5. Correlation and Regression

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4. Sara was studying the relationship between rainfall, r mm, and humidity, h %, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

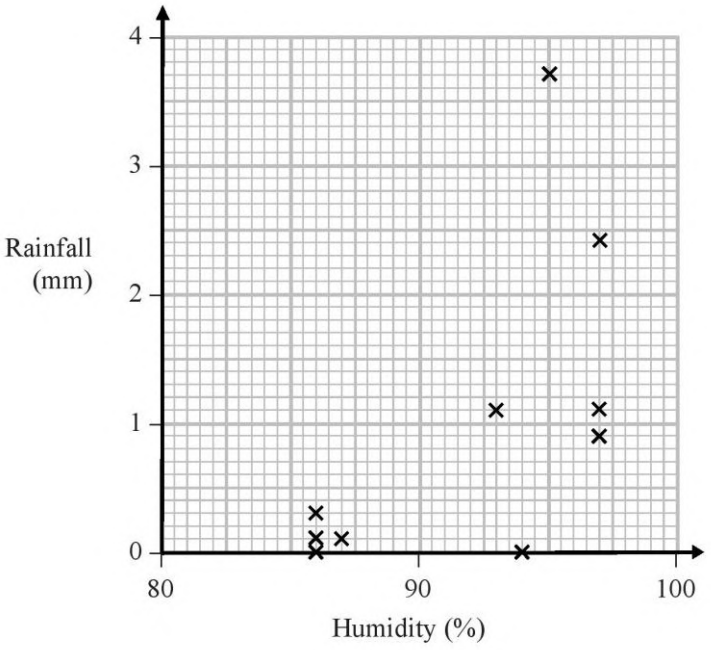
(a) Show that $r = 20.6$ is an outlier.

(1)

(b) Give a reason why Sara might (i) include (ii) exclude this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of r and h.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

Question 4 continued

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$

(d) Give an interpretation of the gradient of this regression line.

(1)

(e) (i) Comment on the suitability of Sara's sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(2)

Total for Question 4 is 7 marks)

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days a location from the large data set gave $\overline{t} = 27.2$ and $\overline{w} = 3.5$

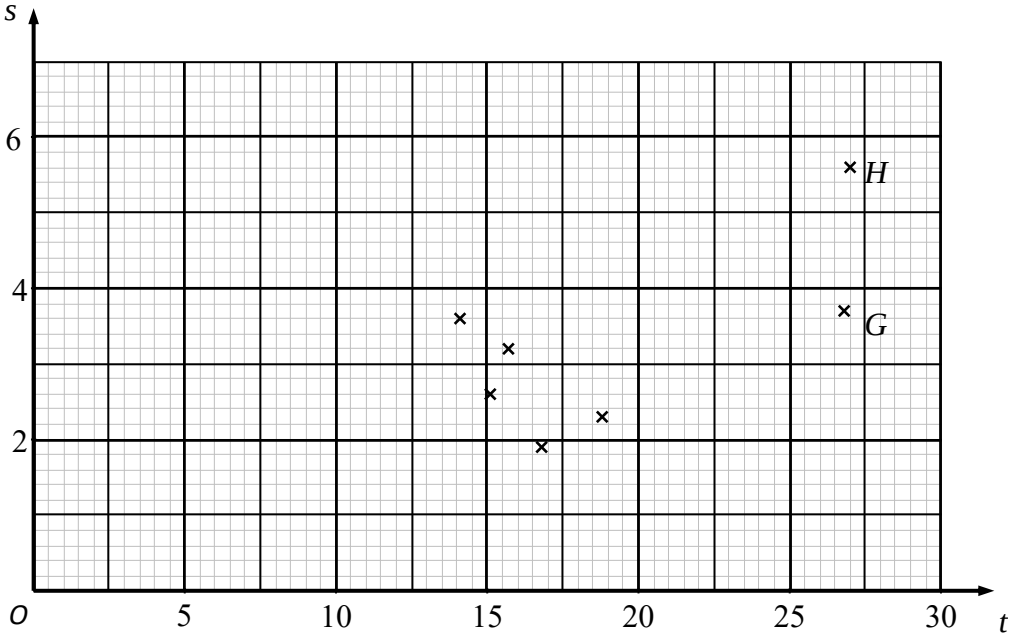
(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

2. A researcher believes that there is a linear relationship between daily mean temperature and daily total rainfall. The 7 places in the northern hemisphere from the large data set are used. The mean of the daily mean temperatures, t °C, and the mean of the daily total rainfall, s mm, for the month of July in 2015 are shown on the scatter diagram below.



