

Chapter 5: Exponential and Logarithms

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8. In a controlled experiment, the number of microbes, N , present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b \quad \text{where } a \text{ and } b \text{ are constants}$$

- (a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b .

(2)

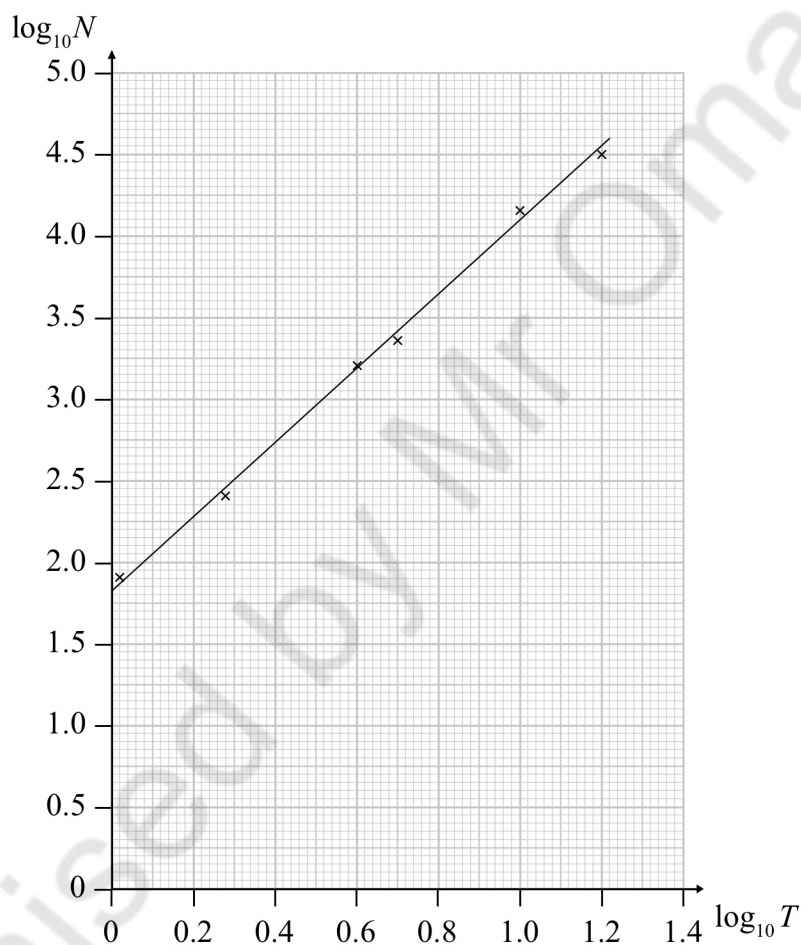


Figure 2

Figure 2 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

- (b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

- (c) With reference to the model, interpret the value of the constant a .

(1)

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- 10.** The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

- (a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

- (b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

- (c) Show that $T = a \ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

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1. A population of a rare species of toad is being studied.

The number of toads, N , in the population, t years after the start of the study, is modelled by the equation

