Chapter 8&9: Energy, Work and Power

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1 A lorry of mass 16 000 kg is travelling along a straight horizontal road. The engine of the lorry is working at constant power. The work done by the driving force in 10 s is 750 000 J.

(a) Find the power of the lorry's engine. [1]

(b) There is a constant resistance force acting on the lorry of magnitude 2400 N.

Find the acceleration of the lorry at an instant when its speed is $25\,\mathrm{m\,s^{-1}}$. [3]

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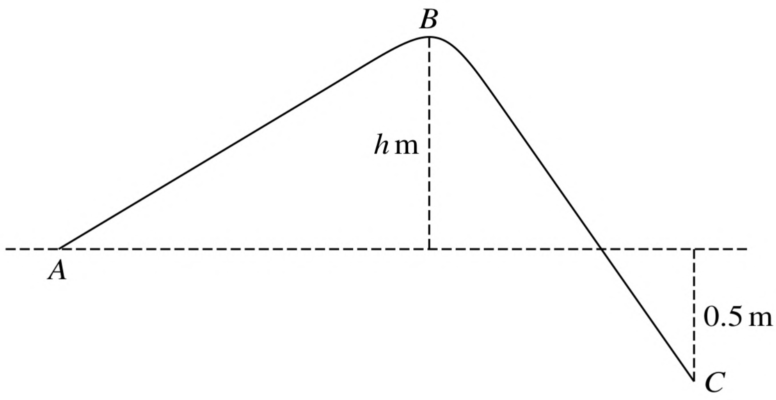

The diagram shows the vertical cross-section of a surface. A, B and C are three points on the cross-section. The level of B is h m above the level of A. The level of C is 0.5 m below the level of A. A particle of mass 0.2 kg is projected up the slope from A with initial speed 5 m s^{-1}. The particle remains in contact with the surface as it travels from A to C.

(a) Given that the particle reaches B with a speed of 3 m s^{-1} and that there is no resistance force, find h. [3]

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(b) It is given instead that there is a resistance force and that the particle does 3.1 J of work against the resistance force as it travels from A to C.

Find the speed of the particle when it reaches C. [3]

2 A car of mass 1800 kg is towing a trailer of mass 400 kg along a straight horizontal road. The car and trailer are connected by a light rigid tow-bar. The car is accelerating at $1.5\ \text{m s}^{-2}$. There are constant resistance forces of 250 N on the car and 100 N on the trailer.

(a) Find the tension in the tow-bar. [2]

(b) Find the power of the engine of the car at the instant when the speed is $20\ \text{m s}^{-1}$. [3]

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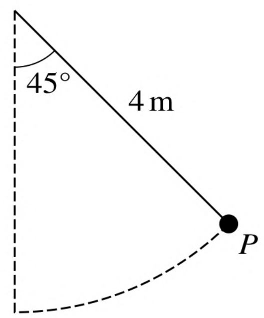

A child of mass 35 kg is swinging on a rope. The child is modelled as a particle P and the rope is modelled as a light inextensible string of length 4 m. Initially P is held at an angle of 45° to the vertical (see diagram).

(a) Given that there is no resistance force, find the speed of P when it has travelled half way along the circular arc from its initial position to its lowest point. [4]

(b) It is given instead that there is a resistance force. The work done against the resistance force as P travels from its initial position to its lowest point is X J. The speed of P at its lowest point is $4\,\mathrm{m\,s^{-1}}$.

Find X. [3]

4 Small smooth spheres A and B, of equal radii and of masses 4 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed $10\,\text{m}\,\text{s}^{-1}$. After the spheres collide A continues to move in the same direction but with half the speed of B.

(a) Find the speed of B after the collision. [2]

A third small smooth sphere C, of mass 1 kg and with the same radius as A and B, is at rest on the plane. B now collides directly with C. After this collision B continues to move in the same direction but with one third the speed of C.

(b) Show that there is another collision between A and B. [3]

(c) *A* and *B* coalesce during this collision.

