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Moments and Static Rigid Bodies

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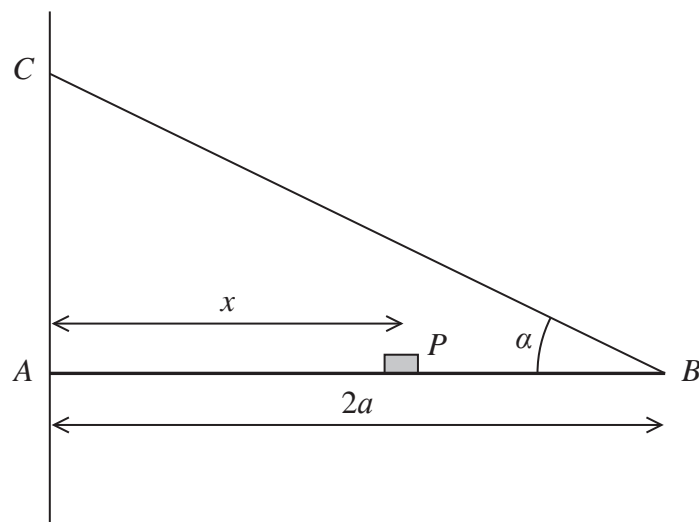


Figure 3

A plank, AB , of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C , which is vertically above A .

A small block of mass $3M$ is placed on the plank at the point P , where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$ (3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a . (2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan \beta$ (5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P . You must justify your answer carefully. (3)

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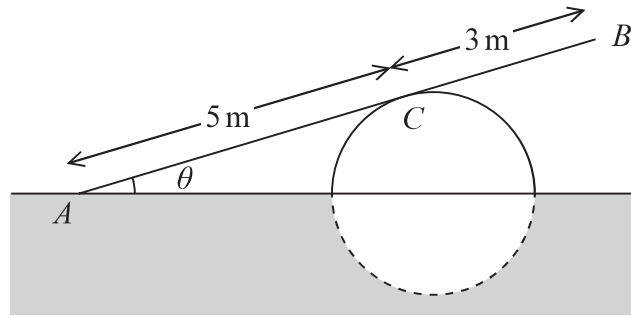


Figure 2

A ramp, AB , of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A .

The point of contact between the ramp and the drum is C , where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan \theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp. (1)

(b) Find the magnitude of the resultant force acting on the ramp at A . (9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B ,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C . (1)

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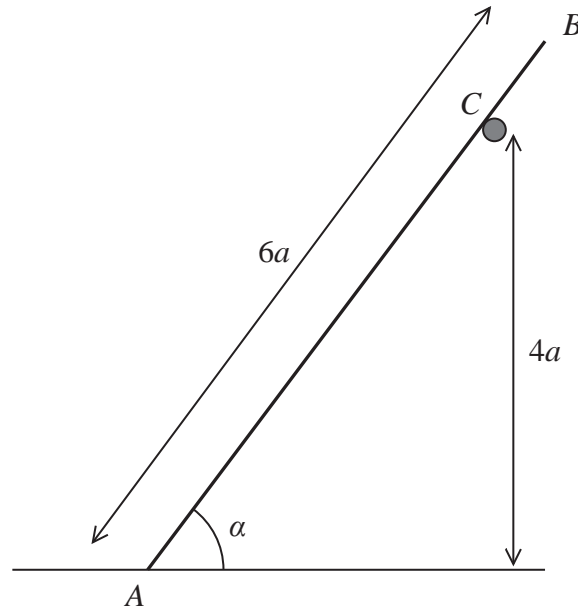


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C .

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ .

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$ (3)

(b) Hence, or otherwise, find the value of μ . (7)

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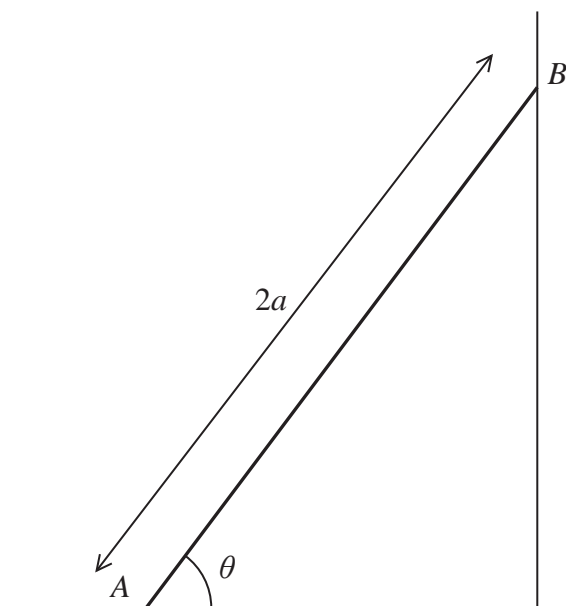


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ , as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geq \frac{1}{2} \cot \theta$ (5)

A horizontal force of magnitude kmg , where k is a constant, is now applied to the beam at A .

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan \theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k . (5)

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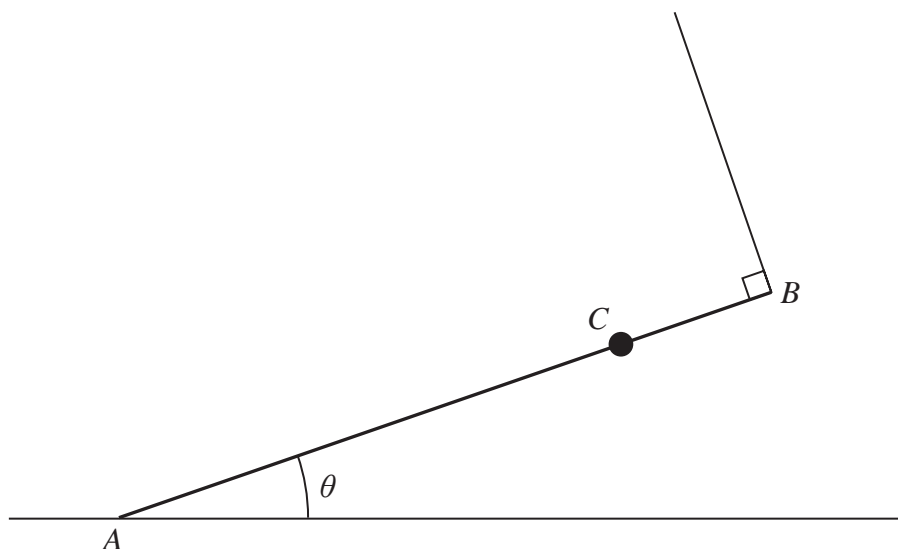


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C , where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

- (a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

- (b) Show that $T = 2Mg \cos \theta$

(3)

Given that $\cos \theta = \frac{3}{5}$

- (c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

- (d) show that $\mu = \frac{8}{19}$

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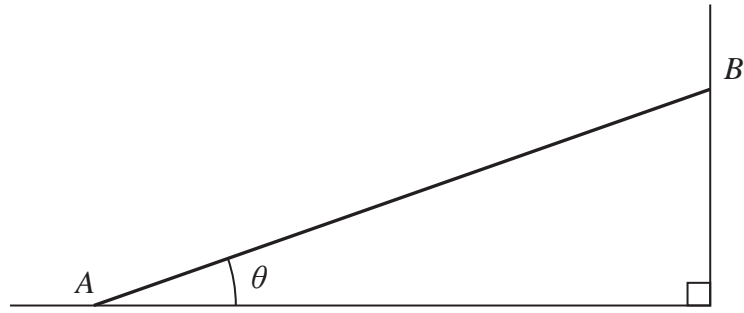


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A . **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S .

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A , show that

$$S = \frac{1}{2} Mg \cot \theta \quad (3)$$

The coefficient of friction between the rod and the ground is μ

Given that $\tan \theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g , the magnitude of the resultant force acting on the rod at A .

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A .

A new value for S is calculated using this new model, with $\tan \theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

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