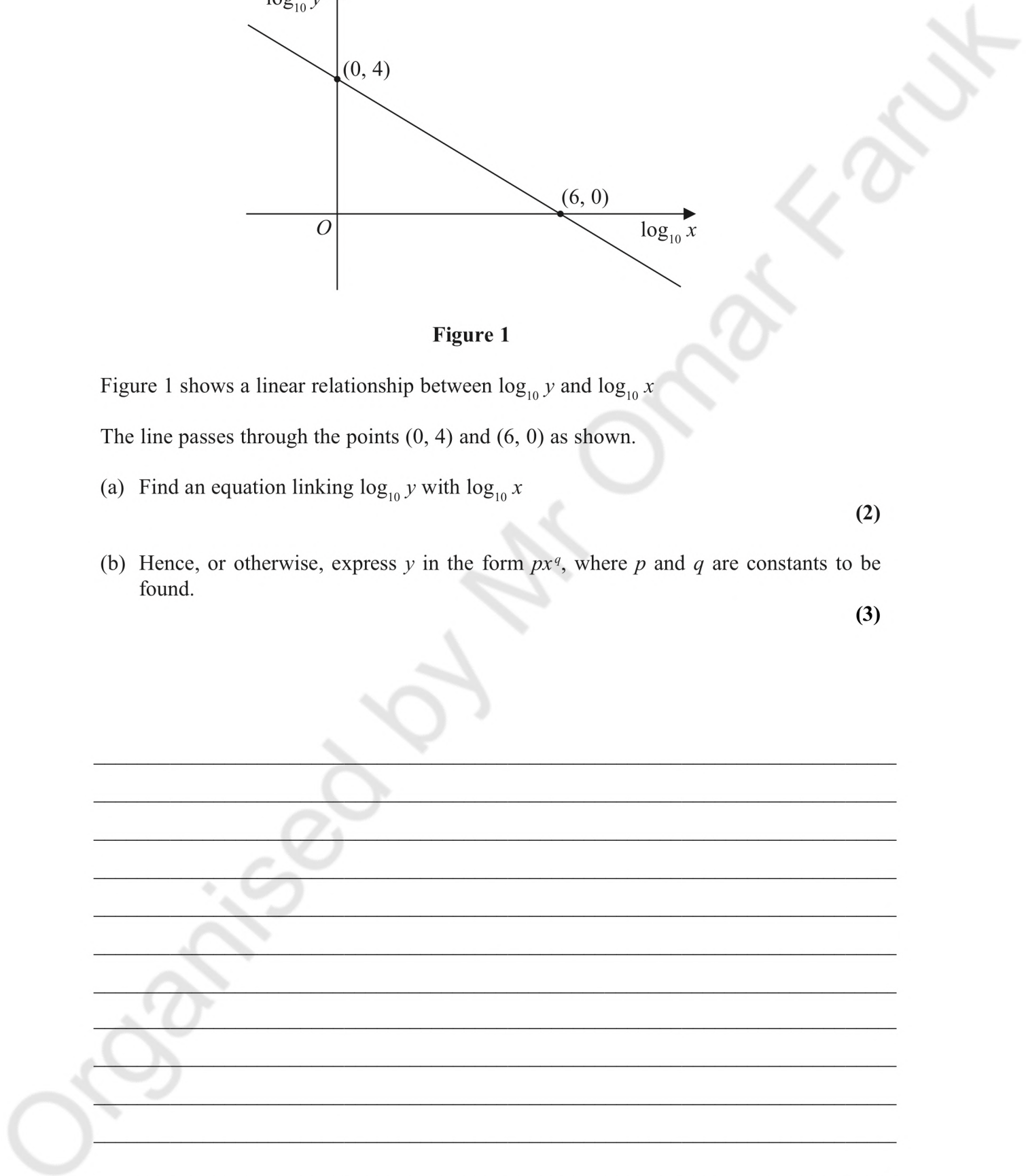
$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \quad t \geq 0, t \in \mathbb{R}$$

According to this model,

- (a) calculate the number of toads in the population at the start of the study, (1)
- (b) find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places. (4)
- (c) Explain why, according to this model, the number of toads in the population can never reach 500 (1)

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2. A scientist monitored the growth of bacteria on a dish over a 30-day period.

The area, $N\text{mm}^2$, of the dish covered by bacteria, t days after monitoring began, is modelled by the equation

$$\log_{10} N = 0.0646t + 1.478 \quad 0 \leq t \leq 30$$

- (a) Show that this equation may be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of a to the nearest integer and give the value of b to 3 significant figures.

(4)

- (b) Use the model to find the area of the dish covered by bacteria 30 days after monitoring began. Give your answer, in mm^2 , to 2 significant figures.

(2)

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- $$\theta = A - 180e^{-kt}$$

Given that the temperature inside the oven is initially 18°C ,

- (2)

(b) Show that $k = p \ln q$ where p and q are rational numbers to be found.

- (4)

(c) the temperature inside the oven 9 minutes after the oven is switched on, giving your answer to 3 significant figures,

- (2)

- (d) the rate of increase of the temperature inside the oven 9 minutes after the oven is switched on. Give your answer in $^{\circ}\text{C min}^{-1}$ to 3 significant figures.