(a) With reference to the scatter diagram, explain why a linear regression model may not be suitable for the relationship between t and s.

(1)

The researcher calculated the product moment correlation coefficient for the 7 places and obtained $r = 0.658$.

(b) Stating your hypotheses clearly, test at the 10% level of significance, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

(c) Using your knowledge of the large data set, suggest the names of the 2 places labelled G and H.

(1)

(d) Using your knowledge from the large data set, and with reference to the locations of the two places labelled G and H, give a reason why these places have the highest temperatures in July.

(2)

(e) Suggest how you could make better use of the large data set to investigate the relationship between daily mean temperature and daily total rainfall.

(1)

(Total 7 marks)

Question 2 continued

1. A company is introducing a job evaluation scheme. Points (x) will be awarded to each job based on the qualifications and skills needed and the level of responsibility. Pay (£y) will then be allocated to each job according to the number of points awarded.

Before the scheme is introduced, a random sample of 8 employees was taken and the linear regression equation of pay on points was $y = 4.5x - 47$

(a) Describe the correlation between points and pay.

(1)

(b) Give an interpretation of the gradient of this regression line.

(1)

(c) Explain why this model might not be appropriate for all jobs in the company.

(1)

2. Tessa owns a small clothes shop in a seaside town. She records the weekly sales figures, £w, and the average weekly temperature, t °C, for 8 weeks during the summer.
The product moment correlation coefficient for these data is −0.915

(a) Stating your hypotheses clearly and using a 5% level of significance, test whether or not the correlation between sales figures and average weekly temperature is negative.

(3)

(b) Suggest a possible reason for this correlation.

(1)

Tessa suggests that a linear regression model could be used to model these data.

(c) State, giving a reason, whether or not the correlation coefficient is consistent with Tessa's suggestion.

(1)

(d) State, giving a reason, which variable would be the explanatory variable.

(1)

Tessa calculated the linear regression equation as $w = 10\,755 - 171t$

(e) Give an interpretation of the gradient of this regression equation.

(1)

Question 2 continued

(Total for Question 2 is 7 marks)

Answer ALL questions. Write your answers in the spaces provided.

1. A sixth form college has 84 students in Year 12 and 56 students in Year 13

The head teacher selects a stratified sample of 40 students, stratified by year group.

(a) Describe how this sample could be taken.

(3)

The head teacher is investigating the relationship between the amount of sleep, s hours, that each student had the night before they took an aptitude test and their performance in the test, p marks.
For the sample of 40 students, he finds the equation of the regression line of p on s to be

$$p = 26.1 + 5.60s$$

(b) With reference to this equation, describe the effect that an extra 0.5 hours of sleep may have, on average, on a student's performance in the aptitude test.

(1)

(c) Describe one limitation of this regression model.

(1)

3. Barbara is investigating the relationship between average income (GDP per capita), x US dollars, and average annual carbon dioxide (CO_2) emissions, y tonnes, for different countries.

She takes a random sample of 24 countries and finds the product moment correlation coefficient between average annual CO_2 emissions and average income to be 0.446

(a) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not the product moment correlation coefficient for all countries is greater than zero.

(3)

Barbara believes that a non-linear model would be a better fit to the data.
She codes the data using the coding $m = \log_{10} x$ and $c = \log_{10} y$ and obtains the model $c = -1.82 + 0.89m$

The product moment correlation coefficient between c and m is found to be 0.882

(b) Explain how this value supports Barbara's belief.

(1)

(c) Show that the relationship between y and x can be written in the form $y = ax^n$ where a and n are constants to be found.

(5)

Question 3 continued

2. Jerry is studying visibility for Camborne using the large data set June 1987.

The table below contains two extracts from the large data set.

It shows the daily maximum relative humidity and the daily mean visibility.

