

Chapter 6: Differentiation

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5. Given that

$$y = \frac{5x^2 - 10x + 9}{(x - 1)^2} \quad x \neq 1$$

show that $\frac{dy}{dx} = \frac{k}{(x-1)^3}$, where k is a constant to be found.

(6)

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7. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p

(1)

The tangent to the curve at P cuts the y -axis at the point A .

(b) Use calculus to find the coordinates of A .

(6)

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4. (i)

- (6)

$$g(x) = x\sqrt{\sin 4x} \quad 0 \leq x < \frac{\pi}{4}$$

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

(5)

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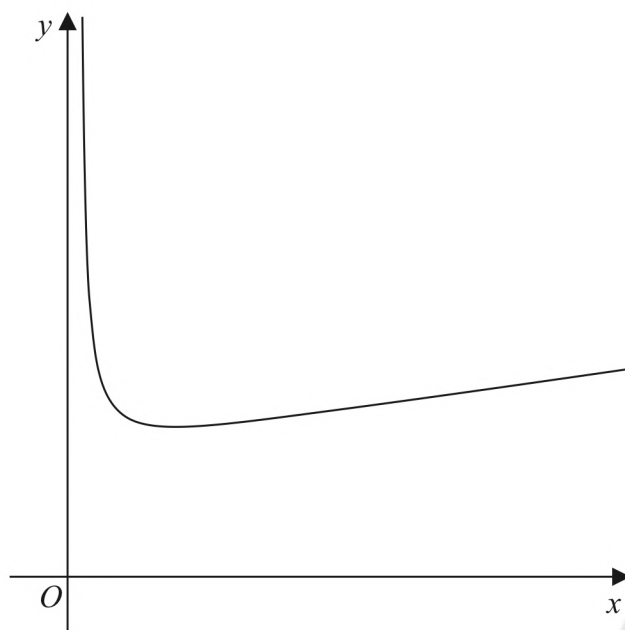
**Figure 1**

Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x + 3}{\sqrt{4x - 1}} \quad x > \frac{1}{4}$$

(a) Find, in simplest form, $f'(x)$.

(4)

(b) Hence find the range of f .

(3)

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(2)

(3)

(b) Hence find the x coordinate of the stationary point of C .

Show that

$$\frac{dy}{dx} = \frac{e^x}{f(x)}$$

where $f(x)$ is a function of e^x that should be found.

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- $$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \quad x > -3$$

(4)

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \quad -3 < x < 3.5 \quad x \in \mathbb{R}$$

- (b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found. (5)

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$$f(x) = x \cos\left(\frac{x}{3}\right) \quad x > 0$$

(a) Find $f'(x)$

(2)

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10. The curve C has equation

$$x = 3 \sec^2 2y \quad x > 3 \quad 0 < y < \frac{\pi}{4}$$

- (a) Find $\frac{dx}{dy}$ in terms of y .

(2)

- (b) Hence show that

$$\frac{dy}{dx} = \frac{p}{qx\sqrt{x-3}}$$

where p is irrational and q is an integer, stating the values of p and q .

(3)

- (c) Find the equation of the normal to C at the point where $y = \frac{\pi}{12}$, giving your answer in the form $y = mx + c$, giving m and c as exact irrational numbers.

(5)

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- $$y = x^2 \cos\left(\frac{1}{2}x\right) \quad 0 < x \leq \pi$$

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = 2 \arctan\left(\frac{4}{x}\right)$$

(4)

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7. Given that

$$x = 6 \sin^2 2y \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{(Bx - x^2)}}$$

where A and B are integers to be found.

(5)

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$$f(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} \quad x > 0$$

(b) Find f^{-1} (3)

(c) (i) Find, in simplest form, $f'(x)$.

(ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer.

(3)

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- (ii) A different curve C_2 has equation

Show that C_1 has a stationary point at $x = \frac{\sqrt{p}}{2}$ where p is a constant to be found. (4)

(a) Show that, for this curve,

where A and B are constants to be found.