Find the total loss of kinetic energy in the system due to the three collisions. [5]

5 A car of mass 1250 kg is moving on a straight road.

(a) On a horizontal section of the road, the car has a constant speed of 32 m s^{-1} and there is a constant force of 750 N resisting the motion.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

(ii) Given that this power is suddenly decreased by 8 kW, find the instantaneous deceleration of the car. [3]

(b) On a section of the road inclined at $\sin^{-1} 0.096$ to the horizontal, the resistance to the motion of the car is $(1000 + 8v)$ N when the speed of the car is $v \text{ m s}^{-1}$. The car travels up this section of the road at constant speed with the engine working at 60 kW.

Find this constant speed. [5]

2 A minibus of mass 4000 kg is travelling along a straight horizontal road. The resistance to motion is 900 N.

(a) Find the driving force when the acceleration of the minibus is $0.5\ \mathrm{m\,s^{-2}}$. [2]

(b) Find the power required for the minibus to maintain a constant speed of $25\ \mathrm{m\,s^{-1}}$. [2]

5 A block B of mass 4 kg is pushed up a line of greatest slope of a smooth plane inclined at 30° to the horizontal by a force applied to B, acting in the direction of motion of B. The block passes through points P and Q with speeds $12\,\mathrm{m\,s^{-1}}$ and $8\,\mathrm{m\,s^{-1}}$ respectively. P and Q are 10 m apart with P below the level of Q.

(a) Find the decrease in kinetic energy of the block as it moves from P to Q. [2]

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(b) Hence find the work done by the force pushing the block up the slope as the block moves from P to Q. [3]

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(c) At the instant the block reaches Q, the force pushing the block up the slope is removed.

Find the time taken, after this instant, for the block to return to P. [4]

1 A particle B of mass 5 kg is at rest on a smooth horizontal table. A particle A of mass 2.5 kg moves on the table with a speed of $6\,\text{m}\,\text{s}^{-1}$ and collides directly with B. In the collision the two particles coalesce.

(a) Find the speed of the combined particle after the collision. [2]

(b) Find the loss of kinetic energy of the system due to the collision. [3]

2 A car of mass 1400 kg is moving along a straight horizontal road against a resistance of magnitude 350 N.

(a) Find, in kW, the rate at which the engine of the car is working when it is travelling at a constant speed of $20\,\mathrm{m\,s^{-1}}$. [2]

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(b) Find the acceleration of the car when its speed is $20\,\mathrm{m\,s^{-1}}$ and the engine is working at 15 kW. [3]

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2 A car of mass 1800 kg is travelling along a straight horizontal road. The power of the car's engine is constant. There is a constant resistance to motion of 650 N.

(a) Find the power of the car's engine, given that the car's acceleration is $0.5\,\mathrm{m\,s^{-2}}$ when its speed is $20\,\mathrm{m\,s^{-1}}$. [3]

(b) Find the steady speed which the car can maintain with the engine working at this power. [2]

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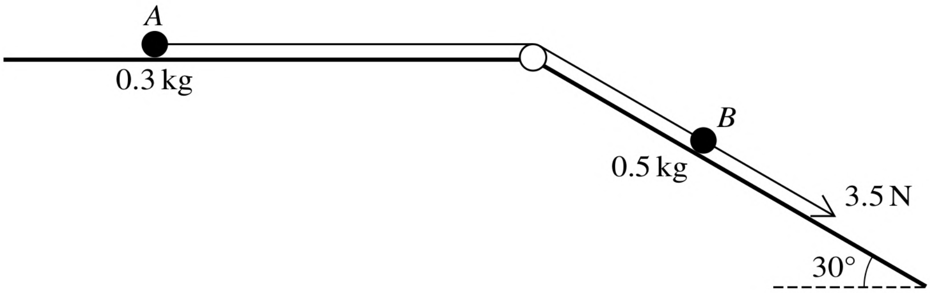

Two particles A and B, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane (see diagram).

(b) It is given instead that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, the total amount of work done against friction is 1.1 J.