Date	Daily Maximum Relative Humidity	Daily Mean Visibility
Units	**%**	
10/06/1987	90	5300
28/06/1987	100	0

(The units for Daily Mean Visibility are deliberately omitted.)

Given that daily mean visibility is given to the nearest 100,

(a) write down the range of distances in metres that corresponds to the recorded value 0 for the daily mean visibility.

(1)

Jerry drew the following scatter diagram, Figure 2, and calculated some statistics using the June 1987 data for Camborne from the large data set.

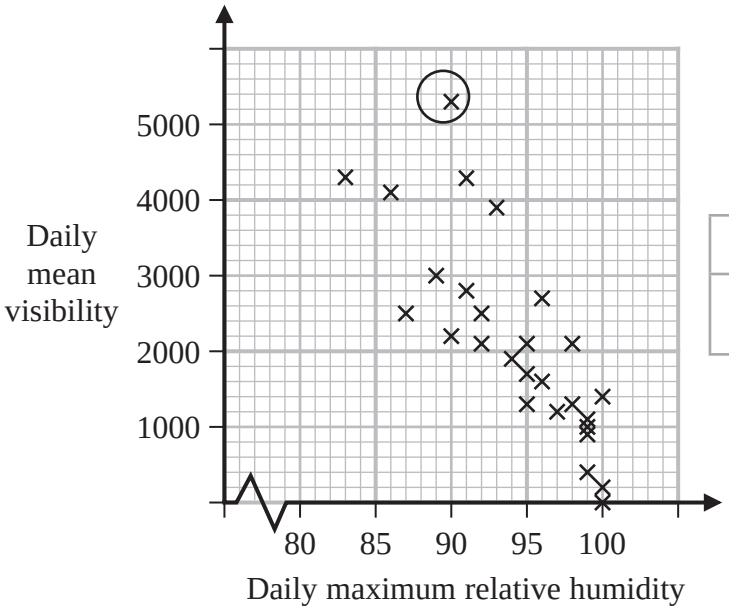


Figure 2

	Q_1	IQR
Daily mean visibility	1100	1600
Daily maximum relative humidity (%)	92	8

Jerry defines an outlier as a value that is more than 1.5 times the interquartile range above Q_3 or more than 1.5 times the interquartile range below Q_1.

(b) Show that the point circled on the scatter diagram is an outlier for visibility.

(2)

(c) Interpret the correlation between the daily mean visibility and the daily maximum relative humidity.

(1)

Jerry drew the following scatter diagram, Figure 3, using the June 1987 data for Camborne from the large data set, but forgot to label the *x*–axis.

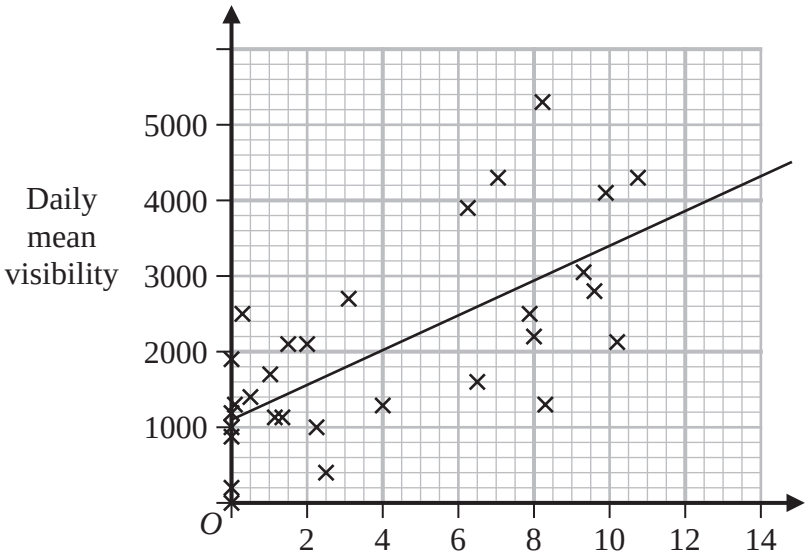


Figure 3

(d) Using your knowledge of the large data set, suggest which variable the *x*-axis on this scatter diagram represents.

