Chapter 1: Projectiles

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7. A particle is projected from a point O with speed U at an angle of elevation α to the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2(1+\tan^2\alpha)}{2U^2}$$

(7)

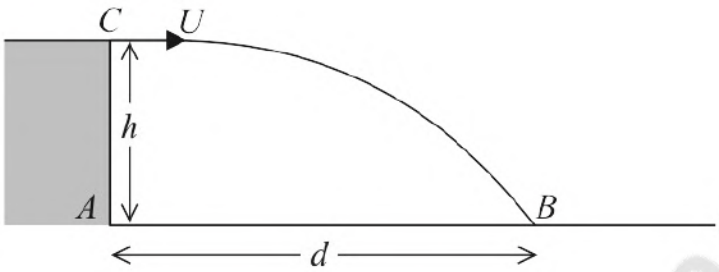

Figure 3

A small stone is projected horizontally with speed U from a point C at the top of a vertical cliff AC so as to hit a fixed target B on the horizontal ground. The point C is a height h above the ground, as shown in Figure 3. The time of flight of the stone from C to B is T, and the stone is modelled as a particle moving freely under gravity.

(b) Find, in terms of U, g and T, the speed of the stone as it hits the target at B.

(4)

It is found that, using the same initial speed U, the target can also be hit by projecting the stone from C at an angle α above the horizontal. The stone is again modelled as a particle moving freely under gravity and the distance $AB = d$.

(c) Using the result in part (a), or otherwise, show that

$$d = \frac{1}{2}gT^2\tan\alpha$$

(6)

6. A particle P is projected from a fixed point A with speed $12\ \text{m s}^{-1}$ at an angle α above the horizontal and moves freely under gravity. As P passes through the point B on its path, P is moving with speed $8\ \text{m s}^{-1}$ at an angle β below the horizontal.

(a) By considering energy, find the vertical distance between A and B.

(4)

Particle P takes 1.5 seconds to travel from A to B.

(b) Find the size of angle α.

(3)

(c) Find the size of angle β.

(3)

(d) Find the length of time for which the speed of P is less than $8\ \text{m s}^{-1}$.

(4)

4. At time $t = 0$ a ball is projected from a fixed point A on horizontal ground to hit a target. The ball is projected from A with speed u m s^{-1} at an angle $\theta°$ to the horizontal. At time $t = 2$ s the ball hits the target. At the instant when it hits the target, the ball is travelling downwards at 30° below the horizontal with speed 12 m s^{-1}. The ball is modelled as a particle moving freely under gravity and the target is modelled as the point T.

(a) Find

(i) the value of θ,

(ii) the value of u.

(7)

The height of T above the ground is h metres.

(b) Find the value of h.

(3)

(c) Find the length of time for which the ball is more than h metres above the ground during the flight from A to T.

(3)

8.

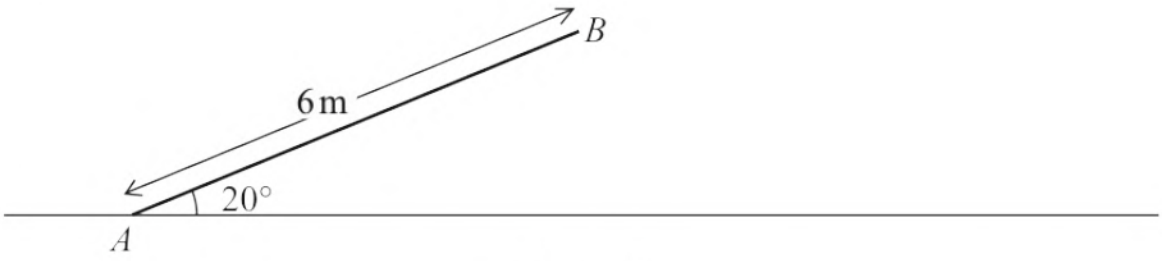

Figure 3

A rough ramp AB is fixed to horizontal ground at A. The ramp is inclined at $20°$ to the ground. The line AB is a line of greatest slope of the ramp and $AB = 6\,\text{m}$. The point B is at the top of the ramp, as shown in Figure 3. A particle P of mass $3\,\text{kg}$ is projected with speed $15\,\text{m s}^{-1}$ from A towards B. At the instant P reaches the point B the speed of P is $10\,\text{m s}^{-1}$. The force due to friction is modelled as a constant force of magnitude F newtons.