Use an energy method to find the speed of B when it has moved this distance down the plane. [You should assume that the string is sufficiently long so that A does not hit the pulley when it moves 0.6 m.] [4]

2 A box of mass 5 kg is pulled at a constant speed a distance of 15 m up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the work done against friction. [1]

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(b) Find the change in gravitational potential energy of the box. [2]

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(c) Find the work done by the pulling force. [1]

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6 A car of mass 1600 kg is pulling a caravan of mass 800 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 400 N and 250 N respectively.

(a) The car and caravan are travelling along a straight horizontal road.

(i) Given that the car and caravan have a constant speed of $25\,\mathrm{m\,s^{-1}}$, find the power of the car's engine. [2]

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(ii) The engine's power is now suddenly increased to 39 kW. Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

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(b) The car and caravan now travel up a straight hill, inclined at an angle of $\sin^{-1} 0.05$ to the horizontal, at a constant speed of $v\,\text{m}\,\text{s}^{-1}$. The car's engine is working at 32.5 kW.

Find v. [3]

2 A car of mass 1400 kg is travelling at constant speed up a straight hill inclined at α to the horizontal, where $\sin\alpha = 0.1$. There is a constant resistance force of magnitude 600 N. The power of the car's engine is 22 500 W.

(a) Show that the speed of the car is $11.25\,\mathrm{m\,s^{-1}}$. [3]

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The car, moving with speed $11.25\,\mathrm{m\,s^{-1}}$, comes to a section of the hill which is inclined at $2°$ to the horizontal.

(b) Given that the power and resistance force do not change, find the initial acceleration of the car up this section of the hill. [3]

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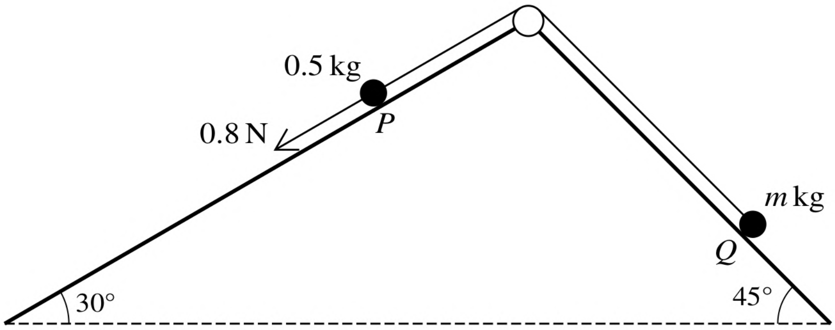

Two particles P and Q of masses 0.5 kg and m kg respectively are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the top of two inclined planes. The particles are initially at rest with P on a smooth plane inclined at 30° to the horizontal and Q on a plane inclined at 45° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes. A force of magnitude 0.8 N is applied to P acting down the plane, causing P to move down the plane (see diagram).

(b) It is given instead that the plane on which Q rests is rough, and that after each particle has moved a distance of 1 m, their speed is $0.6\,\mathrm{m\,s^{-1}}$. The work done against friction in this part of the motion is 0.5 J.

Use an energy method to find the value of m. [5]

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1 A winch operates by means of a force applied by a rope. The winch is used to pull a load of mass 50 kg up a line of greatest slope of a plane inclined at 60° to the horizontal. The winch pulls the load a distance of 5 m up the plane at constant speed. There is a constant resistance to motion of 100 N.

Find the work done by the winch. [3]

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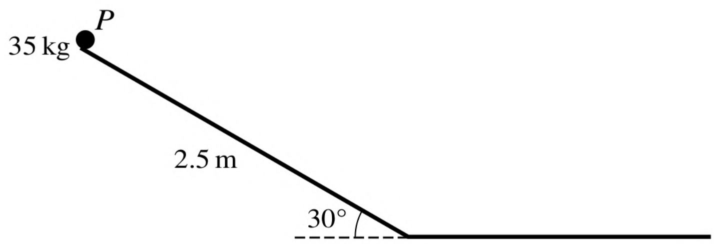

A slide in a playground descends at a constant angle of 30° for 2.5 m. It then has a horizontal section in the same vertical plane as the sloping section. A child of mass 35 kg, modelled as a particle P, starts from rest at the top of the slide and slides straight down the sloping section. She then continues along the horizontal section until she comes to rest (see diagram). There is no instantaneous change in speed when the child goes from the sloping section to the horizontal section.