(1)

2. A random sample of 15 days is taken from the large data set for Perth in June and July 1987.

The scatter diagram in Figure 1 displays the values of two of the variables for these 15 days.

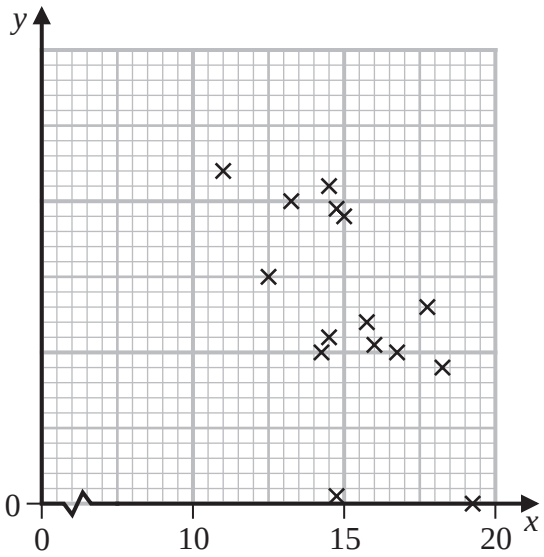


Figure 1

(a) Describe the correlation.

(1)

The variable on the x-axis is Daily Mean Temperature measured in °C.

(b) Using your knowledge of the large data set,

(i) suggest which variable is on the y-axis,

(ii) state the units that are used in the large data set for this variable.

(2)

Stav believes that there is a correlation between Daily Total Sunshine and Daily Maximum Relative Humidity at Heathrow.

He calculates the product moment correlation coefficient between these two variables for a random sample of 30 days and obtains $r = -0.377$

(c) Carry out a suitable test to investigate Stav's belief at a 5% level of significance. State clearly

- your hypotheses
- your critical value

(3)

On a random day at Heathrow the Daily Maximum Relative Humidity was 97%

(d) Comment on the number of hours of sunshine you would expect on that day, giving a reason for your answer.

(1)

Question 2 continued

2. Marc took a random sample of 16 students from a school and for each student recorded

- the number of letters, x, in their last name
- the number of letters, y, in their first name

His results are shown in the scatter diagram on the next page.

(a) Describe the correlation between x and y.

(1)

Marc suggests that parents with long last names tend to give their children shorter first names.

(b) Using the scatter diagram comment on Marc's suggestion, giving a reason for your answer.

(1)

The results from Marc's random sample of 16 observations are given in the table below.

x	3	6	8	7	5	3	11	3	4	5	4	9	7	10	6	6
y	7	7	4	4	6	8	5	5	8	4	7	4	5	5	6	3

(c) Use your calculator to find the product moment correlation coefficient between x and y for these data.

(1)

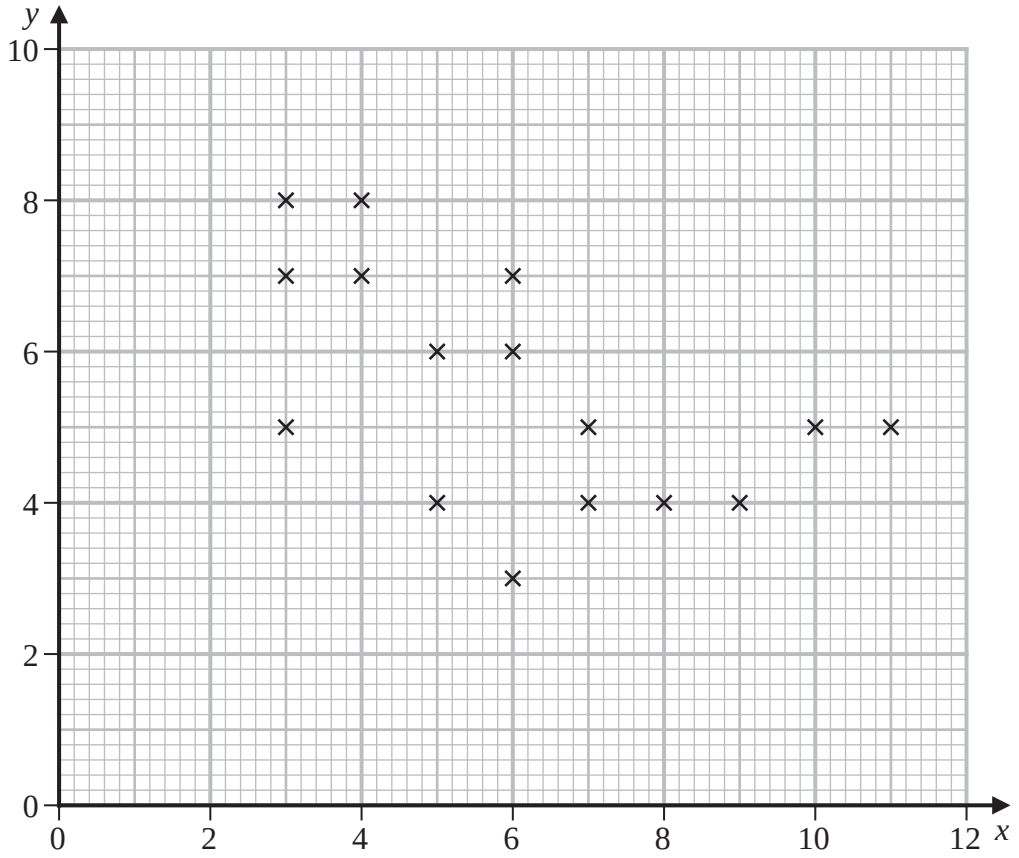
(d) Test whether or not there is evidence of a negative correlation between the number of letters in the last name and the number of letters in the first name.