- (b) Hence, state the maximum gradient of this curve. (4)

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(1)

(3)

(b) Given $x = 2 \sin y$, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{A - x^2}}$$

where A is a constant to be found.

The point P lies on C and has y coordinate $\frac{\pi}{4}$

(c) Find the equation of the tangent to C at P . Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

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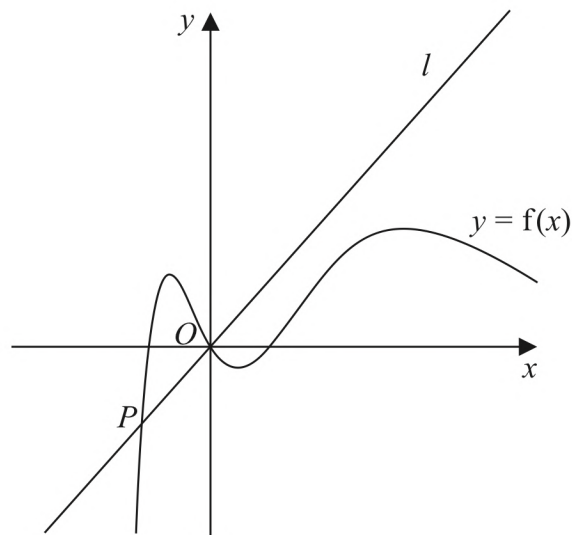
**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$$

(a) Find $f'(x)$.

(2)

The line l is the normal to the curve at O and meets the curve again at the point P .

The point P lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$$

(4)

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- $$y = (2x + 5)e^{3x}$$

(4)

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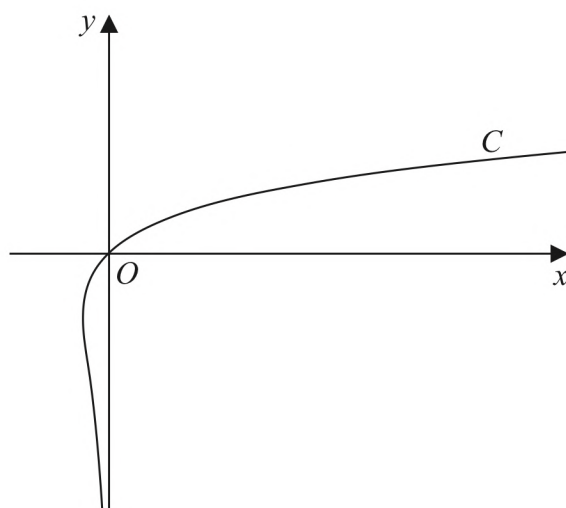
**Figure 3**

Figure 3 shows a sketch of the curve C with equation

$$x = ye^{2y} \quad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)} \quad (4)$$

Given that the straight line with equation $x = k$, where k is a constant, cuts C at exactly two points,

(b) find the range of possible values for k .

(3)

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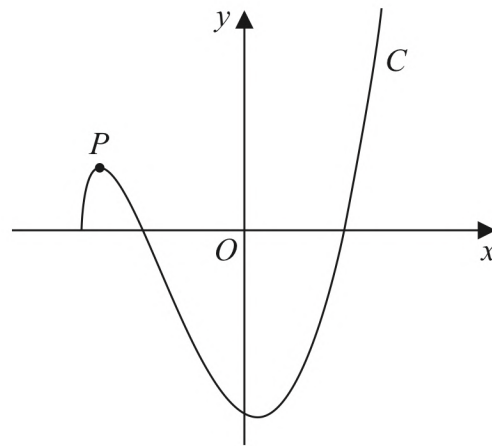


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \quad x \geq -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where k is an integer to be found.

(4)

(b) Hence, find the values of x for which $f'(x) = 0$

(1)

Figure 3 shows a sketch of the curve C with equation $y = f(x)$.