The child experiences a resistance force on the horizontal section of the slide, and the work done against the resistance force on the horizontal section of the slide is 250 J per metre.

(a) It is given that the sloping section of the slide is smooth.

(i) Find the speed of the child when she reaches the bottom of the sloping section. [3]

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(ii) Find the distance that the child travels along the horizontal section of the slide before she comes to rest. [2]

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(b) It is given instead that the sloping section of the slide is rough and that the child comes to rest on the slide 1.05 m after she reaches the horizontal section.

Find the coefficient of friction between the child and the sloping section of the slide. [6]

1 A particle of mass 0.6 kg is projected with a speed of $4\,\text{m}\,\text{s}^{-1}$ down a line of greatest slope of a smooth plane inclined at $10°$ to the horizontal.

Use an energy method to find the speed of the particle after it has moved 15 m down the plane. [3]

5 A car of mass 1250 kg is pulling a caravan of mass 800 kg along a straight road. The resistances to the motion of the car and caravan are 440 N and 280 N respectively. The car and caravan are connected by a light rigid tow-bar.

(a) The car and caravan move along a horizontal part of the road at a constant speed of $30\,\mathrm{m\,s^{-1}}$.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

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(ii) Given that this power is suddenly decreased by 8 kW, find the instantaneous deceleration of the car and caravan and the tension in the tow-bar. [4]

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(b) The car and caravan now travel along a part of the road inclined at $\sin^{-1} 0.06$ to the horizontal. The car and caravan travel up the incline at constant speed with the engine of the car working at 28 kW.

(i) Find this constant speed. [3]

(ii) Find the increase in the potential energy of the caravan in one minute. [2]

2 A cyclist is travelling along a straight horizontal road. She is working at a constant rate of 150 W. At an instant when her speed is 4 m s^{-1}, her acceleration is 0.25 m s^{-2}. The resistance to motion is 20 N.

(a) Find the total mass of the cyclist and her bicycle. [3]

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The cyclist comes to a straight hill inclined at an angle θ above the horizontal. She ascends the hill at constant speed 3 m s^{-1}. She continues to work at the same rate as before and the resistance force is unchanged.

(b) Find the value of θ. [2]

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5 A car of mass 1400 kg is towing a trailer of mass 500 kg down a straight hill inclined at an angle of 5° to the horizontal. The car and trailer are connected by a light rigid tow-bar. At the top of the hill the speed of the car and trailer is $20\,\mathrm{m\,s^{-1}}$ and at the bottom of the hill their speed is $30\,\mathrm{m\,s^{-1}}$.

(a) It is given that as the car and trailer descend the hill, the engine of the car does 150 000 J of work, and there are no resistance forces.

Find the length of the hill. [5]

(b) It is given instead that there is a resistance force of 100 N on the trailer, the length of the hill is 200 m, and the acceleration of the car and trailer is constant.

Find the tension in the tow-bar between the car and trailer. [4]

2 Two small smooth spheres A and B, of equal radii and of masses km kg and m kg respectively, where $k > 1$, are free to move on a smooth horizontal plane. A is moving towards B with speed $6\,\mathrm{m\,s^{-1}}$ and B is moving towards A with speed $2\,\mathrm{m\,s^{-1}}$. After the collision A and B coalesce and move with speed $4\,\mathrm{m\,s^{-1}}$.

(b) Find, in terms of m, the loss of kinetic energy due to the collision. [2]

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5 A car of mass 1600 kg travels at constant speed $20\,\text{m}\,\text{s}^{-1}$ up a straight road inclined at an angle of $\sin^{-1} 0.12$ to the horizontal.

