

Chapter 2: Functions and Graphs

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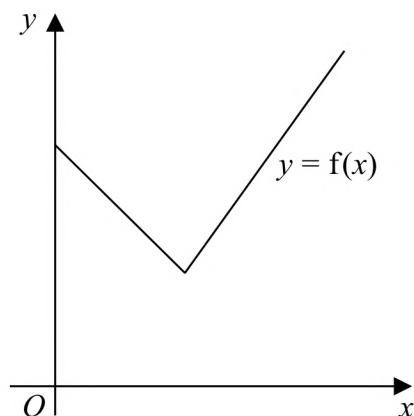
**Figure 1**

Figure 1 shows a sketch of part of the graph $y = f(x)$ where

$$f(x) = 2|3 - x| + 5 \quad x \geq 0$$

(a) Solve the equation

$$f(x) = \frac{1}{2}x + 30 \quad (3)$$

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(b) state the set of possible values for k . (2)

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2. The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \quad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2} \ln x \quad x > 0, x \in \mathbb{R}$$

- (a) Find, in simplest form, the value of $fg(e^2)$ (2)

- (b) Find f^{-1} (3)

- (c) Hence, or otherwise, find all real solutions of the equation

$$\mathbf{f}^{-1}(x) = \mathbf{f}(x) \quad (3)$$

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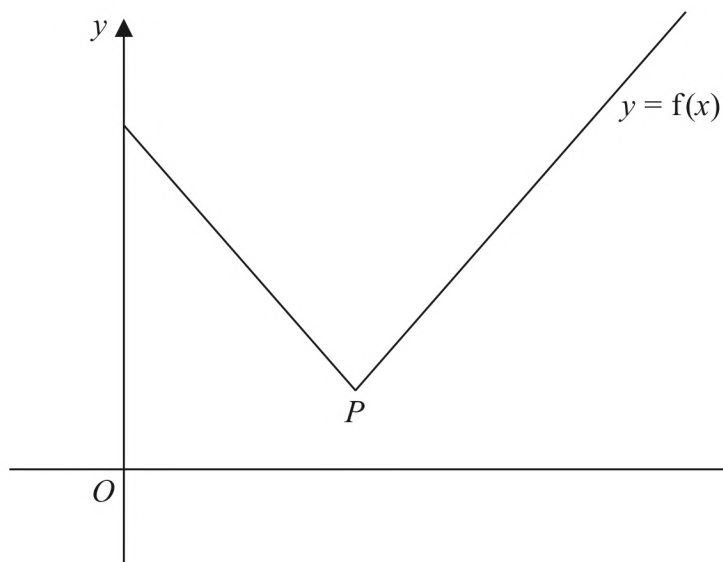
**Figure 2**

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = 2|2x - 5| + 3 \quad x \geq 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P .

(2)

(b) Solve the equation $f(x) = 3x - 2$

(4)

Given that the equation

$$f(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k .

(3)

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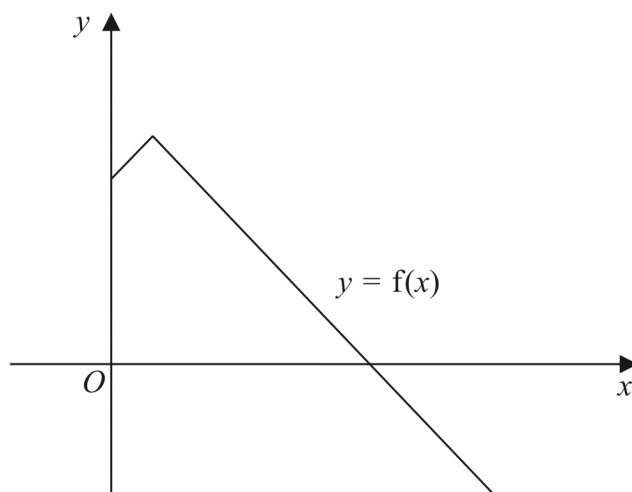
**Figure 2**

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$ where

$$f(x) = 21 - 2|2 - x| \quad x \geq 0$$

(a) Find $ff(6)$

(2)

(b) Solve the equation $f(x) = 5x$

(2)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k .