You should

- state your hypotheses clearly
- use a 5% level of significance

(3)

Question 2 continued.



1. The relationship between two variables p and t is modelled by the regression line with equation

$$p = 22 - 1.1\,t$$

The model is based on observations of the independent variable, t, between 1 and 10

(a) Describe the correlation between p and t implied by this model.

(1)

Given that p is measured in centimetres and t is measured in days,

(b) state the units of the gradient of the regression line.

(1)

Using the model,

(c) calculate the change in p over a 3-day period.

(2)

Tisam uses this model to estimate the value of p when $t = 19$

(d) Comment, giving a reason, on the reliability of this estimate.

(1)

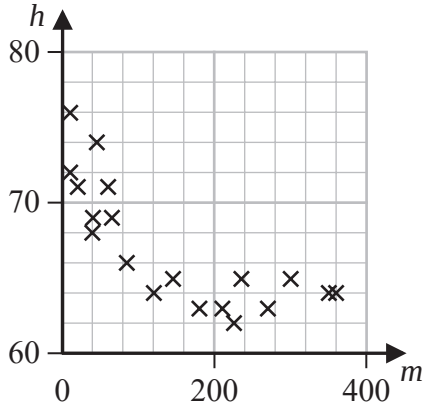
Question 1 continued

(Total for Question 1 is 5 marks)

6. Anna is investigating the relationship between exercise and resting heart rate. She takes a random sample of 19 people in her year at school and records for each person

- their resting heart rate, h beats per minute
- the number of minutes, m, spent exercising each week

Her results are shown on the scatter diagram.



(a) Interpret the nature of the relationship between h and m

(1)

Anna codes the data using the formulae

$$x = \log_{10} m$$
$$y = \log_{10} h$$

The product moment correlation coefficient between x and y is -0.897

(b) Test whether or not there is significant evidence of a negative correlation between x and y
You should

- state your hypotheses clearly
- use a 5% level of significance
- state the critical value used

(3)

The equation of the line of best fit of y on x is

$$y = -0.05x + 1.92$$

(c) Use the equation of the line of best fit of y on x to find a model for h on m in the form

$$h = am^k$$

where a and k are constants to be found.

(5)

Question 6 continued

2. Fred and Nadine are investigating whether there is a linear relationship between Daily Mean Pressure, p hPa, and Daily Mean Air Temperature, t °C, in Beijing using the 2015 data from the large data set.

Fred randomly selects one month from the data set and draws the scatter diagram in Figure 1 using the data from that month.

The scale has been left off the horizontal axis.

