# Chapter 8: Numerical Methods

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**2.** $f(x) = x^3 + 3x^2 + 4x - 12$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3 - x)}{(3 + x)}\right)} \quad x \neq -3$$

**(3)**

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3 - x_n)}{(3 + x_n)}\right)} \quad n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of $x_1$, $x_2$ and $x_3$

**(3)**

The root of $f(x) = 0$ is $\alpha$.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

**(2)**

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**7.**

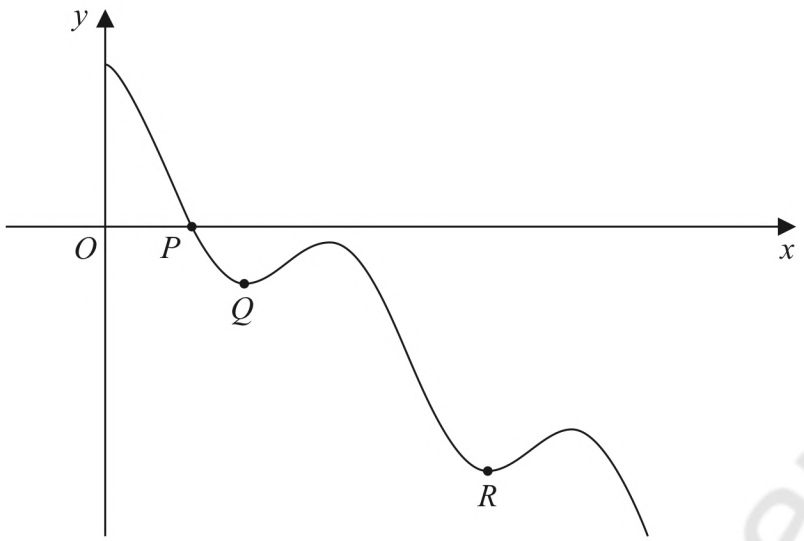


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation

$$y = 2\cos 3x - 3x + 4 \qquad x > 0$$

where $x$ is measured in radians.

The curve crosses the $x$-axis at the point $P$, as shown in Figure 3.

Given that the $x$ coordinate of $P$ is $\alpha$,

(a) show that $\alpha$ lies between 0.8 and 0.9

**(2)**

The iteration formula

$$x_{n+1} = \frac{1}{3}\arccos(1.5x_n - 2)$$

can be used to find an approximate value for $\alpha$.

(b) Using this iteration formula with $x_1 = 0.8$ find, to 4 decimal places, the value of

(i) $x_2$

(ii) $x_5$

**(3)**

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**6.**

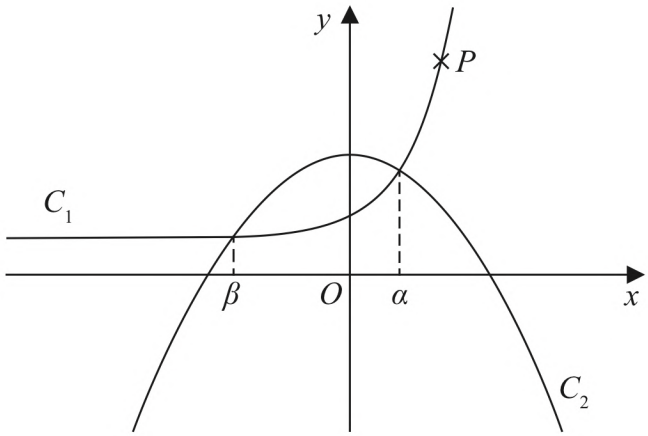


**Figure 3**

Figure 3 shows a sketch of curve $C_1$ with equation $y = 5e^{x-1} + 3$

and curve $C_2$ with equation $y = 10 - x^2$

The point $P$ lies on $C_1$ and has $y$ coordinate 18

(a) Find the $x$ coordinate of $P$, writing your answer in the form $\ln k$, where $k$ is a constant to be found.

**(3)**

The curve $C_1$ meets the curve $C_2$ at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

**(3)**

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$$

is used to find an approximation to $\beta$.

Using this iterative formula with $x_1 = -3$

(c) find the value of $x_2$ and the value of $\beta$, giving each answer to 6 decimal places.

**(3)**

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**6.**

$$f(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find $f'(x)$

**(2)**

(b) Show that the equation $f'(x) = 0$ can be written as

$$x = k\arctan\left(\frac{k}{x}\right)$$

where $k$ is an integer to be found.

