# 5.
# Circles

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**17.** A circle $C$ with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle $C$ also passes through the point $(10, 1)$.

**(3)**

The tangent to the circle $C$ at the point $(10, 11)$ meets the $y$ axis at the point $P$ and the tangent to the circle $C$ at the point $(10, 1)$ meets the $y$ axis at the point $Q$.

(b) Show that the distance $PQ$ is 58 explaining your method clearly.

**(7)**

**Question 17 continued**

**3.** A circle $C$ has equation

$$x^2 + y^2 - 4x + 10y = k$$

where $k$ is a constant.

(a) Find the coordinates of the centre of $C$.

**(2)**

(b) State the range of possible values for $k$.

**(2)**

**(Total for Question 3 is 4 marks)**

**14.** The circle $C$ has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of $C$

(ii) the radius of $C$

**(3)**

The line with equation $y = kx$, where $k$ is a constant, cuts $C$ at two distinct points.

(b) Find the range of values for $k$.

**(6)**

**10.** A circle $C$ has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of $C$,

(ii) the exact radius of $C$.

**(3)**

The straight line with equation $x = k$, where $k$ is a constant, is a tangent to $C$.

(b) Find the possible values for $k$.

**(2)**

**11.** (i) A circle $C_1$ has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line $l$ is the tangent to $C_1$ at the point $P(-5, 7)$.

Find an equation of $l$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(5)**

(ii) A different circle $C_2$ has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where $k$ is a constant.

Given that $C_2$ lies entirely in the 4th quadrant, find the range of possible values for $k$.

**(4)**

**Question 11 continued**

**11.**

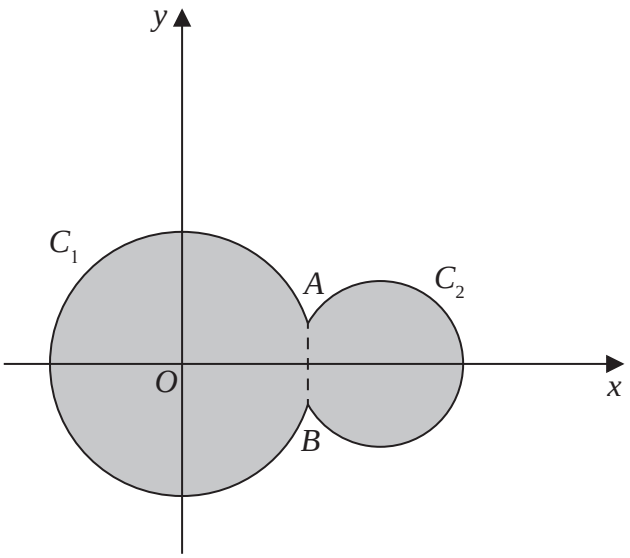


**Figure 3**

Circle $C_1$ has equation $x^2 + y^2 = 100$

Circle $C_2$ has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points $A$ and $B$ as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where $O$ is the origin.

**(4)**

The region shown shaded in Figure 3 is bounded by $C_1$ and $C_2$

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

**(4)**

**Question 11 continued**

**14.** A circle $C$ with radius $r$

- lies only in the 1st quadrant
- touches the $x$-axis and touches the $y$-axis

The line $l$ has equation $2x + y = 12$

(a) Show that the $x$ coordinates of the points of intersection of $l$ with $C$ satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

**(3)**

Given also that $l$ is a tangent to $C$,

(b) find the two possible values of $r$, giving your answers as fully simplified surds.

**(4)**

**Question 14 continued**

**15.**

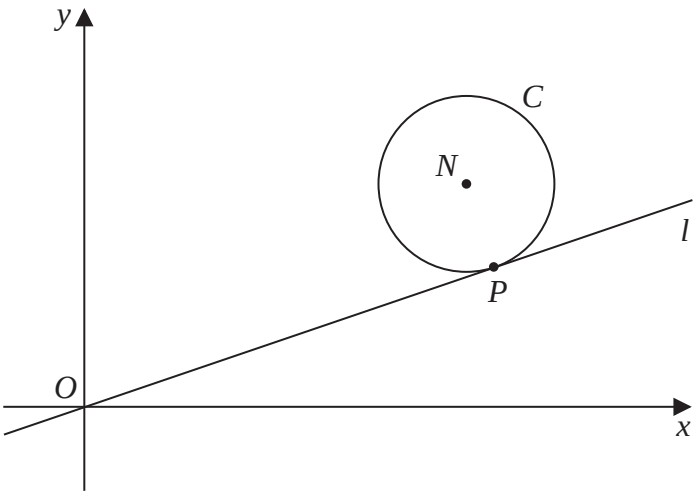


**Figure 4**

Figure 4 shows a sketch of a circle $C$ with centre $N(7, 4)$

The line $l$ with equation $y = \frac{1}{3}x$ is a tangent to $C$ at the point $P$.

Find

(a) the equation of line $PN$ in the form $y = mx + c$, where $m$ and $c$ are constants,

**(2)**

(b) an equation for $C$.

**(4)**

The line with equation $y = \frac{1}{3}x + k$, where $k$ is a non-zero constant, is also a tangent to $C$.

(c) Find the value of $k$.