(a) Find the change in potential energy of the car in 30 s. [3]

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(b) Given that the total work done by the engine of the car in this time is 1960 kJ, find the constant force resisting the motion. [3]

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(c) Calculate, in kW, the power developed by the engine of the car. [2]

(d) Given that this power is suddenly decreased by 15%, find the instantaneous deceleration of the car. [3]

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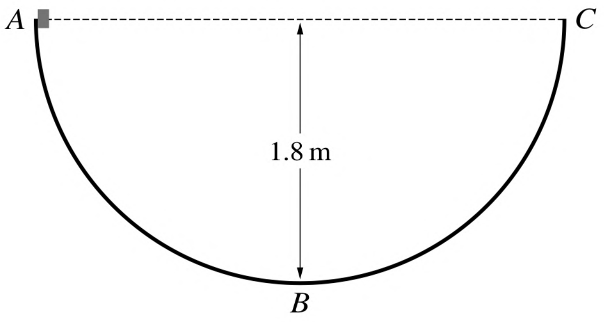

The diagram shows a semi-circular track ABC of radius 1.8 m which is fixed in a vertical plane. The points A and C are at the same horizontal level and the point B is at the bottom of the track. The section AB is smooth and the section BC is rough. A small block is released from rest at A.

(a) Show that the speed of the block at B is $6\,\text{m}\,\text{s}^{-1}$. [2]

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The block comes to instantaneous rest for the first time at a height of 1.2 m above the level of B. The work done against the resistance force during the motion of the block from B to this point is 4.5 J.

(b) Find the mass of the block. [3]

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5 A railway engine of mass 75 000 kg is moving up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = 0.01$. The engine is travelling at a constant speed of $30\,\mathrm{m\,s^{-1}}$. The engine is working at 960 kW. There is a constant force resisting the motion of the engine.

(a) Find the resistance force. [3]

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The engine comes to a section of track which is horizontal. At the start of the section the engine is travelling at $30\,\mathrm{m\,s^{-1}}$ and the power of the engine is now reduced to 900 kW. The resistance to motion is no longer constant, but in the next 60 s the work done against the resistance force is 46 500 kJ.

(b) Find the speed of the engine at the end of the 60 s. [4]

3 A ball of mass 1.6 kg is released from rest at a point 5 m above horizontal ground. When the ball hits the ground it instantaneously loses 8 J of kinetic energy and starts to move upwards.

(a) Use an energy method to find the greatest height that the ball reaches after hitting the ground. [3]

(b) Find the total time taken, from the initial release of the ball until it reaches this greatest height. [3]

4 A car of mass 1400 kg is moving on a straight road against a constant force of 1250 N resisting the motion.

(a) The car moves along a horizontal section of the road at a constant speed of $36\,\text{m}\,\text{s}^{-1}$.

(i) Calculate the work done against the resisting force during the first 8 seconds. [2]

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(ii) Calculate, in kW, the power developed by the engine of the car. [2]

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(iii) Given that this power is suddenly increased by 12 kW, find the instantaneous acceleration of the car. [3]

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(b) The car now travels at a constant speed of $32\,\mathrm{m\,s^{-1}}$ up a section of the road inclined at $\theta°$ to the horizontal, with the engine working at 64 kW.

Find the value of θ. [2]

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1 A crane is used to raise a block of mass 600 kg vertically upwards at a constant speed through a height of 15 m. There is a resistance to the motion of the block, which the crane does 10 000 J of work to overcome.

(a) Find the total work done by the crane. [2]

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(b) Given that the average power exerted by the crane is 12.5 kW, find the total time for which the block is in motion. [2]

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3 A car of mass m kg is towing a trailer of mass 300 kg down a straight hill inclined at 3° to the horizontal at a constant speed. There are resistance forces on the car and on the trailer, and the total work done against the resistance forces in a distance of 50 m is 40 000 J. The engine of the car is doing no work and the tow-bar is light and rigid.

(a) Find the value of m. [3]

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The resistance force on the trailer is 200 N.

(b) Find the tension in the tow-bar between the car and the trailer. [2]

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4 The total mass of a cyclist and her bicycle is 70 kg. The cyclist is riding with constant power of 180 W up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = 0.05$. At an instant when the cyclist's speed is $6\,\mathrm{m\,s^{-1}}$, her acceleration is $-0.2\,\mathrm{m\,s^{-2}}$. There is a constant resistance to motion of magnitude F N.