**(2)**

(c) Starting with $x_1 = 2.5$ use the iteration formula

$$x_{n+1} = k\arctan\left(\frac{k}{x_n}\right)$$

with the value of $k$ found in part (b), to calculate the values of $x_2$ and $x_6$ giving your answers to 3 decimal places.

**(2)**

(d) Using a suitable interval and a suitable function that should be stated, show that a root of $f'(x) = 0$ is 2.581 correct to 3 decimal places.

**(2)**

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**1.**

Using the iteration formula

$$x_{n+1} = 2\arctan\left(\frac{4}{x_n}\right) \qquad x_1 = 2$$

(b) find the value of $x_2$ and the value of $x_6$, giving your answers to 3 decimal places.

**(3)**

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**9.**

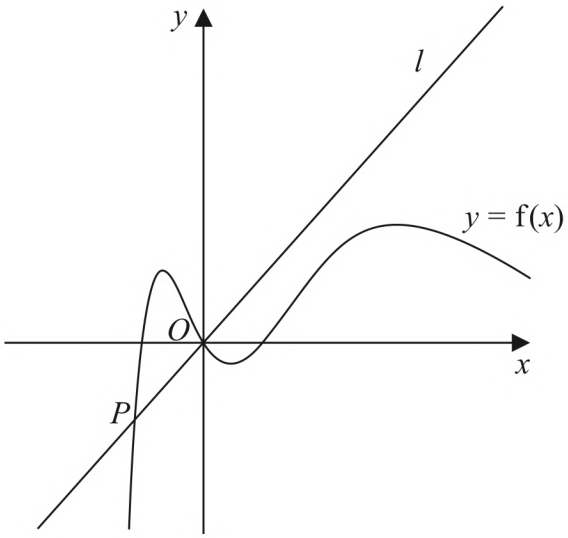


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x(x^2 - 4)\mathrm{e}^{-\frac{1}{2}x}$$

The line $l$ is the normal to the curve at $O$ and meets the curve again at the point $P$.

The point $P$ lies in the 3rd quadrant, as shown in Figure 3.

$x$ coordinate of $P$ is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + \mathrm{e}^{\frac{1}{2}x}}$$

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + \mathrm{e}^{\frac{1}{2}x_n}} \quad \text{with } x_1 = -2$$

find, to 4 decimal places,

(i) the value of $x_2$

(ii) the $x$ coordinate of $P$.

**(3)**

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**5.**

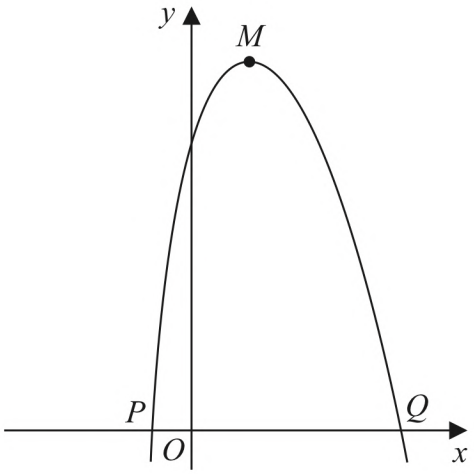


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x + 3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative $x$-axis at the point $P$, as shown in Figure 1.

(a) Show that the $x$ coordinate of $P$ lies in the interval $[-1.25, -1.2]$

**(2)**

The curve cuts the positive $x$-axis at the point $Q$, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of $x_2$

(ii) find, by continued iteration, the $x$ coordinate of $Q$. Give your answer to 4 decimal places.

**(3)**

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**5.**

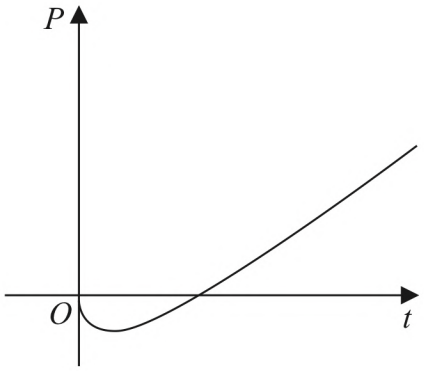


**Figure 2**

The profit made by a company, £$P$ million, $t$ years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4}\ln\left[\frac{t+1}{(2t+1)^2}\right]$$

The graph of $P$ against $t$ is shown in Figure 2.