(2)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = af(x - b)$

The vertex of the graph with equation $y = af(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b .

(2)

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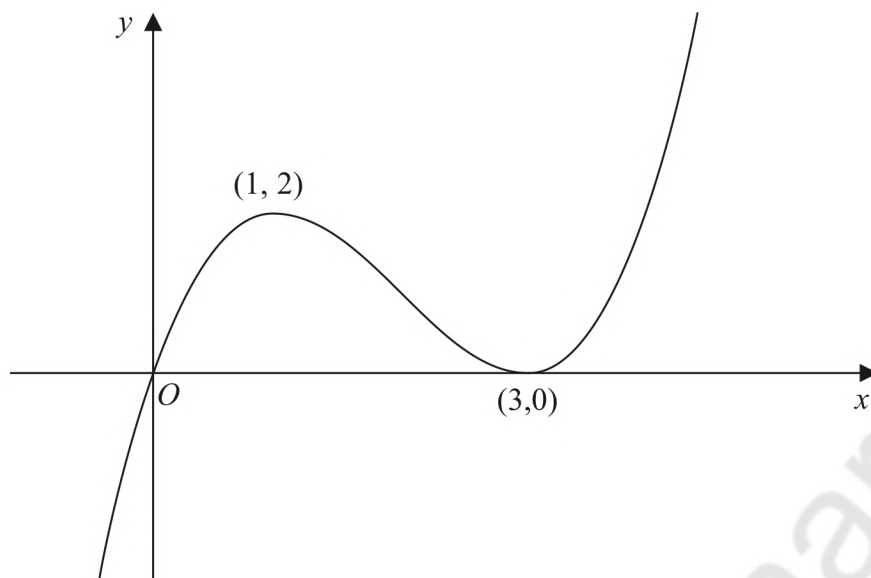
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where $x \in \mathbb{R}$ and $f(x)$ is a polynomial.

The curve passes through the origin and touches the x -axis at the point $(3, 0)$

There is a maximum turning point at $(1, 2)$ and a minimum turning point at $(3, 0)$

On separate diagrams, sketch the curve with equation

(i) $y = 3f(2x)$

(3)

(ii) $y = f(-x) - 1$

(3)

On each sketch, show clearly the coordinates of

- the point where the curve crosses the y -axis
- any maximum or minimum turning points

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(a) Show that

$$f(x) = 3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} \quad x > 4$$

$$f(x) = \frac{ax + b}{cx + d} \quad x > 4$$

(4)

(2)

(2)

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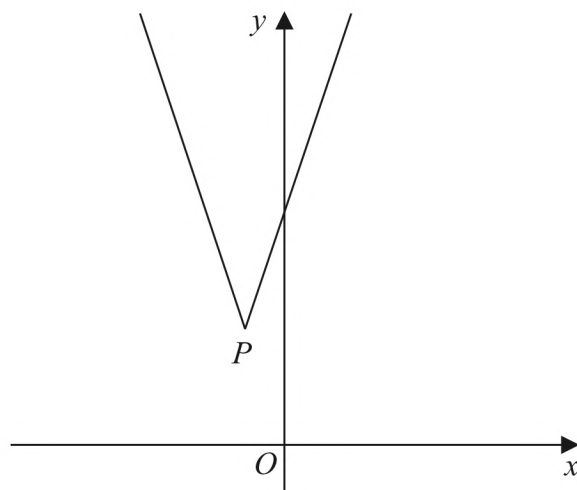
**Figure 2**

Figure 2 shows a sketch of the graph with equation $y = f(x)$, where

$$f(x) = |3x + a| + a$$

and where a is a positive constant.

The graph has a vertex at the point P , as shown in Figure 2.

(a) Find, in terms of a , the coordinates of P .

(2)

(b) Sketch the graph with equation $y = g(x)$, where

$$g(x) = |x + 5a|$$

On your sketch, show the coordinates, in terms of a , of each point where the graph cuts or meets the coordinate axes.

(2)

The graph with equation $y = g(x)$ intersects the graph with equation $y = f(x)$ at two points.

(c) Find, in terms of a , the coordinates of the two points.