After leaving the ramp at B, the particle P moves freely under gravity until it hits the horizontal ground at the point C. The speed of P as it hits the ground at C is $w\,\text{m s}^{-1}$.

Find

(b) (i) the value of w,

(ii) the direction of motion of P as it hits the ground at C,

(5)

(c) the greatest height of P above the ground as P moves from A to C.

(4)

7.

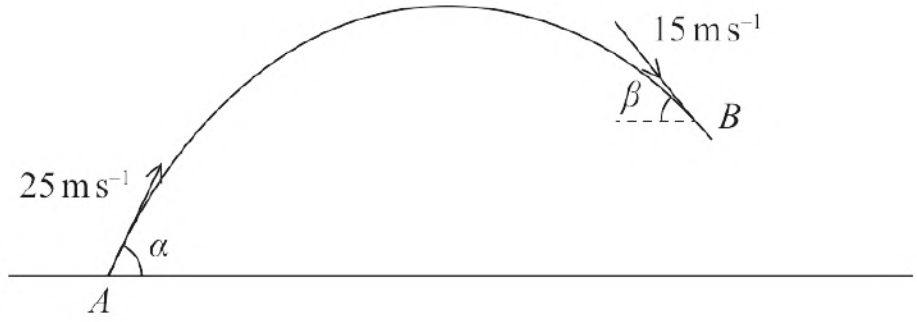

Figure 5

At time $t = 0$ a particle P is projected from a fixed point A on horizontal ground. The particle is projected with speed $25\,\text{m s}^{-1}$ at an angle α to the ground. The particle moves freely under gravity. At time $t = 3$ seconds, P is passing through the point B with speed $15\,\text{m s}^{-1}$ and is moving downwards at an angle β to the horizontal, as shown in Figure 5.

(a) By considering energy, find the height of B above the ground.

(3)

(b) Find the size of angle α.

(3)

(c) Find the size of angle β.

(3)

(d) Find the least speed of P as P travels from A to B.

(2)

As P travels from A to B, the speed, $v\,\text{m s}^{-1}$, of P is such that $v \leqslant 15$ for an interval of T seconds.

(e) Find the value of T.

(3)

8. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane, with* **i** *being horizontal and* **j** *being vertically upwards.*]

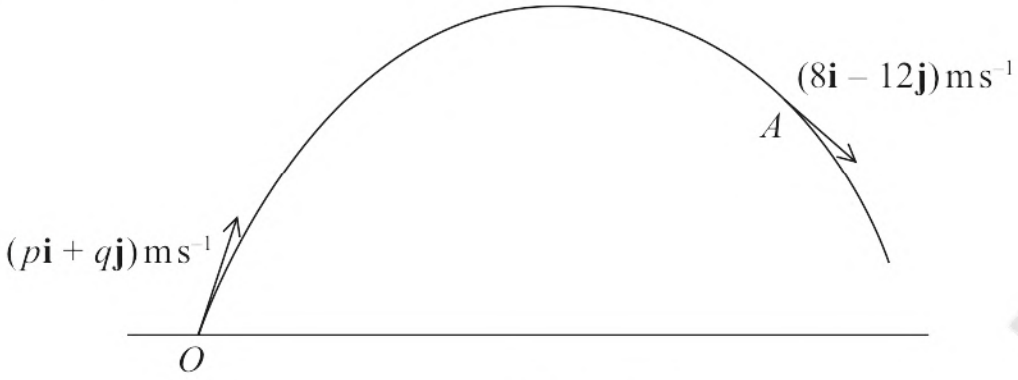

Figure 4

At time $t = 0$, a small ball is projected from a fixed point O on horizontal ground. The ball is projected from O with velocity $(p\mathbf{i} + q\mathbf{j})\,\mathrm{m\,s^{-1}}$, where p and q are positive constants. The ball moves freely under gravity.

At time $t = 3$ seconds, the ball passes through the point A with velocity $(8\mathbf{i} - 12\mathbf{j})\,\mathrm{m\,s^{-1}}$, as shown in Figure 4.

(a) Find the speed of the ball at the instant it is projected from O.

(5)

For an interval of T seconds the speed, $v\,\mathrm{m\,s^{-1}}$, of the ball is such that $v \leqslant 10$

(b) Find the value of T.

(4)

At the point B on the path of the ball, the direction of motion of the ball is perpendicular to the direction of motion of the ball at A.

(c) Find the vertical height of B above A.

(4)

7.

