Chapter 4: Binomial Expansion

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3} \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3

Give each coefficient as a fraction in its simplest form.

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2. (a) Use the binomial expansion to expand

$$(4-5x)^{-\frac{1}{2}} \qquad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2 giving each coefficient as a fully simplified fraction.

(4)

$$f(x) = \frac{2+kx}{\sqrt{4-5x}} \qquad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of f(x), in ascending powers of x, is

$$1 + \frac{3}{10}x + mx^2 + \ldots \qquad \text{where } m \text{ is a constant}$$

(b) find the value of k,

(2)

(c) find the value of m.

(2)

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1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \qquad |x| < \frac{1}{20}$$

giving each coefficient in its simplest form.

(5)

By substituting $x = \frac{1}{100}$ into the answer for (a),

(b) find an approximation for $\sqrt{5}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers to be found.

(2)

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1. Given that k is a constant and the binomial expansion of

$$\sqrt{1+kx} \qquad |kx| < 1$$

in ascending powers of x up to the term in x^3 is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of k,

(ii) find the value of the constant A and the constant B.

(5)

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

(2)

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4. $f(x) = \sqrt{1 - 4x^2}$ $\quad |x| < \frac{1}{2}$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

(4)

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

(2)

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2. (a) Find, in ascending powers of x, the first three non-zero terms of the binomial series expansion of

$$\sqrt[3]{1+4x^3} \qquad |x| < \frac{1}{\sqrt[3]{4}}$$

giving each coefficient as a simplified fraction.

(4)

(b) Use the expansion from part (a) with $x = \frac{1}{3}$ to find a rational approximation to $\sqrt[3]{31}$

(3)

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1. The binomial expansion of

$$(3 + kx)^{-2} \qquad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

(3)

(c) Hence (i) find the value of k

(ii) find the value of D

(3)

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4.

$$g(x) = \frac{1}{\sqrt{4 - x^2}}$$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of $g(x)$. Give each coefficient in simplest form.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

(c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear.

(2)

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1. $$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

(a) Write $f(x)$ in partial fraction form.

(3)

(b) (i) Hence find, in ascending powers of x up to and including the terms in x^2, the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.

(5)

(ii) Find the range of values of x for which this expansion is valid.

(1)

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1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

(5)

Given that

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{n}\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$$

(b) write down the value of n.

(1)

(c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of x, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

(3)

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1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

(4)

(b) Find the range of values of x for which this expansion is valid.

(1)

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1. Find, in ascending powers of x up to and including the term in x^3, the binomial expansion of

$$(1 - 4x)^{-3} \qquad |x| < \frac{1}{4}$$

fully simplifying each term.

(4)

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8. $f(x) = (8 - 3x)^{\frac{4}{3}} \qquad 0 < x < \frac{8}{3}$

(a) Show that the binomial expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \ldots$$

where A and B are constants to be found.

(4)

(b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \qquad 0 < x < \frac{8}{3}$$

where A and B are the constants found in part (a).

(*Solutions relying on calculator technology are not acceptable.*)

(4)

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