

Chapter 7: Integration

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4. (i) Find

$$\int_5^{13} \frac{1}{(2x-1)} \, dx$$

writing your answer in its simplest form.

(4)

(ii) Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin 2x + \sec \frac{1}{3}x \tan \frac{1}{3}x \, dx$$

(3)

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- (ii)

$$h(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \quad x > 1$$

Given $h(x) = Ax + B + \frac{C}{(x-1)^2}$ where A , B and C are constants to be found, find

$$\int h(x) \, dx$$

(4)

(6)

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5. (a) Show that

$$\sin 3x \equiv 3 \sin x - 4 \sin^3 x \quad (4)$$

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx \quad (4)$$

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9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \quad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = g(x)$, where

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \quad -3 < x < 3.5 \quad x \in \mathbb{R}$$

- (b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

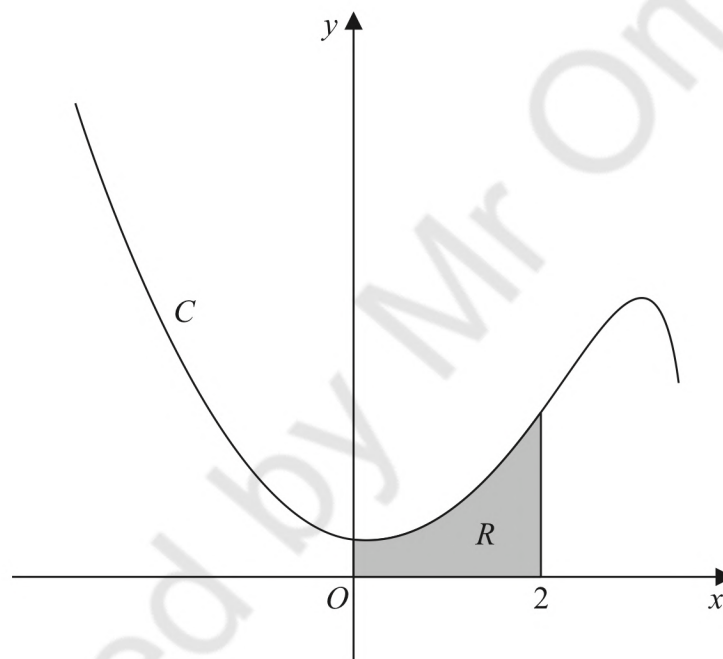


Figure 4

Figure 4 shows a sketch of the curve C .

The region R , shown shaded in Figure 4, is bounded by C , the y -axis, the x -axis and the line with equation $x = 2$

- (c) Find the exact area of R , writing your answer in the form $a + b \ln 2$, where a and b are constants to be found.

(5)

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$$\int \frac{x^2 - 5}{2x^3} dx \quad x > 0$$

(3)

(i) $\int \frac{3x - 2}{3x^2 - 4x + 5} \, dx$

(2)

$$(ii) \int \frac{e^{2x}}{(e^{2x} - 1)^3} dx \quad x \neq 0$$

(2)

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3. (i) Find

$$\int \frac{12}{(2x-1)^2} dx$$

giving your answer in simplest form.

(2)

(ii) (a) Write $\frac{4x+3}{x+2}$ in the form

$$A + \frac{B}{x+2} \quad \text{where } A \text{ and } B \text{ are constants to be found}$$

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2} dx$$

giving your answer in simplest form.

(6)

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- (ii) Find, in simplest form,

(4)

(2)

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- (ii) Show, by algebraic integration, that

(2)

(4)

where a is a rational constant to be found.

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In this question you must show all stages of your working.

Given that k is a positive constant,

$$\int \frac{9x}{3x^2 + k} \, dx$$

(2)

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(4)

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8.

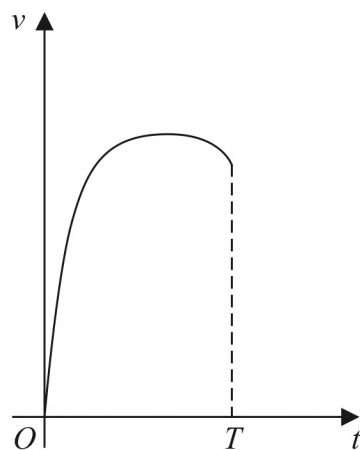


Figure 4

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v \text{ m s}^{-1}$, is modelled by the equation

$$v = 12 - e^{t-10} - 12e^{-0.75t} \quad t \geq 0$$

where t seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.

(5)

Given that the sprinter runs 100 m in T seconds, such that

$$\int_0^T v \, dt = 100$$

(b) show that T is a solution of the equation

$$T = \frac{1}{12} (116 - 16e^{-0.75T} + e^{T-10} - e^{-10})$$

(4)

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1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A , B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x) dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

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(4)

(3)

(b) Hence find,

$$\int f(x) dx$$

(3)

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8. Find, in simplest form,

$$\int (2 \cos x - \sin x)^2 \, dx$$

(5)

3. (i) Find $\frac{d}{dx} \ln(\sin^2 3x)$ writing your answer in simplest form.

(2)

- (b) Hence show that

$$\int_0^{\sqrt{2}} x(3x^2 - 4)^5 \, dx = R$$

where R is an integer to be found.

(Solutions relying on calculator technology are not acceptable.)

(3)

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(b) Hence, using algebraic integration, find the exact value of

(4)

where λ is a constant to be found.

$$\int_1^3 \frac{16}{9(3x - k)} dx = \lambda \ln 10 \quad , \quad K = \frac{7}{3}$$

(4)

$$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$
$$Ax + B + \frac{C}{x - 2}$$

(3)

$$\int_4^8 g(x) \, dx = \alpha + \beta \ln 3$$

(4)

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6.

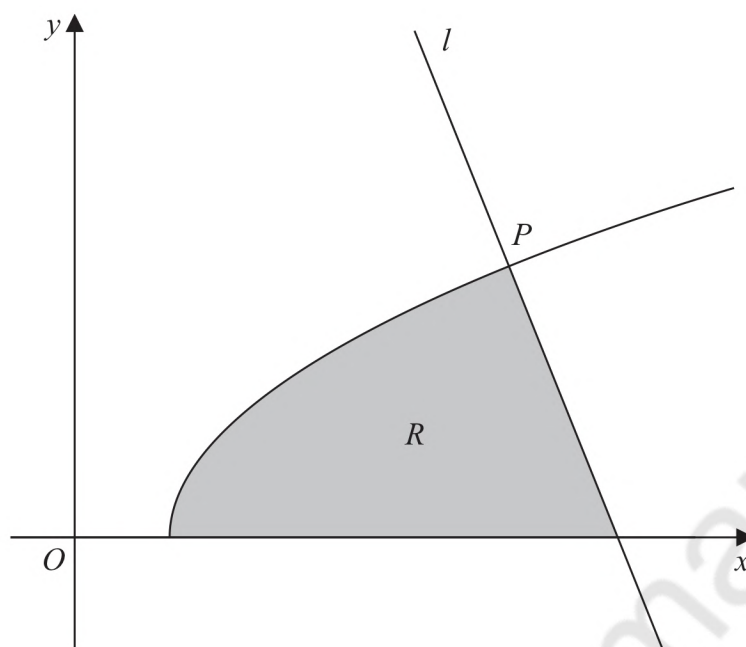


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l , shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

Equation of l is

$$5x + 2y - 50 = 0$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and l .

(b) Use algebraic integration to find the exact area of R .

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