

Chapter 6: Sampling and Sampling Distribution

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2. A farmer sells boxes of eggs.

The eggs are sold in boxes of 6 eggs and boxes of 12 eggs in the ratio $n:1$

A random sample of three boxes is taken.

The number of eggs in the first box is denoted by X_1

The number of eggs in the second box is denoted by X_2

The number of eggs in the third box is denoted by X_3

The random variable $T = X_1 + X_2 + X_3$

Given that $P(T = 18) = 0.729$

- (a) show that $n = 9$

(2)

- (b) find the sampling distribution of T

(4)

The random variable R is the range of X_1, X_2, X_3

- (c) Using your answer to part (b), or otherwise, find the sampling distribution of R

(2)

Question 2 continued

6.2

4. The volume of milk, M litres, in cartons produced by a dairy, has distribution $N(\mu, \sigma^2)$, where μ and σ are unknown. A random sample of 12 cartons is taken and the volume of milk in each carton is measured ($M_1, M_2, \dots, M_{12}$). A statistic X is based on this sample.

(a) Explain what is meant by “a random sample” in this case.

(1)

(b) State the population in this case.

(1)

(c) Write down the distribution of $\frac{M_{12} - \mu}{\sigma}$

(1)

(d) Explain what you understand by the sampling distribution of X .

(1)

(e) State, giving a reason, which of the following is not a statistic based on this sample.

(I) $3M_1 + \frac{2M_{11}}{6}$

(II) $\sum_{i=1}^{12} \left(\frac{M_i - \mu}{\sigma} \right)^2$

(III) $\sum_{i=1}^{12} (2M_i - 3)$

(2)

Question 4 continued

(Total 6 marks)

4. A bag contains a large number of marbles, each of which is blue or red.

A random sample of 3 marbles is taken from the bag.

The random variable D represents the number of blue marbles taken minus the number of red marbles taken.

Given that 20% of the marbles in the bag are blue,

- (a) show that $P(D = -1) = 0.384$ (2)

- (b) find the sampling distribution of D (3)

- (c) write down the mode of D (1)

Takashi claims that the true proportion of blue marbles is greater than 20% and tests his claim by selecting a random sample of 12 marbles from the bag.

- (d) Find the critical region for this test at the 10% level of significance. (2)

- (e) State the actual significance level of this test. (1)

Question 4 continued

6.6

2. The random variable X has probability distribution

x	-1	0	1
$P(X = x)$	0.2	0.4	0.4

Samples of size 2 are selected at random from the distribution and the random variable T is the sum of the 2 values of X .

- (a) Show that $P(T = 0) = 0.32$

(2)

- (b) Find the sampling distribution of T .

(4)

5. A bag contains a large number of 1p coins and 2p coins in the ratio 2 : 3 respectively.

A jar contains a large number of 2p coins and 5p coins in the ratio 1 : 4 respectively.

Two coins are selected at random from the bag and one coin is selected at random from the jar. The random variable T represents the total of the values of the 3 coins selected.

- (a) Find the sampling distribution of T .

(7)

- (b) Find the sampling distribution of the median of the values of the 3 selected coins.

(4)

Question 5 continued

6.9

- (1)

A random sample of 3 bags is taken from the factory.

- (7)

(c) Calculate the minimum value of n such that $P(Y = 0) < 0.2$

(3)

Question 6 continued

6.11

6. The owner of a very large youth club has designed a new method for allocating people to teams. Before introducing the method he decided to find out how the members of the youth club might react.

(a) Explain why the owner decided to take a random sample of the youth club members rather than ask all the youth club members.

(1)

(b) Suggest a suitable sampling frame.

(1)

(c) Identify the sampling units.

(1)

The new method uses a bag containing a large number of balls. Each ball is numbered either 20, 50 or 70

When a ball is selected at random, the random variable X represents the number on the ball where

$$P(X = 20) = p \quad P(X = 50) = q \quad P(X = 70) = r$$

A youth club member takes a ball from the bag, records its number and replaces it in the bag.

He then takes a second ball from the bag, records its number and replaces it in the bag.