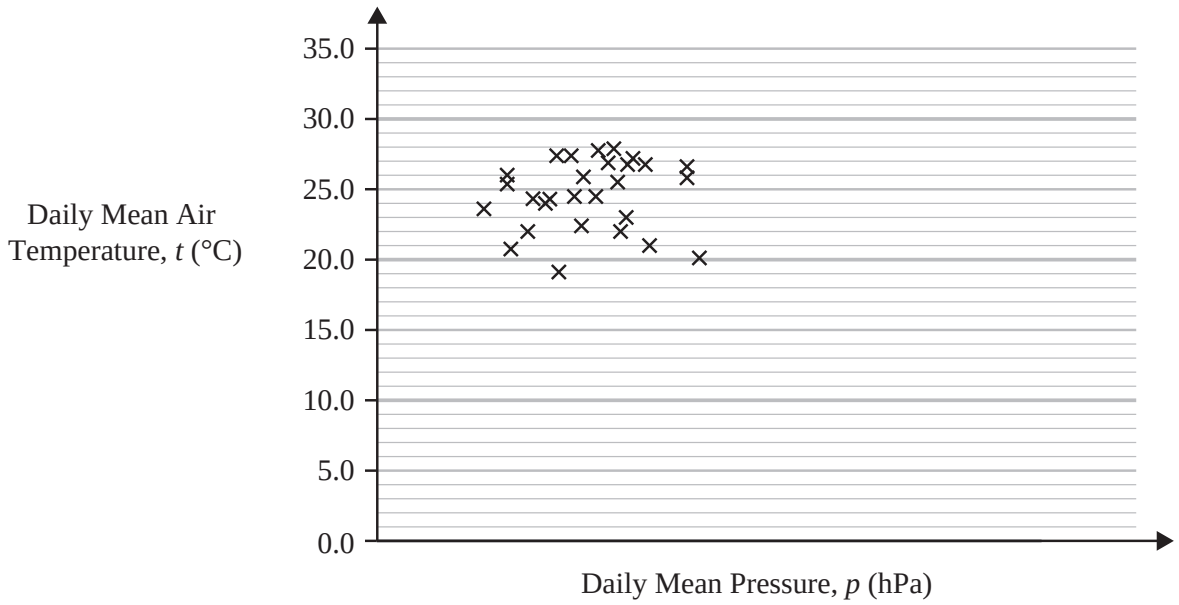


Figure 1

(a) Describe the correlation shown in Figure 1.

(1)

Nadine chooses to use all of the data for Beijing from 2015 and draws the scatter diagram in Figure 2.

She uses the same scales as Fred.