- (3)

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- $$P = ab^t$$

Using the data for the years between the start of 2005 and the start of 2010, a graph is plotted of $\log_{10} P$ against t .

The points are found to lie approximately on a straight line with gradient 0.09 and intercept 0.68 on the $\log_{10} P$ axis.

- (a) Find, according to the model, the value of a and the value of b , giving your answers to 2 decimal places. (4)
- (b) In the context of the model, give a practical interpretation of the constant a . (1)
- (c) Use the model to estimate the percentage of the population who had access to the internet at the start of 2015 (2)

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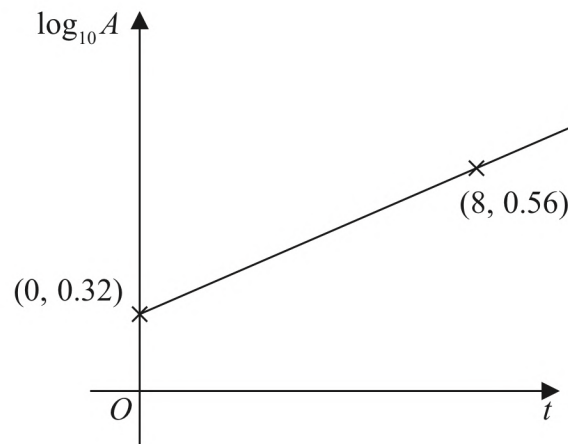


Figure 1

The growth of duckweed on a pond is being studied.

The surface area of the pond covered by duckweed, $A \text{ m}^2$, at a time t days after the start of the study is modelled by the equation

$$A = p q^t \quad \text{where } p \text{ and } q \text{ are positive constants}$$

Figure 1 shows the linear relationship between $\log_{10} A$ and t .

The points $(0, 0.32)$ and $(8, 0.56)$ lie on the line as shown.

(a) Find, to 3 decimal places, the value of p and the value of q .

(4)

Using the model with the values of p and q found in part (a),

(b) find the rate of increase of the surface area of the pond covered by duckweed, in m^2/day , exactly 6 days after the start of the study.
Give your answer to 2 decimal places.

(3)

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- (a) Find the number of fish in the lake at the start of the study. (1)
- (b) Find the upper limit to the number of fish in the lake. (1)
- (c) Find the time, after the start of the study, when there are predicted to be 500 fish in the lake. Give your answer in years and months to the nearest month. (4)

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3.

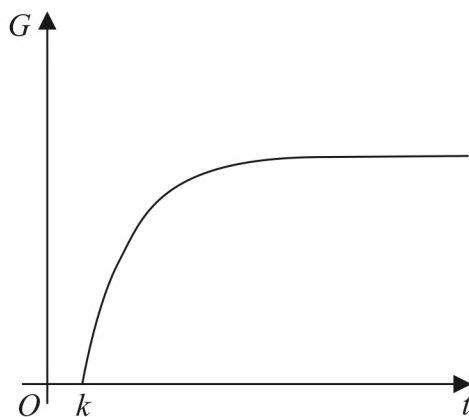


Figure 2

The total mass of gold, G tonnes, extracted from a mine is modelled by the equation

$$G = 40 - 30e^{1-0.05t} \quad t \geq k \quad G \geq 0$$

where t is the number of years after 1st January 1800.

Figure 2 shows a sketch of G against t .

Use the equation of the model to answer parts (a), (b) and (c).

(a) (i) Find the value of k .

(ii) Hence find the year and month in which gold started being extracted from the mine.

(3)

(b) Find the total mass of gold extracted from the mine up to 1st January 1870.

(2)

There is a limit to the mass of gold that can be extracted from the mine.

(c) State the value of this limit.

(1)

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- 7** The mass, M kg, of a species of tree can be modelled by the equation

$$\log_{10} M = 1.93 \log_{10} r + 0.684$$

where r cm is the base radius of the tree.