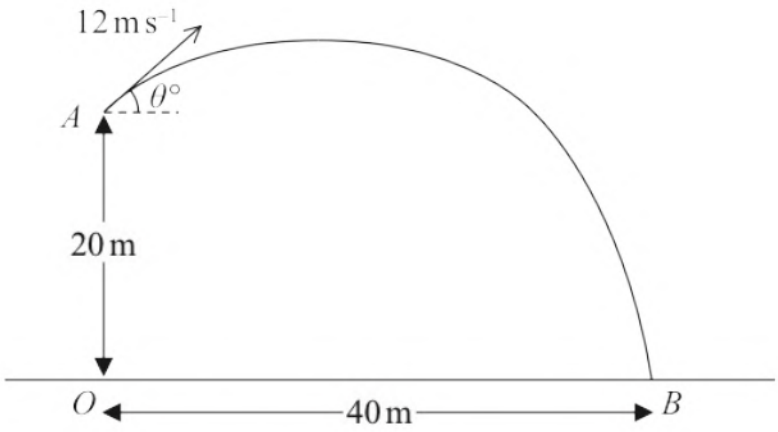

Figure 4

The fixed point A is 20 m vertically above the point O which is on horizontal ground. At time $t = 0$, a particle P is projected from A with speed 12 m s^{-1} at an angle $\theta°$ above the horizontal. The particle moves freely under gravity. At time $t = 5$ seconds, P strikes the ground at the point B, where $OB = 40$ m, as shown in Figure 4.

(a) By considering energy, find the speed of P as it hits the ground at B.

(4)

(b) Find the least speed of P as it moves from A to B.

(2)

(c) Find the length of time for which the speed of P is more than 10 m s^{-1}.

(6)

7. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

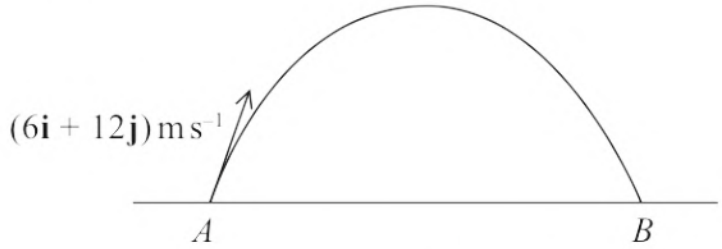

Figure 5

A small ball is projected with velocity $(6\mathbf{i} + 12\mathbf{j})\,\text{m s}^{-1}$ from a fixed point A on horizontal ground. The ball hits the ground at the point B, as shown in Figure 5. The motion of the ball is modelled as a particle moving freely under gravity.

(a) Find the distance AB.

(4)

When the height of the ball above the ground is more than h metres, the speed of the ball is less than $10\,\text{m s}^{-1}$

(b) Find the smallest possible value of h.

(4)

When the ball is at the point C on its path, the direction of motion of the ball is perpendicular to the direction of motion of the ball at the instant before it hits the ground at B.

(c) Find, in terms of **i** and **j**, the velocity of the ball when it is at C.

(3)

8.

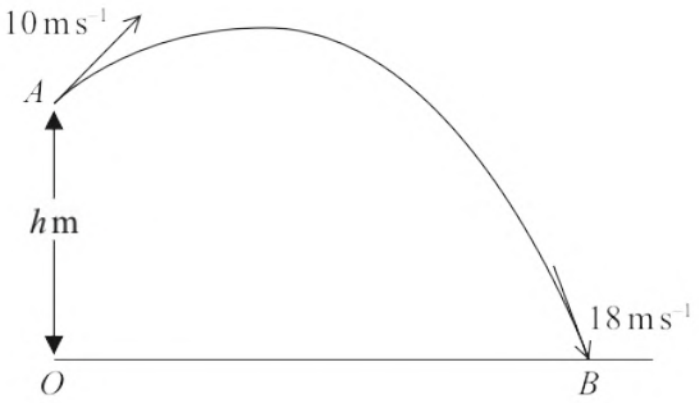

Figure 4

The fixed point A is h metres vertically above the point O that is on horizontal ground. At time $t = 0$, a particle P is projected from A with speed $10\,\text{m s}^{-1}$. The particle moves freely under gravity. At time $t = 2.5$ seconds, P strikes the ground at the point B. At the instant when P strikes the ground, the speed of P is $18\,\text{m s}^{-1}$, as shown in Figure 4.

(a) By considering energy, find the value of h.

(3)

(b) Find the distance OB.

(5)

As P moves from A to B, the speed of P is less than or equal to $8\,\text{m s}^{-1}$ for T seconds.