The curve has a local maximum at the point P

(c) Find the exact coordinates of P

(2)

The function g is defined by

$$g(x) = 2f(x) + 4 \quad -\frac{9}{4} \leq x \leq 0$$

(d) Find the range of g

(3)

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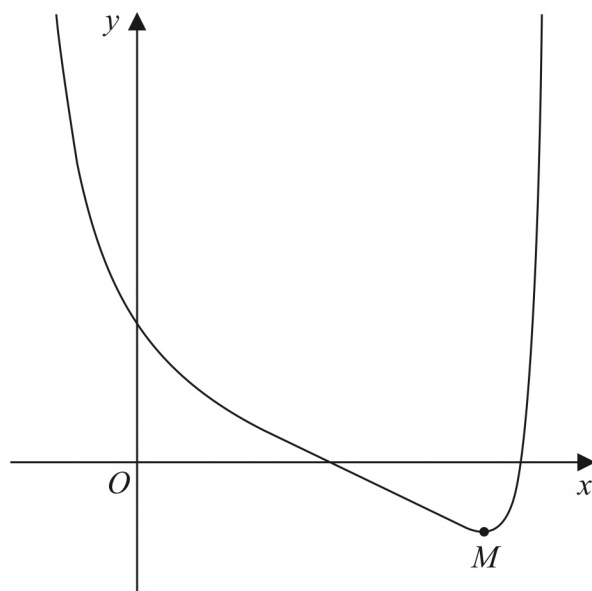


Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows the curve with equation

$$y = \frac{1 + 2 \cos x}{1 + \sin x} \quad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point M , shown in Figure 5, is the minimum point on the curve.

(a) Show that the x coordinate of M is a solution of the equation

$$2 \sin x + \cos x = -2$$

(4)

(b) Hence find, to 3 significant figures, the x coordinate of M .

(5)

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3. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

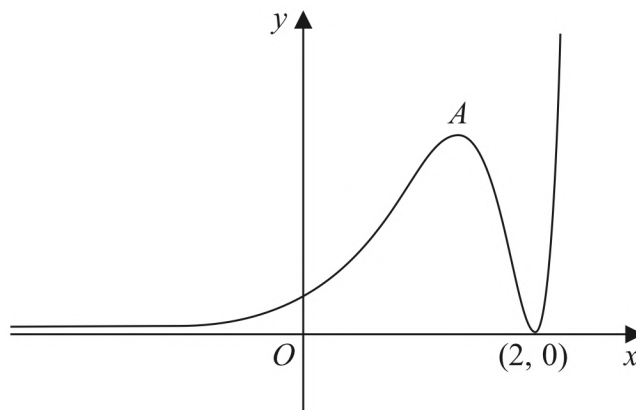


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x - 2)^2 e^{3x} \quad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at $(2, 0)$

(a) Use calculus to find the exact coordinates of A .

(5)

Given that the equation $f(x) = k$, where k is a constant, has **at least** two distinct roots,

(b) state the range of possible values for k .

(2)

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$$y = \log_{10}(2x + 1)$$

(2)

(3)

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$$y = \frac{2 + 3\sin x}{\cos x + \sin x}$$
$$\frac{dy}{dx} = \frac{a \tan x + b \sec x + c}{\sec x + 2 \sin x}$$

(6)

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7. The curve C has equation

$$x = 3 \tan\left(y - \frac{\pi}{6}\right) \quad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C .

Given that the tangent to C at P crosses the x -axis at the point Q .

(b) find, in simplest form, the exact x coordinate of Q .

