and b are constants to be found.

(7)

7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{\mathrm{d}x}{\mathrm{d}t} = x(9 - 2x)$$

(c) Write $\dfrac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + \mathrm{e}^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

8.

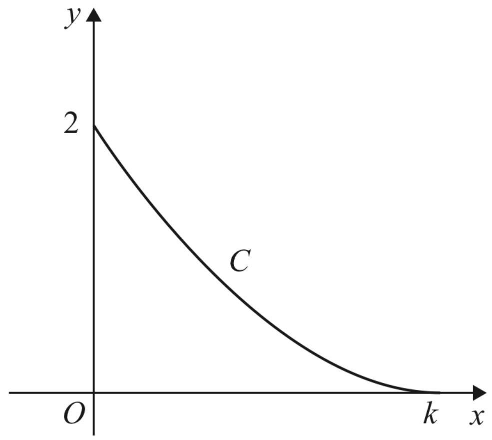

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

The region bounded by the curve, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta\left(1 - \cos 4t\right) \mathrm{d}t$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

2. Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,\mathrm{d}x$$

giving your answer in the form $p\ln q + r$ where p, q and r are rational numbers.

(6)

5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y}\right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)

7. (a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x\,dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

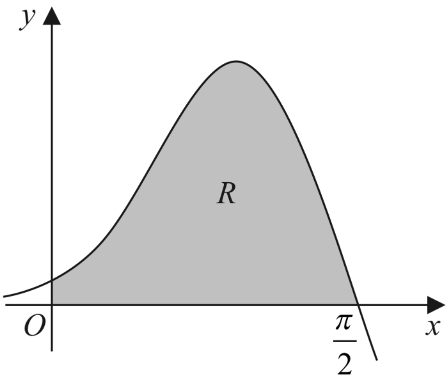

Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x}\cos x$$

The region R, shown shaded in Figure 3, is bounded by the positive x-axis, the positive y-axis and the curve.

The region R is rotated through 2π radians about the x-axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

8.

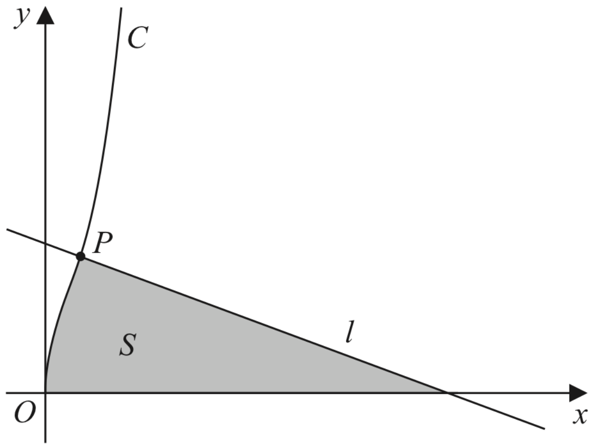

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

equation for l is **(6)**

$$8y + 2x = 17$$

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx$$

giving your answer in simplest form.

(5)

5.

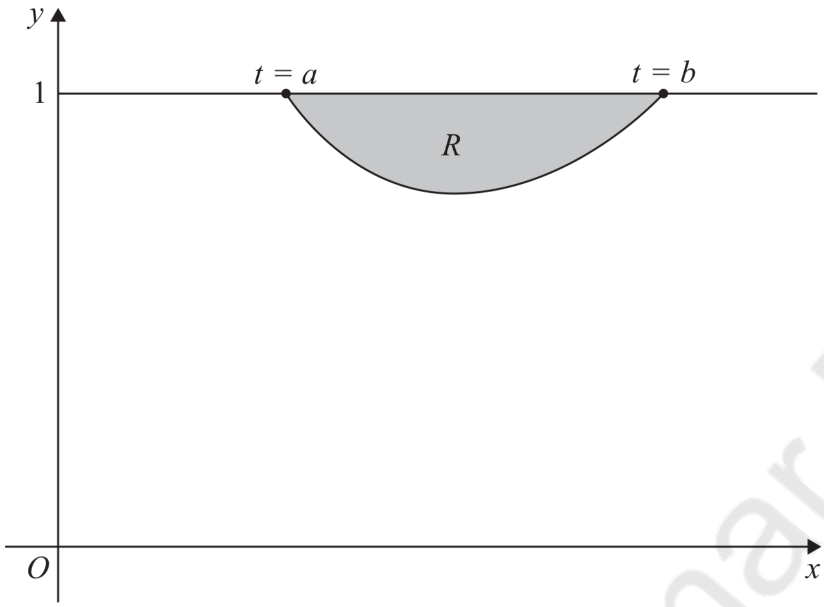

Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where M and k are constants to be found.

(5)

(c) (i) Write $\dfrac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R, giving your answer in simplest form.

(6)

7. The current, x amps, at time t seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where k is a constant.

Initially there is zero current in the circuit.

(a) Solve the differential equation to find an equation, in terms of k, for the current in the circuit at time t seconds.
Give your answer in the form $x = f(t)$.

(6)

Given that in the long term the current in the circuit approaches 7 amps,

(b) find the value of k.

(2)

(c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

(3)

9.

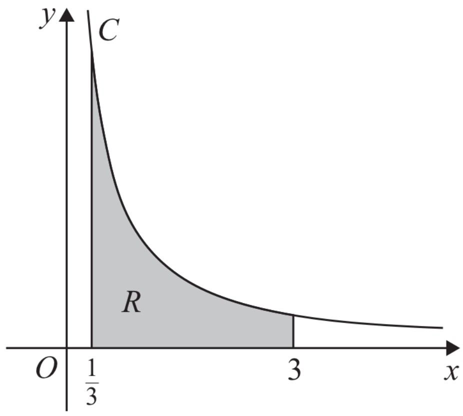

Figure 3

The curve C, shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\left(\arctan\sqrt{x}\right)}$$

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = 3$, the x-axis and the line with equation $x = \frac{1}{3}$

The region R is rotated through 360° about the x-axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume V of the solid formed is given by

$$k\int_a^b \frac{1}{u^2}\,\mathrm{d}u$$

where k, a and b are constants to be found.

(6)

(b) Hence, using algebraic integration, find the value of V in simplest form.

(3)