

Chapter 2: Poisson Distribution

Mr Faruk

Teacher of Mathematics
BSc/MSc/PGCE Mathematics

✉ cieigcsolutions@gmail.com



5. A delivery company loses packages randomly at a mean rate of 10 per month.

The probability that the delivery company loses more than 12 packages in a randomly selected month is p

- (a) Find the value of p

(1)

The probability that the delivery company loses more than k packages in a randomly selected month is at least $2p$

- (b) Find the largest possible value of k

(2)

In a randomly selected month,

- (c) find the probability that exactly 4 packages were lost in each half of the month.

(3)

In a randomly selected two-month period, 21 packages were lost.

- (d) Find the probability that at least 10 packages were lost in each of these two months.

(4)

- (e) Using a suitable approximation, find the probability that more than 27 packages are lost during a randomly selected 4-month period.

(5)

Question 5 continued

2.2

5. Cars stop at a service station randomly at a rate of 3 every 5 minutes.
- (a) Calculate the probability that in a randomly selected 10 minute period,
- (i) exactly 7 cars will stop at the service station,
 - (ii) more than 7 cars will stop at the service station.

(4)

1. Each day a restaurant opens between 11 am and 11 pm. During its opening hours, the restaurant receives calls for reservations at an average rate of 6 per hour.
 - (a) Find the probability that the restaurant receives exactly 1 call for a reservation between 6pm and 7pm.

(2)

7. Members of a conservation group record the number of sightings of a rare animal. The number of sightings follows a Poisson distribution with a rate of 1 every 2 months.
- (a) Find the smallest value of n such that the probability that there are at least n sightings in 2 months is less than 0.05
(2)
 - (b) Find the smallest number of months, m , such that the probability of no sightings in m months is less than 0.05
(2)
 - (c) Find the probability that there is at least 1 sighting per month in each of 3 consecutive months.
(3)
 - (d) Find the probability that the number of sightings in an 8 month period is equal to the expected number of sightings for that period.
(2)
 - (e) Given that there were 4 sightings in a 4 month period, find the probability that there were more sightings in the last 2 months than in the first 2 months.
(3)

Question 7 continued

2.6

2. During morning hours, employees arrive randomly at an office drinks dispenser at a rate of 2 every 10 minutes.

The number of employees arriving at the drinks dispenser is assumed to follow a Poisson distribution.

- (a) Find the probability that fewer than 5 employees arrive at the drinks dispenser during a 10-minute period one morning.

(2)

During a 30-minute period one morning, the probability that n employees arrive at the drinks dispenser is the same as the probability that $n + 1$ employees arrive at the drinks dispenser.

- (b) Find the value of n

(3)

During a 45-minute period one morning, the probability that between c and 12, inclusive, employees arrive at the drinks dispenser is 0.8546

- (c) Find the value of c

(3)

- (d) Find the probability that exactly 2 employees arrive at the drinks dispenser in exactly 4 of the 6 non-overlapping 10-minute intervals between 10 am and 11 am one morning.

(4)

Question 2 continued

2.8

Answer ALL questions. Write your answers in the spaces provided.

1. A company produces NuttyChok bars that are made of chocolate and whole hazelnuts.

A 40 kg batch of the chocolate and hazelnut mixture used to make NuttyChok bars will contain 4750 hazelnuts.

- (a) Show that the mean number of hazelnuts in an 80 g bar of NuttyChok is 9.5 (1)

A Poisson model is suggested for the number of hazelnuts in each 80 g bar of NuttyChok.

- (b) State two assumptions that need to be made for a Poisson model to be suitable in this case. (2)

- (c) Find the probability that an 80 g bar of NuttyChok

(i) contains exactly 12 hazelnuts,

(ii) contains fewer than 8 hazelnuts. (4)

- (d) Find the probability that a pack of three 80 g bars of NuttyChok includes exactly one bar with fewer than 8 hazelnuts. (3)

Question 1 continued

2.10

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Question 1 continued

2.13

4. In a peat bog, Common Spotted-orchids occur at a mean rate of 4.5 per m^2
- (a) Give an assumption, not already stated, that is required for the number of Common Spotted-orchids per m^2 of the peat bog to follow a Poisson distribution. (1)

Given that the number of Common Spotted-orchids in 1 m^2 of the peat bog can be modelled by a Poisson distribution,

- (b) find the probability that in a randomly selected 1 m^2 of the peat bog
- (i) there are exactly 6 Common Spotted-orchids,
- (ii) there are fewer than 10 but more than 4 Common Spotted-orchids. (4)

Question 4 continued

2.15

3. The number of water fleas, in 100ml of pond water, has a Poisson distribution with mean 7

- (a) Find the probability that a sample of 100ml of the pond water does **not** contain exactly 4 water fleas.

(2)

Aja collects 5 separate samples, each of 100 ml, of the pond water.

- (b) Find the probability that exactly 1 of these samples contains exactly 4 water fleas.