(5)

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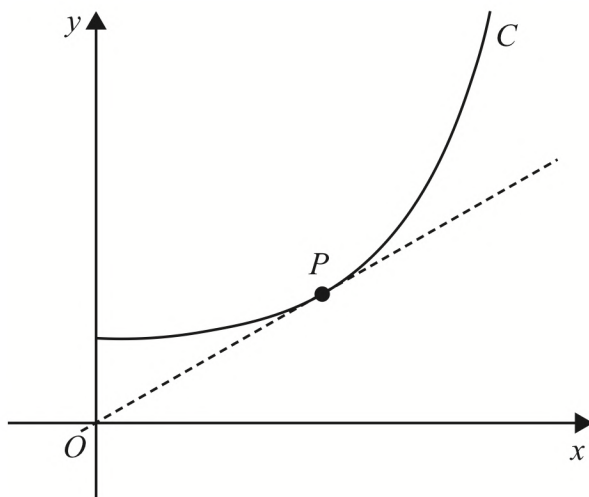
**Figure 3**

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \quad x \geq 0$$

- (a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

The point P with x coordinate α lies on C .

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

- (b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

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(2)

(2)

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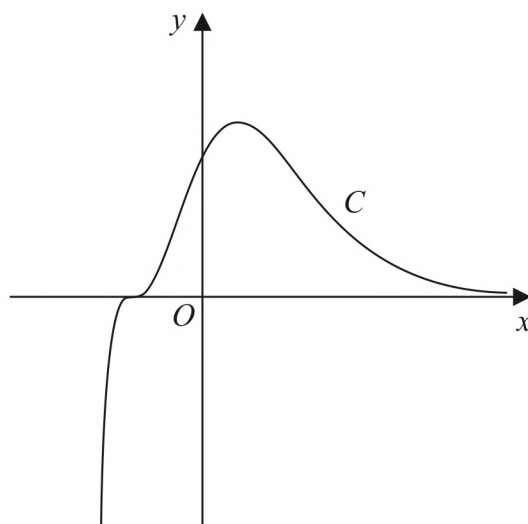


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (2x + 1)^3 e^{-4x}$$

(a) Show that

$$f'(x) = A(2x + 1)^2 (1 - 4x) e^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C .

(3)

The function g is defined by

$$g(x) = 8f(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = g(x)$.

(2)

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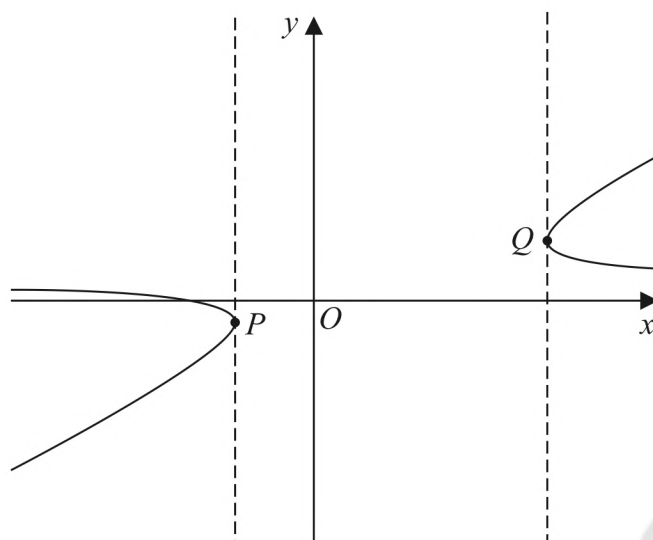
**Figure 4**

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

- (a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y -axis, as shown in Figure 4.

- (b) Use the answer to part (a) to find the equations of these two tangents.

(4)

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- $$y = \frac{\ln(x^2 + k)}{x^2 + k} \quad x \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

(3)

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

- (c) find the upper limit to the value for k .