(5)

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- (a) Solve the equation

$$f(x) = \frac{4x + 6}{x - 5} \quad x \in \mathbb{R}, \quad x \neq 5$$

$$g(x) = 5 - 2x^2 \quad x \in \mathbb{R}, \ x \leq 0$$

$$\text{fg}(x) = 3 \quad (4)$$

- (b) Find f^{-1} (3)

- (c) Sketch and label, on the same axes, the curve with equation $y = g(x)$ and the curve with equation $y = g^{-1}(x)$. Show on your sketch the coordinates of the points where each curve meets or cuts the coordinate axes. (3)

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6. Given that k is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left| 2x - \frac{k}{3} \right|$

Show on each sketch the coordinates, in terms of k , of each point where the graph meets or cuts the axes.

(4)

(b) Hence find, in terms of k , the values of x for which

$$\left| 2x - \frac{k}{3} \right| = k - 2|x|$$

giving your answers in simplest form.

(4)

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- (a) Show that $f(x) = \frac{5x}{x+3}$ (3)
- (b) Find f^{-1} (3)
- (c) (i) Find, in simplest form, $f'(x)$.
- (ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer. (3)

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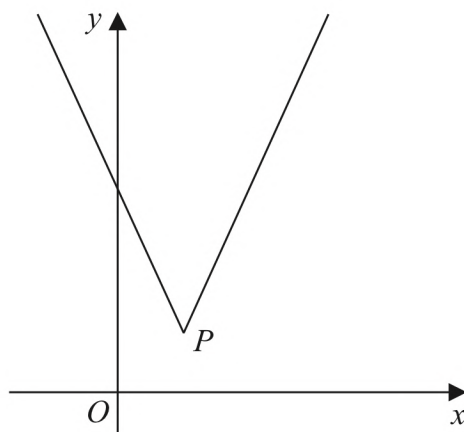


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |3x - 13| + 5 \quad x \in \mathbb{R}$$

The vertex of the graph is at point P , as shown in Figure 1.

(a) State the coordinates of P .

(2)

(b) (i) State the range of f .

(ii) Find the value of $ff(4)$

(2)

(c) Solve, using algebra and showing your working,

$$16 - 2x > |3x - 13| + 5$$

(4)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = af(x + b)$

The vertex of the graph with equation $y = af(x + b)$ is $(4, 20)$

Given that a and b are constants,

(d) find the value of a and the value of b .

(2)

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6. The function f is defined by

$$f(x) = \frac{5x - 3}{x - 4} \quad x > 4$$

- (a) Show, by using calculus, that f is a decreasing function.

(3)

- (b) Find f^{-1}

(3)

- (c) (i) Show that $ff(x) = \frac{ax + b}{x + c}$ where a , b and c are constants to be found.

- (ii) Deduce the range of ff .

(5)

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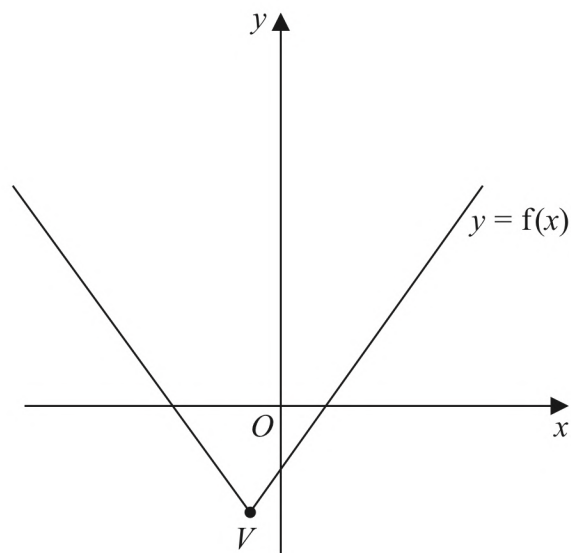


Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = \frac{1}{2}|2x + 7| - 10$$

(a) State the coordinates of the vertex, V , of the graph.

(2)

(b) Solve, using algebra,

$$\frac{1}{2}|2x + 7| - 10 \geq \frac{1}{3}x + 1$$

(4)

(c) Sketch the graph with equation

$$y = |f(x)|$$

stating the coordinates of the local maximum point and each local minimum point.