(c) Find the value of T

(6)

7. A particle P is projected from a fixed point O on horizontal ground. The particle is projected with speed u at an angle α above the horizontal. At the instant when the horizontal distance of P from O is x, the vertical distance of P above the ground is y.
The motion of P is modelled as that of a particle moving freely under gravity.

(a) Show that $y = x\tan\alpha - \dfrac{gx^2}{2u^2}\left(1 + \tan^2\alpha\right)$

(6)

A small ball is projected from the fixed point O on horizontal ground. The ball is projected with speed $20\,\text{m s}^{-1}$ at angle θ° above the horizontal. A vertical pole AB, of height $2\,\text{m}$, stands on the ground with $OA = 10\,\text{m}$, as shown in Figure 3.

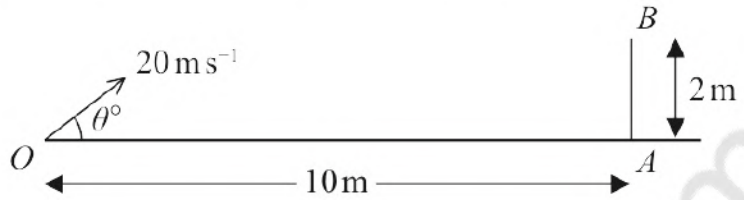

Figure 3

The ball is modelled as a particle moving freely under gravity and the pole is modelled as a rod.
The path of the ball lies in the vertical plane containing O, A and B.

Using the model,

(b) find the range of values of θ for which the ball will pass over the pole.

(3)

Given that $\theta = 40$ and that the ball first hits the ground at the point C

(c) find the speed of the ball at the instant it passes over the pole,

(5)

(d) find the distance OC.

(2)

8. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane, with* **i** *being horizontal and* **j** *being vertically upwards.*]

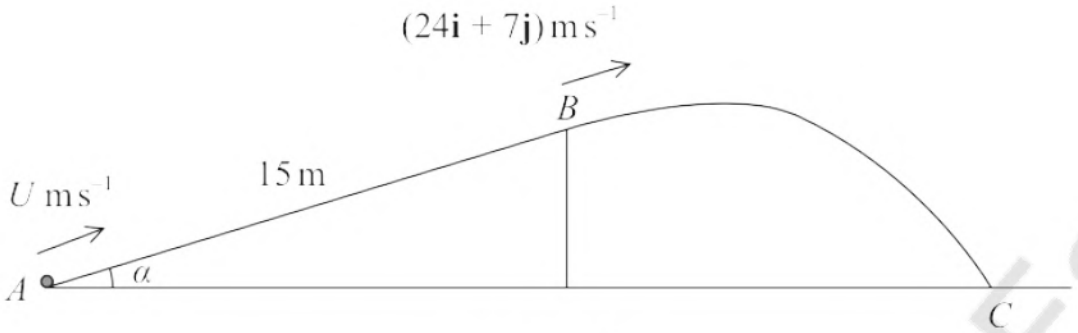

Figure 3

A rough ramp is fixed to horizontal ground.

The ramp is inclined to the ground at an angle α, where $\tan\alpha = \frac{7}{24}$

The point A is at the bottom of the ramp and the point B is at the top of the ramp.
The line AB is a line of greatest slope of the ramp and $AB = 15\,\text{m}$, as shown in Figure 3.

A particle P of mass 0.3 kg is projected with speed $U\,\text{m s}^{-1}$ from A directly towards B.
At the instant P reaches the point B, the velocity of P is $(24\mathbf{i} + 7\mathbf{j})\,\text{m s}^{-1}$
The particle leaves the ramp at B, and moves freely under gravity until it hits the horizontal ground at the point C.

The coefficient of friction between P and the ramp is $\frac{1}{5}$

(c) Find the time taken by P to move from B to C.

(3)

At the instant immediately before P hits the ground at C, the particle is moving downwards at $\theta°$ to the horizontal.

(d) Find the value of θ

(4)

8. A particle P is projected from a fixed point O. The particle is projected with speed $u\,\text{m s}^{-1}$ at angle α above the horizontal. The particle moves freely under gravity. At the instant when the horizontal distance of P from O is x metres, P is y metres vertically above the level of O.