(1)

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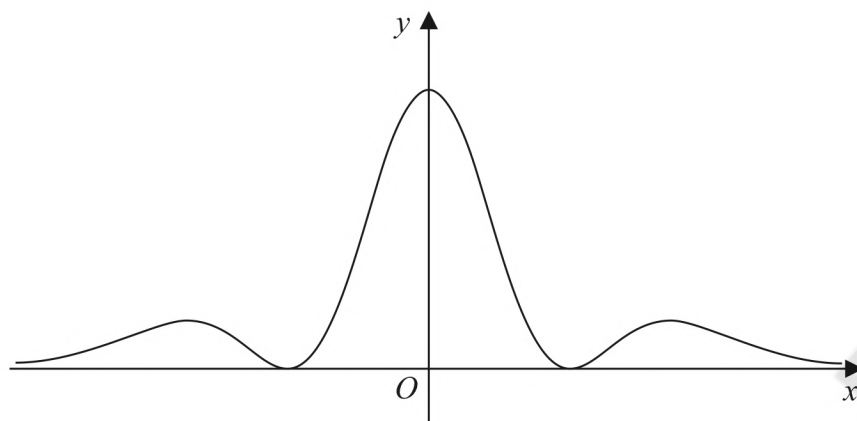
**Figure 1**

Figure 1 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = e^{-x^2} (2x^2 - 3)^2$$

(a) Find the range of f

(2)

(b) Show that

$$f'(x) = 2x(2x^2 - 3)e^{-x^2}(A - Bx^2)$$

where A and B are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \quad 0 \leq y \leq \frac{\pi}{8} \quad 0 \leq x \leq 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P (2)

(b) Find $\frac{dx}{dy}$ (2)

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x + s)^2}}$$

where q , r and s are constants to be found. (3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,

(ii) state the value of $\frac{dy}{dx}$ at this point. (2)

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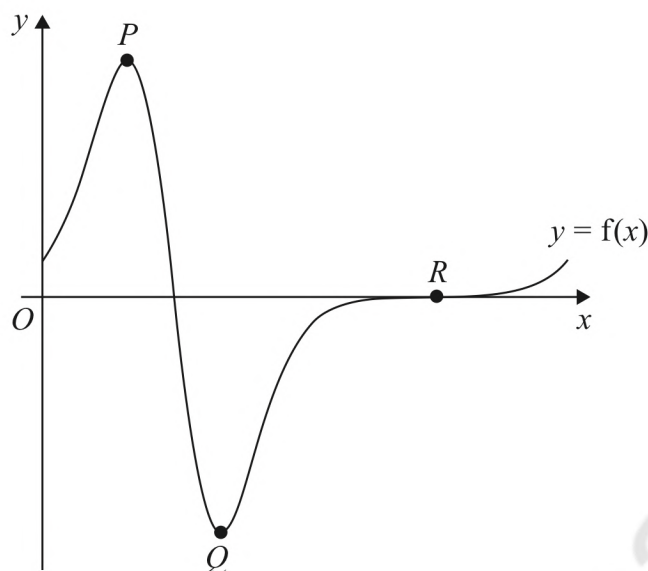


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2e^{3\sin x} \cos x \quad 0 \leq x \leq 2\pi$$

The curve intersects the x -axis at point R , as shown in Figure 1.

(a) State the coordinates of R

(1)

The curve has two turning points, at point P and point Q , also shown in Figure 1.

(b) Show that, at points P and Q ,

$$a \sin^2 x + b \sin x + c = 0$$

where a , b and c are integers to be found.

(4)

(c) Hence find the x coordinate of point Q , giving your answer to 3 decimal places.

(2)

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{16}{9(3x - k)} \quad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

- (a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of k . (2)

The point P with x coordinate 1 lies on C .