(4)

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$$\begin{aligned} f(x) &= \frac{5-x}{3x+2} & x \in \mathbb{R}, x \neq -\frac{2}{3} \\ g(x) &= 2x-7 & x \in \mathbb{R} \end{aligned}$$

(2)

(3)

$$f\left(\frac{1}{a}\right) = g(a + 3) \quad (4)$$

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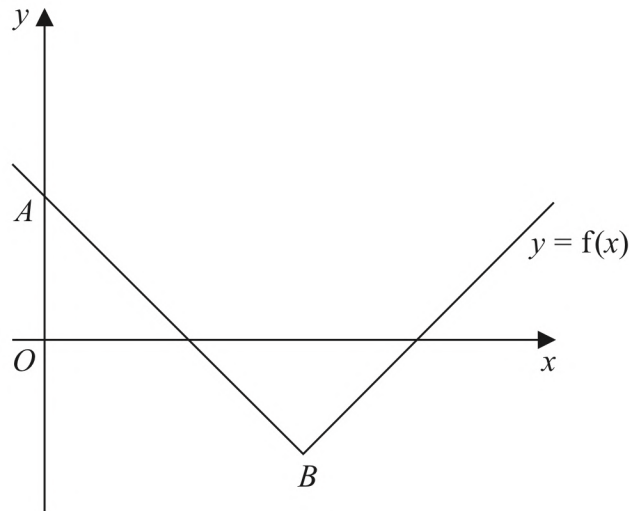


Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 9| - 2 \quad x \in \mathbb{R}$$

and k is a positive constant.

The graph intersects the y -axis at the point A and has a minimum point at B as shown.

(a) (i) Find the y coordinate of A

(ii) Find, in terms of k , the x coordinate of B

(2)

(b) Find, in terms of k , the range of values of x that satisfy the inequality

$$|kx - 9| - 2 < 0$$

(3)

Given that the line $y = 3 - 2x$ intersects the graph $y = f(x)$ at two distinct points,

(c) find the range of possible values of k

(3)

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2. The functions f and g are defined by

$$f(x) = 5 - \frac{4}{3x + 2} \quad x \geq 0$$

$$g(x) = \left| 4 \sin\left(\frac{x}{3} + \frac{\pi}{6}\right) \right| \quad x \in \mathbb{R}$$

- (a) Find the range of f (2)
- (b) (i) Find $f^{-1}(x)$
- (ii) Write down the domain of f^{-1} (3)
- (c) Find $fg(-\pi)$ (2)

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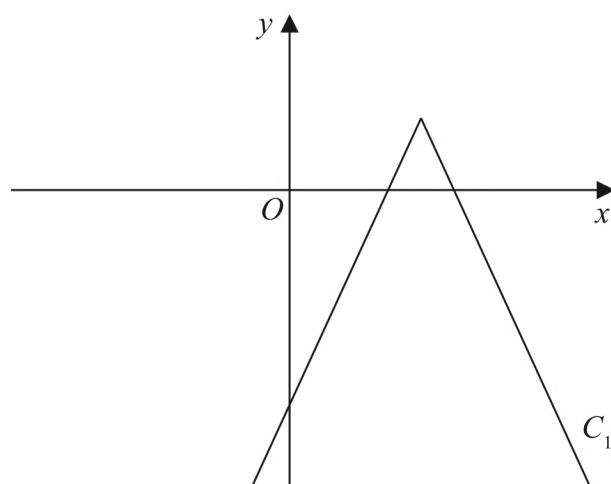
**Figure 3**

Figure 3 shows a sketch of the graph of C_1 with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

- (i) the vertex of C_1
- (ii) the intersection of C_1 with the y -axis.

(2)

(b) Find the x coordinates of the intersections of C_1 with the x -axis.

(2)

Diagram 1, shown on page 21, is a copy of Figure 3.

(c) On Diagram 1, sketch the curve C_2 with equation

$$y = \frac{1}{9}x^2 - 9$$

Identify clearly the coordinates of any points of intersection of C_2 with the coordinate axes.

(3)

(d) Find the coordinates of the points of intersection of C_1 and C_2
(Solutions relying entirely on calculator technology are not acceptable.)