(a) Find the value of F. [4]

(b) Find the steady speed that the cyclist could maintain up the hill when working at this power. [2]

5 Two racing cars A and B are at rest alongside each other at a point O on a straight horizontal test track. The mass of A is 1200 kg. The engine of A produces a constant driving force of 4500 N. When A arrives at a point P its speed is $25\,\mathrm{m\,s^{-1}}$. The distance OP is d m. The work done against the resistance force experienced by A between O and P is 75 000 J.

(a) Show that $d = 100$. [3]

1 Small smooth spheres A and B, of equal radii and of masses 5 kg and 3 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed $8.5\,\text{m}\,\text{s}^{-1}$. The spheres collide and after the collision A continues to move in the same direction but with a quarter of the speed of B.

(a) Find the speed of B after the collision. [3]

(b) Find the loss of kinetic energy of the system due to the collision. [2]

6 A car of mass 900 kg is moving up a hill inclined at $\sin^{-1} 0.12$ to the horizontal. The initial speed of the car is $11\,\text{m}\,\text{s}^{-1}$. After 12 s, the car has travelled 150 m up the hill and has speed $16\,\text{m}\,\text{s}^{-1}$. The engine of the car is working at a constant rate of 24 kW.

(a) Find the work done against the resistive forces during the 12 s. [5]

The car then travels along a straight horizontal road. There is a resistance to the motion of the car of $(1520 + 4v)$ N when the speed of the car is v m s^{-1}. The car travels at a constant speed with the engine working at a constant rate of 32 kW.

(b) Find this speed. [3]

5 A cyclist is riding along a straight horizontal road. The total mass of the cyclist and her bicycle is 70 kg. At an instant when the cyclist's speed is $4\,\mathrm{m\,s^{-1}}$, her acceleration is $0.3\,\mathrm{m\,s^{-2}}$. There is a constant resistance to motion of magnitude 30 N.

(a) Find the power developed by the cyclist. [3]

The cyclist comes to the top of a hill inclined at 5° to the horizontal. The cyclist stops pedalling and freewheels down the hill (so that the cyclist is no longer supplying any power). The magnitude of the resistance force remains at 30 N. Over a distance of d m, the speed of the cyclist increases from $6\,\text{m}\,\text{s}^{-1}$ to $12\,\text{m}\,\text{s}^{-1}$.

(b) Find the change in kinetic energy. [2]

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(c) Use an energy method to find d. [3]

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2 Small smooth spheres A and B, of equal radii and of masses 6 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially A is moving towards B with speed $5\,\text{m}\,\text{s}^{-1}$ and B is moving towards A with speed $3\,\text{m}\,\text{s}^{-1}$. After the spheres collide, both A and B move in the same direction and the difference in the speeds of the spheres is $2\,\text{m}\,\text{s}^{-1}$.

Find the loss of kinetic energy of the system due to the collision. [5]

3 A constant resistance of magnitude 1400 N acts on a car of mass 1250 kg.

(a) The car is moving along a straight level road at a constant speed of $28\,\text{m}\,\text{s}^{-1}$.

Find, in kW, the rate at which the engine of the car is working. [2]

(b) The car now travels at a constant speed up a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.12$, with the engine working at 43.5 kW.

Find this speed. [3]

(c) On another occasion, the car pulls a trailer of mass 600 kg up the same hill. The system of the car and the trailer is modelled as particles connected by a light inextensible cable. The car's engine produces a driving force of 5000 N and the resistance to the motion of the trailer is 300 N. The resistance to the motion of the car remains 1400 N.

Find the acceleration of the system and the tension in the cable. [4]

1 A cyclist is riding a bicycle along a straight horizontal road AB of length 50 m. The cyclist starts from rest at A and reaches a speed of $6\,\mathrm{m\,s^{-1}}$ at B. The cyclist produces a constant driving force of magnitude 100 N. There is a resistance force, and the work done against the resistance force from A to B is 3560 J.