Given that the gradient of the curve at P is -12

- (b) find the two possible values of k (3)

Given also that $k < 3$

- (c) find the equation of the normal to C at P , writing your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found. (3)

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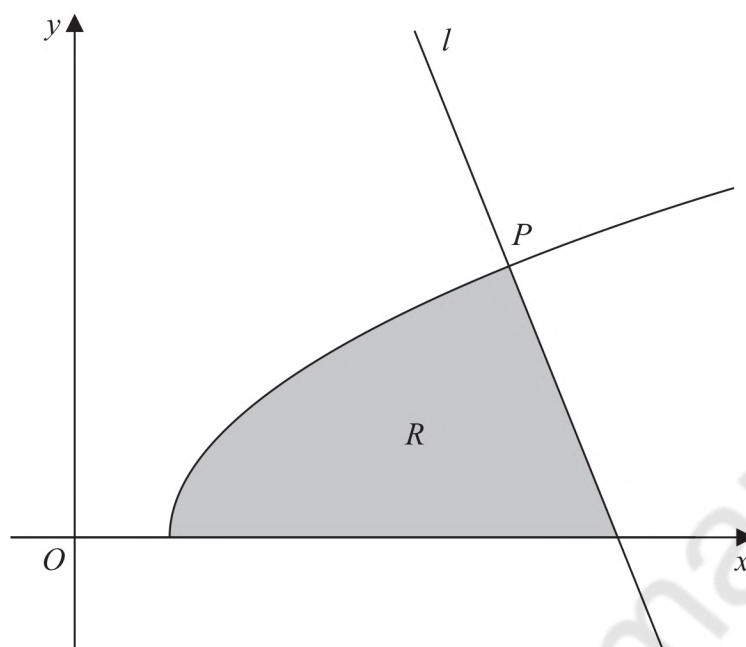


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l , shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

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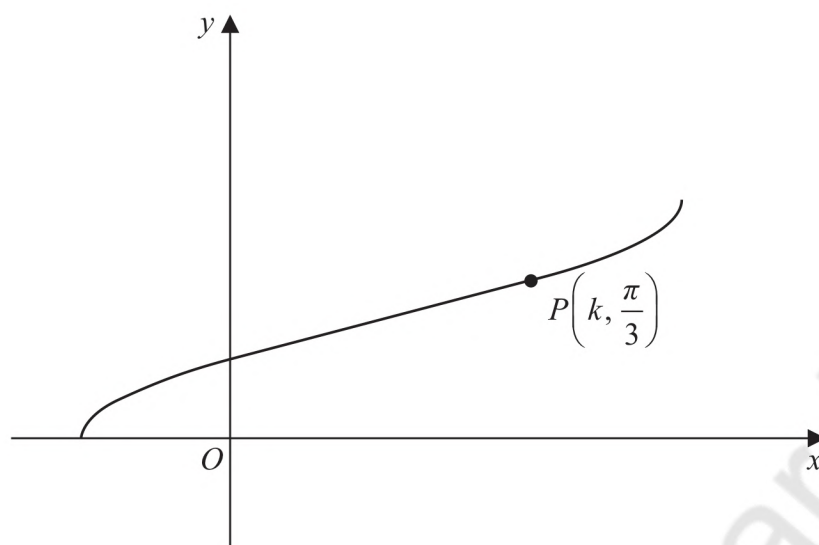


Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \quad 0 \leq y \leq \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x -axis at the point N .

(c) Find the exact area of triangle OPN , where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

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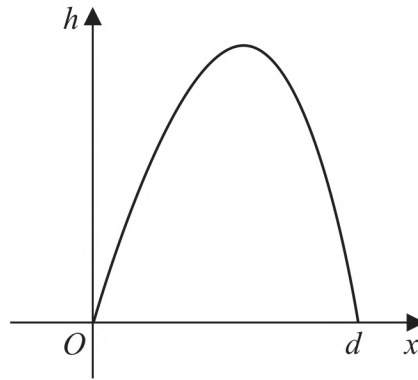
**Figure 4**

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5xe^{0.02x} \quad 0 \leq x \leq d$$

Use the model to answer parts (a), (b) and (c).

(a) Find the value of d , giving your answer to 2 decimal places.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

(b) Show that the maximum value of h occurs when

$$x = 50 \ln \left(\frac{150}{x + 50} \right)$$

(4)

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