(a) Show that $y = x\tan\alpha - \dfrac{gx^2}{2u^2}(1 + \tan^2\alpha)$

(6)

A small ball is projected from a fixed point A with speed $U\,\text{m s}^{-1}$ at $\theta°$ above the horizontal.
The point B is on horizontal ground and is vertically below the point A, with $AB = 20\,\text{m}$.
The ball hits the ground at the point C, where $BC = 30\,\text{m}$, as shown in Figure 4.

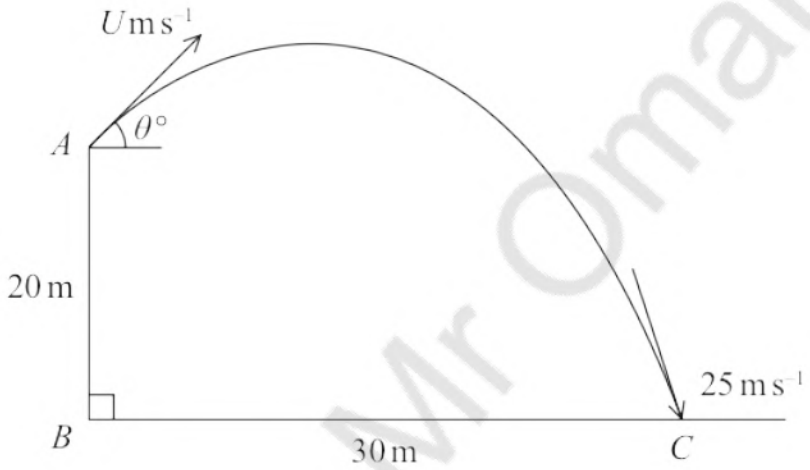

Figure 4

The speed of the ball immediately before it hits the ground is $25\,\text{m s}^{-1}$
The motion of the ball is modelled as that of a particle moving freely under gravity.

(b) Use the principle of conservation of mechanical energy to find the value of U.

(3)

(c) Find the value of θ

(3)

7. [*In this question, the perpendicular unit vectors* **i** *and* **j** *are in a vertical plane with* **i** *being horizontal and* **j** *being vertically upwards.*]

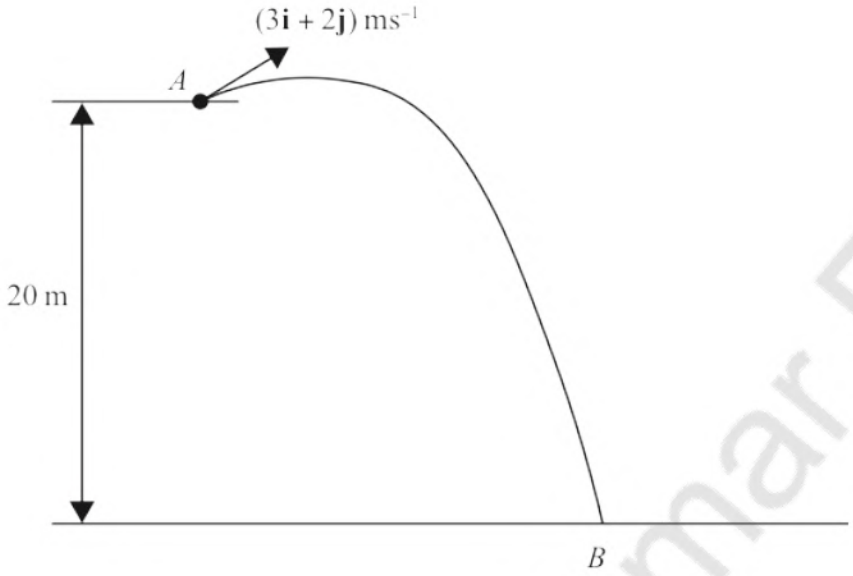

Figure 4

A small ball is projected with velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} from the fixed point A.

The point A is 20 m above horizontal ground.

The ball hits the ground at the point B, as shown in Figure 4.

The ball is modelled as a particle moving freely under gravity.

(a) By considering energy, find the speed of the ball at the instant immediately before it hits the ground.

(3)

(b) Find the direction of motion of the ball at the instant immediately before it hits the ground.

(3)

(c) Find the time taken for the ball to travel from A to B.

(3)

At the instant when the direction of motion of the ball is perpendicular to $(3\mathbf{i} + 2\mathbf{j})$ the ball is h metres above the ground.

(d) Find the value of h.