(5)

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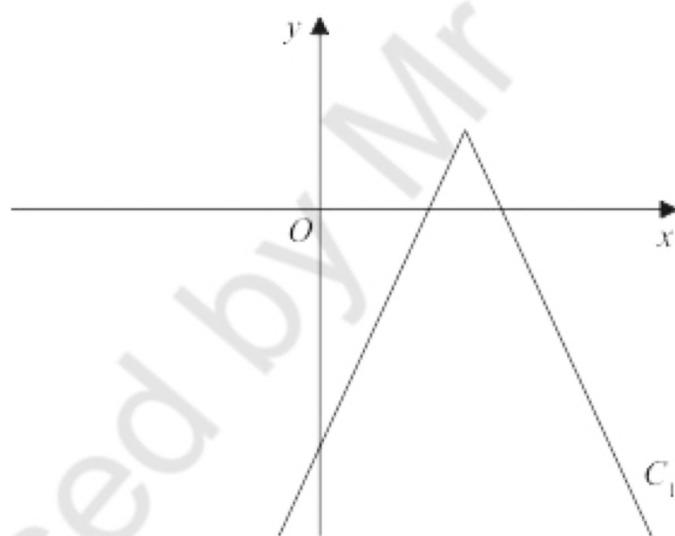


Diagram 1

- (c) Find g^{-1}

$$g(x) = \frac{3}{2x+1} \quad x \in \mathbb{R} \quad x \geq 0$$

(3)

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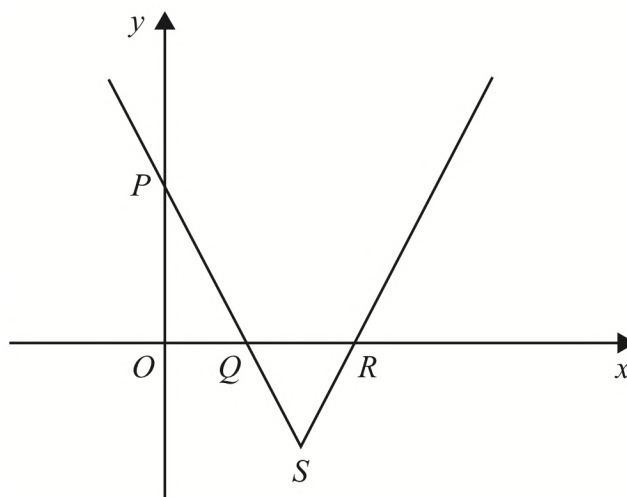
**Figure 2**

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y -axis at the point P
- cuts the x -axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a , the coordinates of

(i) point P

(ii) points Q and R

(iii) point S

(4)

(b) Find, in simplest form in terms of a , the values of x for which

$$|3x - 5a| - 2a = |x - 2a|$$

(4)

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[illegible]