The base radius of a particular tree of this species is 45 cm.

According to the model,

- (a) find the mass of this tree, giving your answer to 2 significant figures.

(2)

- (b) Show that the equation of the model can be written in the form

$$M = pr^q$$

giving the values of the constants p and q to 3 significant figures.

(3)

- (c) With reference to the model, interpret the value of the constant p .

(1)

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- The surface area of the pond covered by the weed, $A \text{ m}^2$, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

Given that

- 30m^2 of the surface of the pond was covered by the weed at the start of the study
- 50m^2 of the surface of the pond was covered by the weed T days after the start of the study

- (a) show that $p = 2.4$ (2)

- (b) find the value of T , giving your answer to one decimal place.

(Solutions relying entirely on graphical or numerical methods are not acceptable.)

The weed grows until it covers the surface of the pond.

- (c) Find, according to the model, the maximum possible surface area of the pond. (1)

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The amount of the antibiotic, x milligrams, in the patient's bloodstream t hours after the dose was given, is found to satisfy the equation

(a) Show that this equation can be written in the form

where p and q are constants to be found. Give the value of p to the nearest whole number and the value of q to 2 significant figures.

(4)

(1)

$$x = 400 \times 1.4^{-t}$$

(3)

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4.

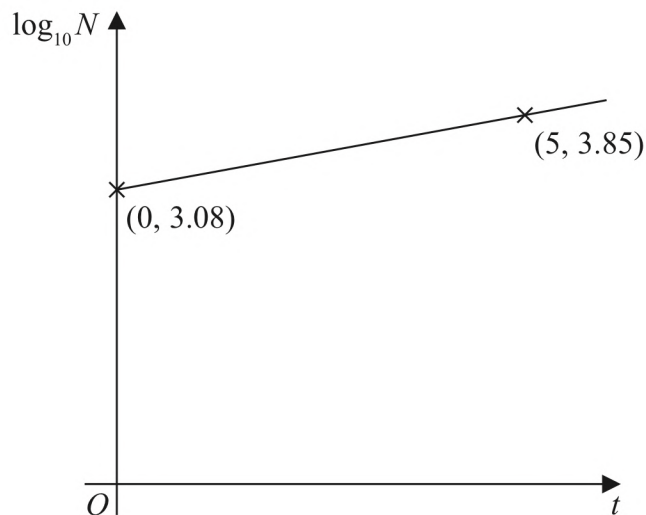


Figure 1

The number of subscribers to an online video streaming service, N , is modelled by the equation

$$N = ab^t$$

where a and b are constants and t is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between t and $\log_{10} N$

The line passes through the points $(0, 3.08)$ and $(5, 3.85)$

Using this information,

(a) find an equation for this line. (2)

(b) Find the value of a and the value of b , giving your answers to 3 significant figures. (3)

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of T (2)

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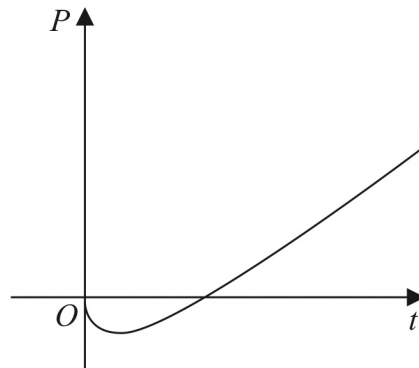
$$y = \log_{10}(2x + 1)$$

(2)

(3)

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5.

**Figure 2**

The profit made by a company, £ P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4} \ln \left[\frac{t+1}{(2t+1)^2} \right]$$

The graph of P against t is shown in Figure 2.

According to the model,

- (a) show that exactly one year after it started trading, the company had made a loss of approximately £830 000 (2)

A manager of the company wants to know the value of t for which $P = 0$

- (b) Show that this value of t occurs in the interval $[6, 7]$ (2)
- (c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t+1)^2}{t+1} \right] \quad (2)$$

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The line passes through the points $(0, 1.5)$ and $(-4.8, 0)$ as shown.