(6)

4. [*In this question* **i** *and* **j** *are unit vectors, with* **i** *horizontal and* **j** *vertical.*]

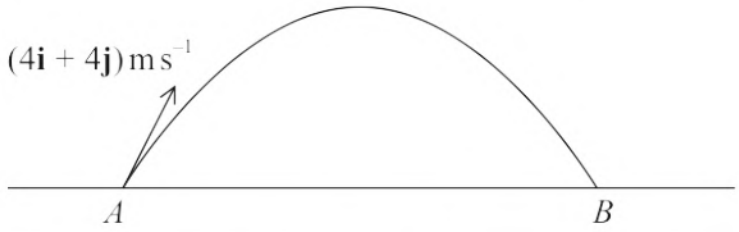

Figure 3

The fixed points A and B lie on horizontal ground.

At time $t = 0$, a particle P is projected from A with velocity $(4\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

Particle P moves freely under gravity and hits the ground at B, as shown in Figure 3.

At time T_1 seconds, P is at its highest point above the ground.

(a) Find the value of T_1

(2)

At time $t = 0$, a particle Q is also projected from A but with velocity $(5\mathbf{i} + 7\mathbf{j})\,\mathrm{m\,s^{-1}}$

Particle Q moves freely under gravity.

(b) Find the vertical distance between Q and P at time T_1 seconds, giving your answer to 2 significant figures.

(3)

At the instant when particle P reaches B, particle Q is moving at $\alpha°$ below the horizontal.

(c) Find the value of α.

(4)

At time T_2 seconds, the direction of motion of Q is perpendicular to the initial direction of motion of Q.

(d) Find the value of T_2

(3)

8. [*In this question* **i** *and* **j** *are unit vectors, with* **i** *horizontal and* **j** *vertical.*]

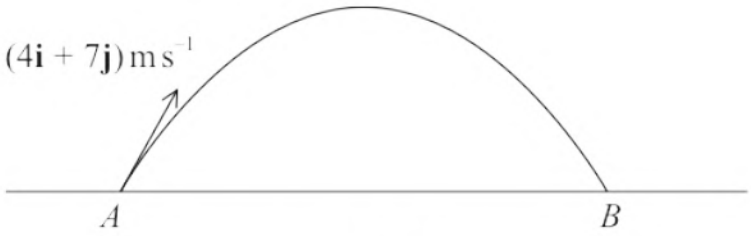

Figure 6

The fixed points A and B lie on horizontal ground.

At time $t = 0$, a particle P is projected from A with velocity $(4\mathbf{i} + 7\mathbf{j})\,\mathrm{m\,s^{-1}}$

Particle P moves freely under gravity and hits the ground at B, as shown in Figure 6.

(a) Find the distance AB.

(4)

The speed of P is less than $5\,\mathrm{m\,s^{-1}}$ for an interval of length T seconds.

(b) Find the value of T

(3)

At the instant when the direction of motion of P is perpendicular to the initial direction of motion of P, the particle is h metres above the ground.

(c) Find the value of h.

(4)

7. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards*.]

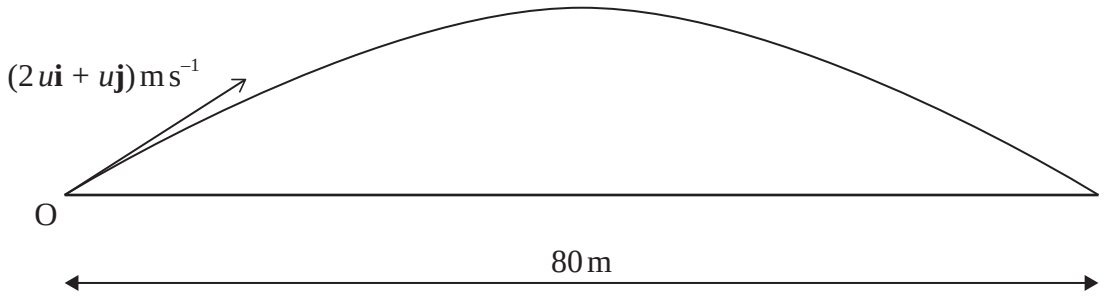

Figure 3

A golf ball is hit from a point O on horizontal ground and is modelled as a particle moving freely under gravity. The initial velocity of the ball is $(2u\mathbf{i} + u\mathbf{j})\,\text{m s}^{-1}$
The ball first hits the horizontal ground at a point which is 80 m from O, as shown in Figure 3.

Use the model to

(a) show that $u = 14$

(6)

(b) find the total time, while the ball is in the air, for which the speed of the ball is greater than $7\sqrt{17}\ \text{m s}^{-1}$

(5)