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Question 4 continued

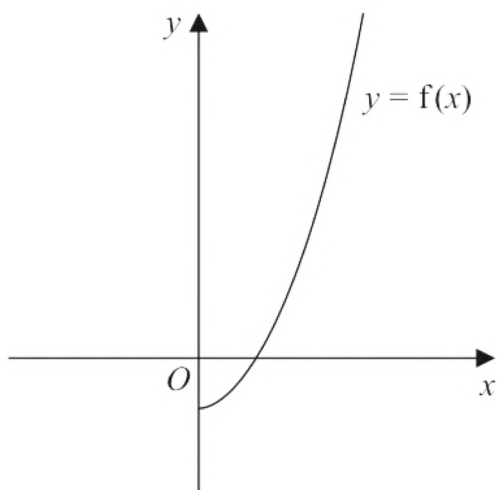


Diagram 1

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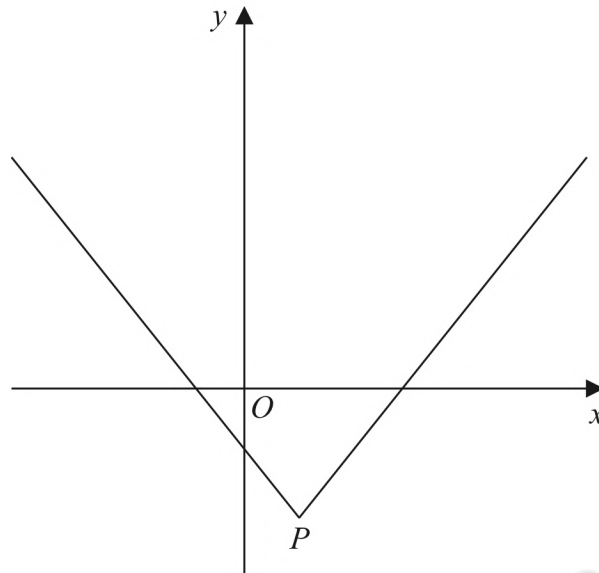
**Figure 2**

Figure 2 shows a sketch of the graph $y = f(x)$, where

$$f(x) = 3|x - 2| - 10$$

The vertex of the graph is at point P , shown in Figure 2.

(a) Find the coordinates of P

(2)

(b) Find $ff(0)$

(2)

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

$$f(|x|) = 0$$

(3)

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$f(x) = \frac{x+5}{x-4}$
 $x \in \mathbb{R}, x \neq 4$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$g(x) = x^2 + 5$
 $x \in \mathbb{R}, x > 0$

(c) Find the exact value of a for which

$gf(a) = 7$

(3)

[illegible]

$f(x) = \frac{x+5}{x-4}$
 $x \in \mathbb{R}, x \neq 4$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

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 $x \in \mathbb{R}, x > 0$

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[illegible]

$f(x) = \frac{x+5}{x-4}$
 $x \in \mathbb{R}, x \neq 4$

(a) Find $ff(6)$

(2)

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(3)

[illegible]

$f(x) = \frac{x+5}{x-4}$
 $x \in \mathbb{R}, x \neq 4$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$g(x) = x^2 + 5$
 $x \in \mathbb{R}, x > 0$

(c) Find the exact value of a for which

$gf(a) = 7$

(3)

$f(x) = \frac{x+5}{x-4}$
 $x \in \mathbb{R}, x \neq 4$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

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 $x \in \mathbb{R}, x > 0$

(c) Find the exact value of a for which

$gf(a) = 7$

(3)

[illegible]

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9.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

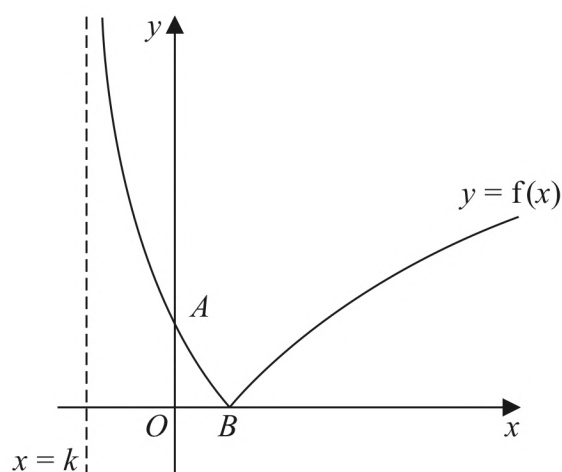


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4 \ln(x + 1)| \quad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y -axis at point A
- meets the x -axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4 \ln(x + 1)| > 3$$

(5)

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- Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(1)

(2)

(1)

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[illegible]

$f(x) = \frac{2x - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \quad x \in \mathbb{R} \quad x > 2$

(a) Show that $f(x) = \frac{2x}{3x - 5}$ (3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear. (3)

The function g is defined by

$$g(x) = 3 + 2 \ln x \quad x \geq 1$$

(c) Find g^{-1} (3)

(d) Find the exact value of a for which

$$gf(a) = 5$$
 (4)

$f(x) = \frac{2x - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \quad x \in \mathbb{R} \quad x > 2$

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 (4)

$f(x) = \frac{2x - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5}$ $x \in \mathbb{R} \quad x > 2$

(a) Show that $f(x) = \frac{2x}{3x - 5}$ (3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear. (3)

The function g is defined by

$$g(x) = 3 + 2 \ln x \qquad x \geqslant 1$$

(c) Find g^{-1} (3)

