

Chapter 1: Binomial Distribution

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4. A sweet shop produces different coloured sweets and sells them in bags.

The proportion of green sweets produced is p

Each bag is filled with a random sample of n sweets.

The mean number of green sweets in a bag is 4.2 and the variance is 3.57

- (a) Find the value of n and the value of p

(4)

The proportion of red sweets produced by the shop is 0.35

- (b) Find the probability that, in a random sample of 25 sweets, the number of red sweets exceeds the expected number of red sweets.

(3)

The shop claims that 10% of its customers buy more than two bags of sweets. A random sample of 40 customers is taken and 1 customer buys more than two bags of sweets.

- (c) Test, at the 5% level of significance, whether or not there is evidence that the proportion of customers who buy more than two bags of sweets is less than the shop's claim. State your hypotheses clearly.

(4)

1. A salesman sells insurance to people. Each day he chooses a number of people to contact. The probability that the salesman sells insurance to a person he contacts is 0.05

On Monday he chooses to contact 10 people.

- (a) Find the probability that on Monday the salesman sells insurance to
(i) exactly 1 person,
(ii) at least 3 people. (3)
- (b) Find the number of people he should contact each day in order to sell insurance, on average, to 3 people per day. (2)
- (c) Calculate the least number of people he must choose to contact on Friday, so that the probability of selling insurance to at least 1 person on Friday exceeds 0.99 (4)

Question 1 continued

(Total 9 marks)

2. At a cafe, customers ordering hot drinks order either tea or coffee.
Of all customers ordering hot drinks, 80% order tea and 20% order coffee.
Of those who order tea, 35% take sugar and of those who order coffee 60% take sugar.

- (a) A random sample of 12 customers ordering hot drinks is selected.

Find the probability that fewer than 3 of these customers order coffee.

(3)

- (b) (i) A randomly selected customer who orders a hot drink is chosen. Show that the probability that the customer takes sugar is 0.4

(1)

- (ii) Write down the distribution for the number of customers who take sugar from a random sample of n customers ordering hot drinks.

(1)

- (c) A random sample of 10 customers ordering hot drinks is selected.

- (i) Find the probability that exactly 4 of these 10 customers take sugar.

(1)

- (ii) Given that at least 3 of these 10 customers take sugar, find the probability that no more than 6 of these 10 customers take sugar.

(3)

Question 2 continued

1.5

- (3)

Question 1 continued

1.7

6. Sarah makes clay pots in batches of size n . The clay pots need to be fired in a kiln. Sarah finds that 30% of the clay pots crack when they are fired. The remainder do not crack.

- (a) Explain what assumptions need to be made to model with a binomial distribution the number of clay pots in a batch that crack when fired.

(2)

Sarah makes and fires a batch of 8 clay pots.

- (b) Write down the expected number of clay pots that crack.

(1)

- (c) Find the probability that exactly 2 clay pots crack.

(2)

The probability that at least k clay pots crack is 0.0113

- (d) Find the value of k .

(2)

Question 6 continued

1.9

- (a) Show that $P(X > 2) = p^3(4 - 3p)$

(3)

(b) find $P(X > 2)$

(4)

(c) find the probability that exactly 3 people in the sample wear glasses.

(2)

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Question 1 continued

2. Duck eggs are classed as large or small.

Kiyoshi knows that 35% of the eggs laid by his ducks are classed as large and the rest are classed as small. The eggs are randomly allocated to boxes each containing 6 eggs.

A box is selected at random.

Calculate the probability that this box contains

- (a) (i) exactly 2 large eggs,

- (ii) more large eggs than small eggs.

(5)

Kiyoshi believes that the proportion of large eggs produced by the ducks will increase if he adds a supplement to the ducks' food. To check his belief he takes a random sample of 50 eggs after the supplement has been added to the ducks' food and finds that 25 of them are large eggs.

Kiyoshi sells boxes that contain more large eggs than small eggs for a profit of £1.20 and the rest of the boxes for a profit of £0.60

The proportion of large eggs produced by Kiyoshi's ducks has increased to 45% and the cost of the supplement is £0.10 per box of eggs. Given that the eggs are still randomly allocated to boxes,

- (c) explain whether or not Kiyoshi should continue to add the supplement to the ducks' food in order to make a greater profit.

(5)

Question 2 continued

1.13

- (ii) at least 6 flawed plates.

(4)

Question 3 continued

1.15

1. Jim farms oysters in a particular lake. He knows from past experience that 5% of young oysters do not survive to be harvested.

In a random sample of 30 young oysters, the random variable X represents the number that do not survive to be harvested.

- (a) Write down a suitable model for the distribution of X .

(1)

- (b) State an assumption that has been made for the model in part (a).

(1)

- (c) Find the probability that

- (i) exactly 24 young oysters **do survive** to be harvested,
(ii) at least 3 young oysters do not survive to be harvested.

(4)

Question 1 continued

1.17

- (5)

(2)

Question 1 continued

1.19

- A random sample of 30 people who bought eggs is taken. The random variable F denotes the number of people who do **not** buy free range eggs.

(2)

(2)

A random sample of 30 customers who buy eggs from the farm shop is taken.

- (3)

Question 1 continued

1.21

- (1)

7. A bag contains 10 counters each with exactly one number written on it.

There are 6 counters with the number 7 on them

There are 3 counters with the number 8 on them

There is 1 counter with the number 9 on it

A random sample of 3 counters is taken from the bag (without replacement).
These counters are then put back in the bag.

This process is then repeated until 20 samples have been taken.

The random variable Y represents the number of these 20 samples that contain the counter with the number 9 on it.

(a) (i) Find the mean of Y

(ii) Find the variance of Y

(5)

Question 7 continued

4. The probability that a person completes a particular task in less than 15 minutes is 0.4

Jeffrey selects 20 people at random and asks them to complete the task. The random variable, X , represents the number of people who complete the task in less than 15 minutes.

- (a) Find $P(5 \leq X < 8)$

(3)

3. *Superbounce* is a manufacturer of tennis balls.

It knows from past records that 10% of its tennis balls fail a bounce test.

(a) Find the probability that from a random sample of 10 of these tennis balls

(i) at least 4 fail the bounce test

(ii) more than 1 but fewer than 5 fail the bounce test.

(4)

Question 3 continued

1.27

- (1)

Question 1 continued

1.29

- For each call to a customer

- Past records show that, for each call to a customer, the probability that Sam makes a sale is 0.2

- (i) exactly 2 sales in 14 calls,

- (4)

(b) Find the minimum value of n

- (i) so that the expected number of sales each day is at least 6

- (ii) so that the probability of at least 1 sale in a randomly selected day exceeds 0.95

- (2)

- (4)

Question 1 continued

1.31

- (3)

Question 6 continued

1.33