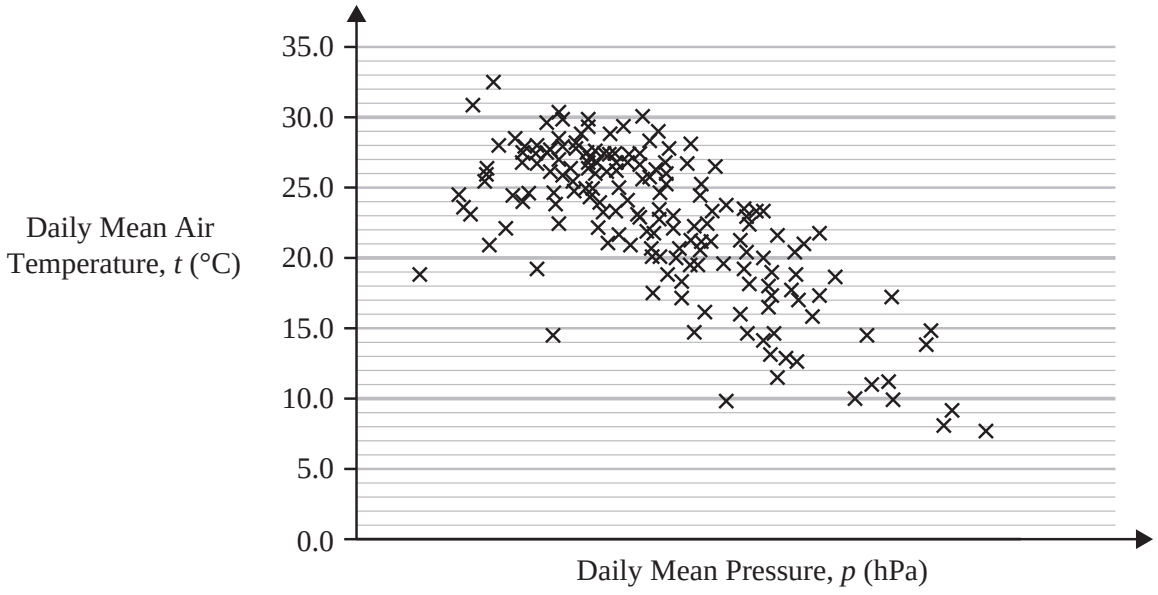


Figure 2

Question 2 continued

(b) Explain, in context, what Nadine can infer about the relationship between p and t using the information shown in Figure 2.

(1)

(c) Using your knowledge of the large data set, state a value of p for which interpolation can be used with Figure 2 to predict a value of t.

(1)

(d) Using your knowledge of the large data set, explain why it is not meaningful to look for a linear relationship between Daily Mean Wind Speed (Beaufort Conversion) and Daily Mean Air Temperature in Beijing in 2015.

(1)

(Total for Question 2 is 4 marks)

Question 2 continued

2. Amar is studying the flight of a bird from its nest.

He measures the bird's height above the ground, h metres, at time t seconds for 10 values of t
Amar finds the equation of the regression line for the data to be $h = 38.6 - 1.28t$

(a) Interpret the gradient of this line.

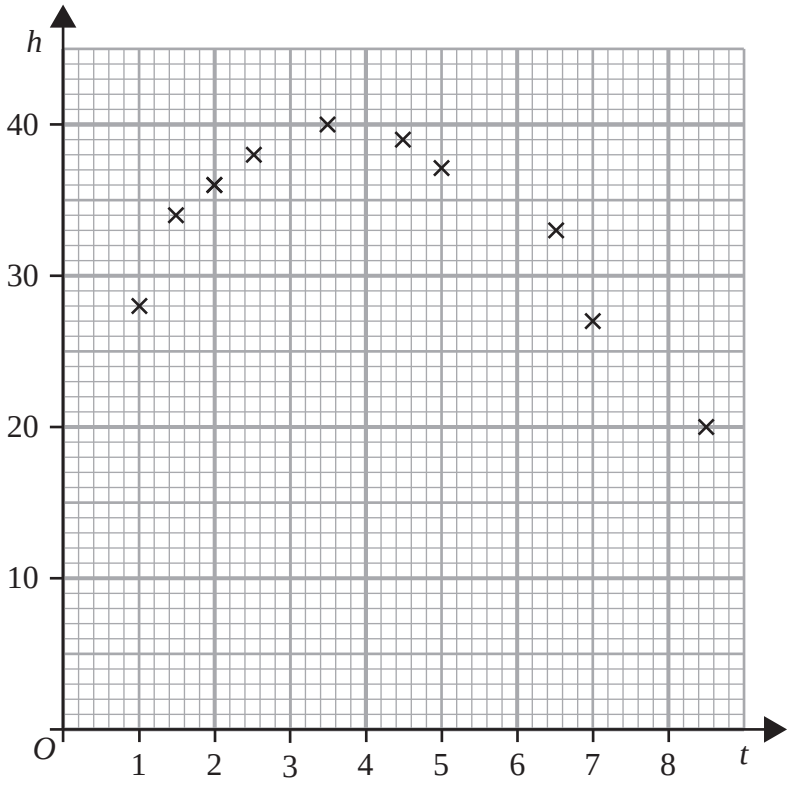
(1)

The product moment correlation coefficient between h and t is -0.510

(b) Test whether or not there is evidence of a negative correlation between the height above the ground and the time during the flight.
You should
- state your hypotheses clearly
- use a 5% level of significance
- state the critical value used

(3)

Jane draws the following scatter diagram for Amar's data.



(c) With reference to the scatter diagram, state, giving a reason, whether or not the regression line $h = 38.6 - 1.28t$ is an appropriate model for these data.

(1)

Jane suggests an improved model using the variable $u = (t - k)^2$ where k is a constant.

She obtains the equation $h = 38.1 - 0.78u$

(d) Choose a suitable value for k to write Jane's improved model for h in terms of t only.

(1)

Question 2 continued