(c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t+1)^2}{t+1}\right]$$

**(2)**

(d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t_n+1)^2}{t_n+1}\right] \quad \text{with} \quad t_1 = 6$$

find the value of $t_2$ and the value of $t_6$, giving your answers to 3 decimal places.

**(3)**

(e) Hence find, according to the model, how many months it takes in total, from when the company started trading for it to make a profit.

**(2)**

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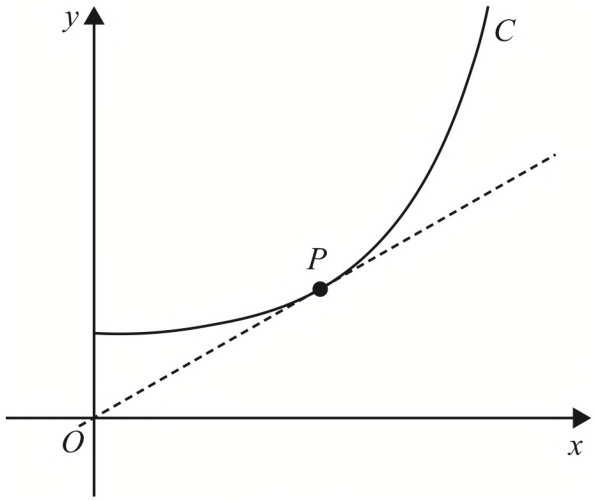


**Figure 3**

Figure 3 shows a sketch of part of the curve $C$ with equation

$$y = \sqrt{3 + 4e^{x^2}} \qquad x \geqslant 0$$

The point $P$ with $x$ coordinate $\alpha$ lies on $C$.

$x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(c) Hence show that $\alpha$ lies between 1 and 2

**(2)**

(d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

**(1)**

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for $\alpha$.

(e) Use the iteration formula to find, to 4 decimal places, the value of

(i) $x_3$

(ii) $\alpha$

**(3)**

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**1.** $g(x) = x^6 + 2x - 1000$

(a) Show that $g(x) = 0$ has a root $\alpha$ in the interval $[3, 4]$

**(2)**

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \qquad \text{with } x_1 = 3$$

(b) (i) find, to 4 decimal places, the value of $x_2$

(ii) find, by repeated iteration, the value of $\alpha$.
Give your answer to 4 decimal places.

**(3)**

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**1.** A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, $\alpha$, in the interval $[1, 2]$

**(2)**

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root $\alpha$.

(b) (i) Find the value of $x_2$ to 4 decimal places.

(ii) Find, by repeated iteration, the value of $\alpha$, giving your answer to 4 decimal places.

**(3)**

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**2.** A curve has equation $y = f(x)$ where

$$f(x) = x^4 - 5x^2 + 4x - 7 \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, $\alpha$, in the interval $[2, 3]$

**(2)**

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$$

**(1)**

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{5x_n^{\,2} - 4x_n + 7}{x_n}}$$

is used to find $\alpha$

(c) Starting with $x_1 = 2$ and using the iterative formula,

(i) find, to 4 decimal places, the value of $x_2$

(ii) find, to 4 decimal places, the value of $\alpha$

**(3)**

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**8.**

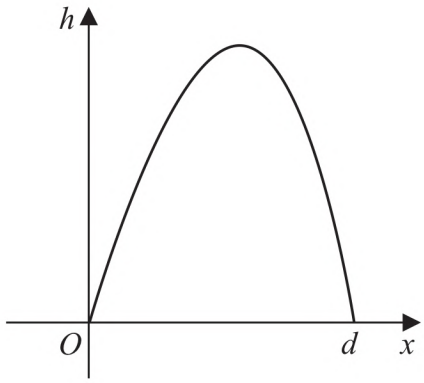


**Figure 4**

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, $h$ metres, of the ball above the ground has been plotted against the horizontal distance travelled, $x$ metres, measured from where the ball was hit.

The ball travels a horizontal distance of $d$ metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,e^{0.02x} \qquad 0 \leqslant x \leqslant d$$

**Use the model to answer parts (a), (b) and (c).**

**(3)**

the maximum value of $h$ occurs when

$$x = 50\ln\left(\frac{150}{x + 50}\right)$$

**(4)**

Using the iteration formula

$$x_{n+1} = 50\ln\left(\frac{150}{x_n + 50}\right) \qquad \text{with } x_1 = 30$$

(c) (i) find the value of $x_2$ to 2 decimal places,

(ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

**(3)**

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