**(3)**

**Question 15 continued**

**7.** The circle $C$ has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of $C$,

(ii) the exact radius of $C$, giving your answer as a simplified surd.

**(4)**

The line $l$ has equation $y = 3x + k$ where $k$ is a constant.

Given that $l$ is a tangent to $C$,

(b) find the possible values of $k$, giving your answers as simplified surds.

**(5)**

**Question 7 continued**

**11.**

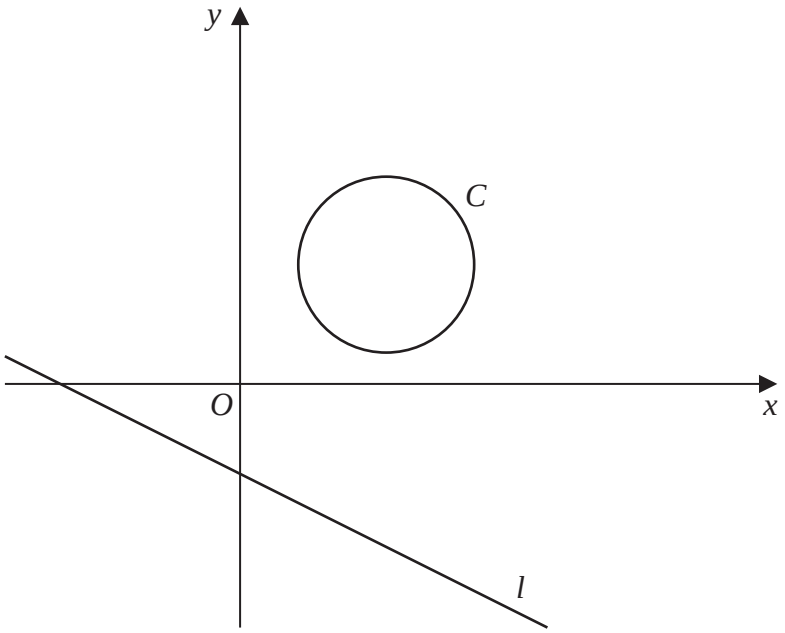


**Figure 3**

Figure 3 shows the circle $C$ with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line $l$ with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of $C$,

(ii) the radius of $C$.

**(3)**

(b) Find the shortest distance between $C$ and $l$.

**(5)**

**Question 11 continued**

**3.** A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

**(3)**

Given that $P$ is the point on the circle that is furthest away from the origin $O$,

(b) find the exact length $OP$

**(2)**

**6.** The circle $C$ has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where $k$ is a constant.

(a) Find the coordinates of the centre of $C$.

**(2)**

Given that $C$ does not cut or touch the $x$-axis,

(b) find the range of possible values for $k$.

**(3)**

**10.** A circle $C$ has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where $k$ is a constant.

(a) Find in terms of $k$,

(i) the coordinates of the centre of $C$

(ii) the radius of $C$

**(3)**

The line with equation $y = 2x - 1$ intersects $C$ at 2 distinct points.

(b) Find the range of possible values of $k$.

**(6)**

**Question 10 continued**

**10.**

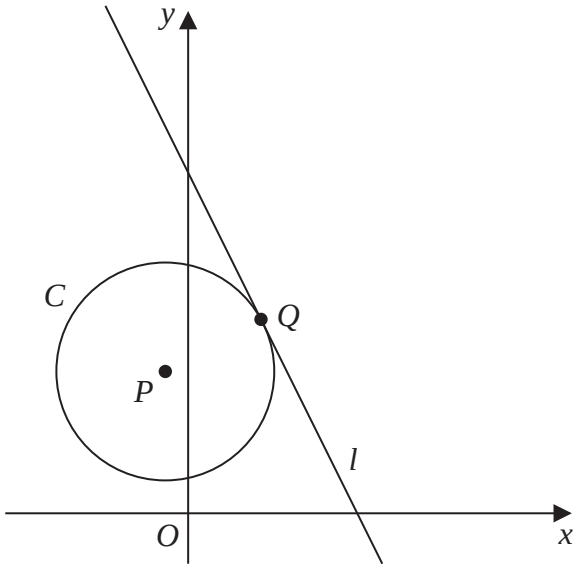


**Figure 4**

Figure 4 shows a sketch of the circle $C$

- the point $P$ $(-1, k + 8)$ is the centre of $C$
- the point $Q$ $(3, k^2 - 2k)$ lies on $C$
- $k$ is a positive constant
- the line $l$ is the tangent to $C$ at $Q$

Given that the gradient of $l$ is $-2$

(a) show that

$$k^2 - 3k - 10 = 0$$

**(4)**

(b) Hence find an equation for $C$

**(4)**

**Question 10 continued**

**14.** The circle $C_1$ has equation

$$x^2 + y^2 - 6x + 14y + 33 = 0$$

(a) Find

(i) the coordinates of the centre of $C_1$

(ii) the radius of $C_1$

**(3)**

A different circle $C_2$

- has centre with coordinates $(-6, -8)$
- has radius $k$, where $k$ is a constant

Given that $C_1$ and $C_2$ intersect at 2 distinct points,

(b) find the range of values of $k$, writing your answer in set notation.

**(5)**

**Question 14 continued**