(3)

Question 3 continued

2.17

Question 2 continued

2.19

4. The number of cars entering a safari park per 10-minute period can be modelled by a Poisson distribution with mean 6

(a) Find the probability that in a given 10-minute period exactly 8 cars will enter the safari park.

(2)

(b) Find the smallest value of n such that the probability that at least n cars enter the safari park in 10 minutes is less than 0.05

(2)

The probability that no cars enter the safari park in m minutes, where m is an integer, is less than 0.05

(c) Find the smallest value of m

(2)

A car enters the safari park.

(d) Find the probability that there is less than 5 minutes before the next car enters the safari park.

(3)

Given that exactly 15 cars entered the safari park in a 30-minute period,

(e) find the probability that exactly 1 car entered the safari park in the first 5 minutes of the 30-minute period.

(4)

Question 4 continued

2.21

- 1 A local pottery makes cups. The number of faulty cups made by the pottery in a week follows a Poisson distribution with a mean of 6

In a randomly chosen week, the probability that there will be at least x faulty cups made is 0.1528

- (a) Find the value of x

(3)

1. The independent random variables W and X have the following distributions.

$$W \sim \text{Po}(4) \quad X \sim \text{B}(3, 0.8)$$

- (a) Write down the value of the variance of W

(1)

- (b) Determine the mode of X
Show your working clearly.

(2)

One observation from each distribution is recorded as W_1 and X_1 respectively.

- (c) Find $P(W_1 = 2 \text{ and } X_1 = 2)$

(3)

- (d) Find $P(X_1 < W_1)$

(4)

Question 1 continued

2.24

- A randomly selected 50 millilitre sample of the solution is taken.

(a) Find the probability that

- (i) exactly 10 particles are found,
- (ii) between 6 and 11 particles (inclusive) are found.

(4)

Petra takes 12 independent samples of m millilitres of the solution.

The probability that at least 2 of these samples contain no particles is 0.1184

(b) Using the Statistical Tables provided, find the value of m

(6)

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Question 5 continued

2.26

- (3)

She selects 15 rolls at random.

- (4)

She sells each piece of her cloth that contains faults for £2.00

- (4)

Question 1 continued

2.28

5. A company produces steel cable.

Defects in the steel cable produced by this company occur at random, at a constant rate of 1 defect per 16 metres.

On one day the company produces a piece of steel cable 80 metres long.

(a) Find the probability that there are at most 5 defects in this piece of steel cable.

(2)

7. A bakery sells muffins individually at an average rate of 8 muffins per hour.

- (a) Find the probability that, in a randomly selected one-hour period, the bakery sells at least 4 but not more than 8 muffins.

(3)

A sample of 5 non-overlapping **half-hour** periods is selected at random.

- (b) Find the probability that the bakery sells fewer than 3 muffins in exactly 2 of these periods.

(5)

Given that 4 muffins were sold in a one-hour period,

- (c) find the probability that more muffins were sold in the first 15 minutes than in the last 45 minutes.

(4)

Question 7 continued

2.32

5. A supermarket receives complaints at a mean rate of 6 per week.

(a) State one assumption necessary, in order for a Poisson distribution to be used to model the number of complaints received by the supermarket.

(1)

(b) Find the probability that, in a given week, there are

(i) fewer than 3 complaints received by the supermarket,

(ii) at least 6 complaints received by the supermarket.

(3)

1. The manager of a supermarket is investigating the number of complaints per day received from customers.

A random sample of 180 days is taken and the results are shown in the table below.

Number of complaints per day	0	1	2	3	4	5	6	≥ 7
Frequency	12	28	37	38	29	17	19	0

- (a) Calculate the mean and the variance of these data. (3)
- (b) Explain why the results in part (a) suggest that a Poisson distribution may be a suitable model for the number of complaints per day. (1)

The manager uses a Poisson distribution with mean 3 to model the number of complaints per day.

- (c) For a randomly selected day find, using the manager’s model, the probability that there are
- (i) at least 3 complaints,
- (ii) more than 4 complaints but less than 8 complaints. (4)

Question 1 continued

2.35

- 1 A garage sells tyres. The number of customers arriving at the garage to buy tyres in a 10-minute period is modelled by a Poisson distribution with mean 2

(a) Find the probability that

- (i) fewer than 4 customers arrive to buy tyres in the next 10 minutes,
- (ii) more than 5 customers arrive to buy tyres in the next 10 minutes.

(3)

The manager randomly selects 20 non-overlapping, 30-minute periods.

- (b) Find the probability that there are between 4 and 7 (inclusive) customers arriving to buy tyres in exactly 15 of these 30-minute periods.

(4)

The manager believes that placing an advert in the local paper will lead to a significant increase in the number of customers arriving at the garage.

A week after the advert is placed, the manager randomly selects a 25-minute period and finds that 10 customers arrive at the garage to buy tyres.

- (c) Test, at the 5% level of significance, whether or not there is evidence to support the manager's belief.
State your hypotheses clearly.

(5)

- (d) Explain why the Poisson distribution is unlikely to be valid for the number of tyres sold during a 10-minute period.

(1)

Question 1 continued

2.37