The random variable M is the mean of the 2 numbers recorded.

Given that

$$P(M = 20) = \frac{25}{64} \quad P(M = 60) = \frac{1}{16} \quad \text{and} \quad q > r$$

(d) show that $P(M = 50) = \frac{1}{16}$

(7)

Question 6 continued

4. A bag contains a large number of balls, each with one of the numbers 1, 2 or 5 written on it in the ratio 2 : 3 : 4 respectively.

A random sample of 3 balls is taken from the bag.

The random variable B represents the range of the numbers written on the balls in the sample.

(i) Find $P(B = 4)$

(ii) Find the sampling distribution of B .

(10)

Question 4 continued

- 40% of the counters are numbered 1
35% of the counters are numbered 2
25% of the counters are numbered 3

(a) Show that Alif gets a score of 8 with probability 0.0875

(1)

- (5)

- (2)

Question 5 continued

(Total 8 marks)

- 6 (a) Explain what you understand by the sampling distribution of a statistic.

(1)

At Sam's cafe a standard breakfast consists of 6 breakfast items. Customers can then choose to upgrade to a medium breakfast by adding 1 extra breakfast item or they can upgrade to a large breakfast by adding 2 extra breakfast items. Standard, medium and large breakfasts are sold in the ratio 6:3:2 respectively.

A random sample of 2 customers is taken from customers who have bought a breakfast from Sam's cafe on a particular day.

- (b) Find the sampling distribution for the total number, T , of breakfast items bought by these 2 customers. Show your working clearly.

(7)

- (c) Find $E(T)$

(2)

Question 6 continued

7. A bag contains 10 counters each with exactly one number written on it.

There are 6 counters with the number 7 on them

There are 3 counters with the number 8 on them

There is 1 counter with the number 9 on it

A random sample of 3 counters is taken from the bag (without replacement).

These counters are then put back in the bag.

This process is then repeated until 20 samples have been taken.

The random variable Y represents the number of these 20 samples that contain the counter with the number 9 on it.

(a) (i) Find the mean of Y

(ii) Find the variance of Y

(5)

A random sample of 3 counters is chosen from the bag (without replacement).

(b) List all possible samples where the median of the numbers on the 3 counters is 7

(2)

(c) Find the sampling distribution of the median of the numbers on the 3 counters.

(5)

Question 7 continued

6. A bag contains a large number of counters with one of the numbers 5, 10 or 20 written on each of them in the ratio $5:2:a$

A jar contains a large number of counters with one of the numbers 5 or 10 written on each of them in the ratio $1:3$

One counter is selected at random from the bag and then two counters are selected at random from the jar.

The random variable R represents the range of the numbers on the 3 counters.

Given that $P(R = 15) = \frac{63}{256}$

- (a) by forming and solving an equation in a , show that $a = 9$

(3)

- (b) find the sampling distribution of R

(6)

Question 6 continued

(2)

- (1)

- (2)

- (3)

- (3)

- (1)

It asks a sample of 300 members for their opinions on the fees it charges for using the centre.

(b) (i) identify a suitable sampling frame,

- (2)

A is the list of the different types of membership that can be paid for by members.

C is the number in the **sample** of 300 members who are satisfied with the fees they pay.

- (1)

- (2)

(2)

- (6)

- (6)

(3)

Question 6 continued

- 4 A bag contains 50 counters, each with one of the numbers 4, 7 or 10 written on it in the ratio 2:3:5 respectively.

A random sample of 2 counters is taken from the bag. The numbers on the 2 counters are recorded as D_1 and D_2

The random variable M represents the mean of D_1 and D_2

- (a) Show that $P(M = 4) = \frac{9}{245}$ (1)
- (b) Find the sampling distribution of M (6)

A random sample of n sets of 2 counters is taken. The random variable T represents the number of these n sets of 2 counters that have a mean of 4

Given that each set of 2 counters is replaced after it is drawn,

- (c) calculate the minimum value of n such that $P(T = 0) < 0.15$ (3)

Question 4 continued

Organised by Mr Omar Faruk