- (a) Write down an equation for l . (2)
- (b) Hence, or otherwise, express y in the form kb^x , giving the values of the constants k and b to 3 significant figures. (3)

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In this question you must show all stages of your working.

A population of fruit flies is being studied.

$$F = \frac{350e^{kt}}{9 + e^{kt}}$$

Use the equation of the model to answer parts (a), (b) and (c).

- Given that there are 200 fruit flies in the population 15 days after the start of the study,

- Given also that, when $t = T$, the number of fruit flies in the population is increasing at a rate of 10 per day,

- (c) find the possible values of T , giving your answers to one decimal place. (5)

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A graph showing the relationship between $\log_6 T$ (vertical axis) and $\log_6 x$ (horizontal axis). The line passes through the points $(0, 4)$ and $(2, 0)$. The origin is labeled O .

Figure 1

The line passes through the points $(0, 4)$ and $(2, 0)$ as shown.

- (a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$
- (ii) Hence find the exact value of T when $x = 216$
- (b) Find an equation, not involving logs, linking T with x

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7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \quad t \geq 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T , giving your answer to 3 significant figures.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

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The total sales N of this phone, in **thousands**, is modelled by the equation

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

- (1)

Given that when $t = T$ the total sales of the phone was 100 000

- (3)

- (Solutions relying entirely on calculator technology are not acceptable.)*

(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

- (1)

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6. An area of sea floor is being monitored.

The area of the sea floor, $S \text{ km}^2$, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where p and q are constants and t is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

- (a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$ (2)

- (b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of p and the value of q each to 3 significant figures. (3)

- (c) With reference to the model, interpret the value of the constant q (1)

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The total amount raised in the t months after monitoring began, $\pounds D$, is modelled by the equation

(a) Write this equation in the form

where a and b are constants to be found. Give each value to 4 significant figures.

(3)

According to the model,

(b) find the value of T , giving your answer to 3 significant figures.

(2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.

(2)

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Solutions relying entirely on calculator technology are not acceptable.

$$T = 10 + Ae^{-Bt}$$

Given that the temperature of the air in the room at the instant the heat source was switched off was 18°C ,

- Given also that, exactly 45 minutes after the heat source was switched off, the temperature of the air in the room was 16°C ,

- Using the values for A and B ,

- (d) Explain why, according to the model, the temperature of the air in the room cannot fall to 5°C

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3. (i) The variables x and y are connected by the equation

$$y = \frac{10^6}{x^3} \quad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

(3)

(ii)

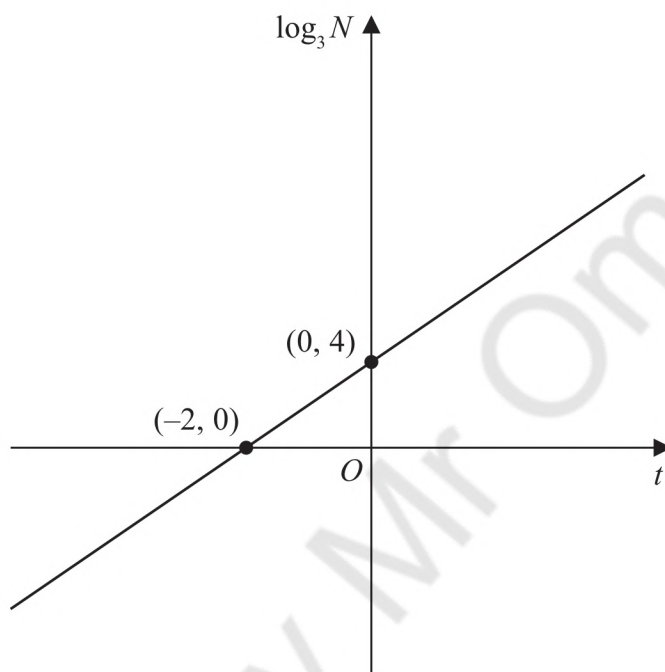


Figure 2

Figure 2 shows the linear relationship between $\log_3 N$ and t .

Show that $N = ab^t$ where a and b are constants to be found.

(3)

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