(d) Find the exact value of a for which

$$gf(a) = 5$$

(4)

$f(x) = \frac{2x - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \quad x \in \mathbb{R} \quad x > 2$

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(c) Find g^{-1} (3)

(d) Find the exact value of a for which

$$gf(a) = 5$$
 (4)

[illegible]

[illegible]

$f(x) = \frac{2x - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \quad x \in \mathbb{R} \quad x > 2$

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You must make your reasoning clear. (3)

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(d) Find the exact value of a for which

$$gf(a) = 5$$
 (4)

[illegible]

$f(x) = \frac{2x - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \quad x \in \mathbb{R} \quad x > 2$

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The function g is defined by

$$g(x) = 3 + 2 \ln x \quad x \geq 1$$

(c) Find g^{-1} (3)

(d) Find the exact value of a for which

$$gf(a) = 5$$
 (4)

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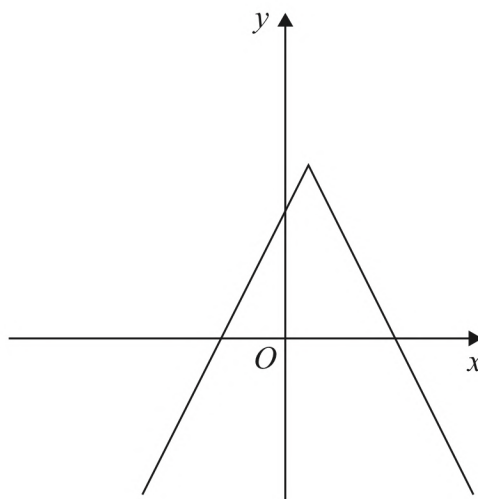


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b ,

- (i) the coordinates of the maximum point of the graph,
- (ii) the coordinates of the point of intersection of the graph with the y -axis,
- (iii) the coordinates of the points of intersection of the graph with the x -axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)

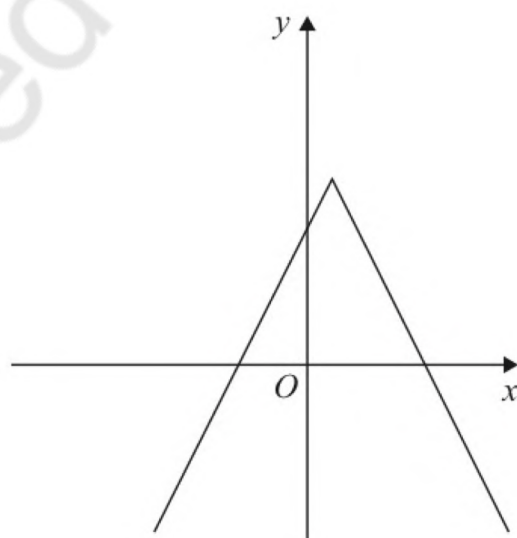


Diagram 1

1.

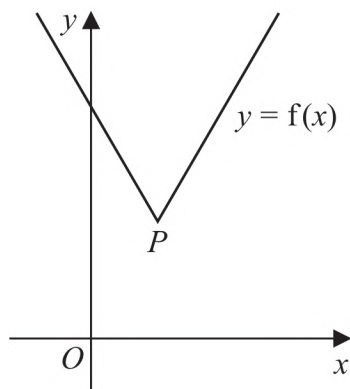
**Figure 1**

Figure 1 shows a sketch of the graph with equation $y = f(x)$ where

$$f(x) = 2|x - 5| + 10$$

The point P , shown in Figure 1, is the vertex of the graph.

(a) State the coordinates of P

(2)

(b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(Solutions relying on calculator technology are not acceptable.)

(2)

(c) Find the point to which P is mapped, when the graph with equation $y = f(x)$ is transformed to the graph with equation $y = 3f(x - 2)$

(2)

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$$g(x) = \frac{6x - 2}{2x + 1} \quad x > \frac{1}{3}$$

- (a) Find $f^{-1}(22)$ (2)
- (b) Use differentiation to prove that g is an increasing function. (3)
- (c) Find g^{-1} (3)
- (d) Find the range of fg (2)

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