Find the total mass of the cyclist and bicycle. [3]

4 A car of mass 1200 kg is travelling along a straight horizontal road AB. There is a constant resistance force of magnitude 500 N. When the car passes point A, it has a speed of $15\,\text{m}\,\text{s}^{-1}$ and an acceleration of $0.8\,\text{m}\,\text{s}^{-2}$.

(a) Find the power of the car's engine at the point A. [3]

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The car continues to work with this power as it travels from A to B. The car takes 53 seconds to travel from A to B and the speed of the car at B is $32\,\text{m}\,\text{s}^{-1}$.

(b) Show that the distance AB is 1362.6 m. [3]

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2 A box of mass 5 kg is pulled at a constant speed of $1.8\,\mathrm{m\,s^{-1}}$ for 15 s up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the change in gravitational potential energy of the box. [2]

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(b) Find the work done by the pulling force. [2]

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6 A car of mass 1750 kg is pulling a caravan of mass 500 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 650 N and 150 N respectively.

(a) The car and caravan are moving along a straight horizontal road at a constant speed of $24\,\text{m s}^{-1}$.

(i) Find the power of the car's engine. [2]

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(ii) The engine's power is now suddenly increased to 40 kW.

Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

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(b) The car and caravan now travel up a straight hill, inclined at an angle $\sin^{-1} 0.14$ to the horizontal, at a constant speed of $v \text{ m s}^{-1}$. The car's engine is working at 31 kW. The resistances to the motion of the car and caravan are unchanged.

Find v. [3]

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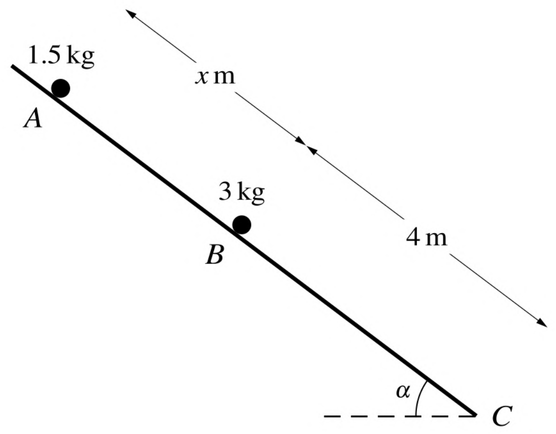

Particles of masses 1.5 kg and 3 kg lie on a plane which is inclined at an angle of α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The section of the plane from A to B is smooth and the section of the plane from B to C is rough. The 1.5 kg particle is held at rest at A and the 3 kg particle is in limiting equilibrium at B. The distance AB is x m and the distance BC is 4 m (see diagram).

(a) Show that the coefficient of friction between the particle at B and the plane is 0.75. [3]

The 1.5 kg particle is released from rest. In the subsequent motion the two particles collide and coalesce. The time taken for the combined particle to travel from B to C is 2 s. The coefficient of friction between the combined particle and the plane is still 0.75.

(b) Find x. [6]

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(c) Find the total loss of energy of the particles from the time the 1.5 kg particle is released until the combined particle reaches C. [3]

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1 A crate of mass 200 kg is being pulled at constant speed along horizontal ground by a horizontal rope attached to a winch. The winch is working at a constant rate of 4.5 kW and there is a constant resistance to the motion of the crate of magnitude 600 N.

(a) Find the time that it takes for the crate to move a distance of 15 m. [2]

The rope breaks after the crate has moved 15 m.

(b) Find the time taken, after the rope breaks, for the crate to come to rest. [3]

4 A toy railway locomotive of mass 0.8 kg is towing a truck of mass 0.4 kg on a straight horizontal track at a constant speed of $2\,\mathrm{m\,s^{-1}}$. There is a constant resistance force of magnitude 0.2 N on the locomotive, but no resistance force on the truck. There is a light rigid horizontal coupling connecting the locomotive and the truck.

(a) State the tension in the coupling. [1]

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(b) Find the power produced by the locomotive's engine. [1]

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The power produced by the locomotive's engine is now changed to 1.2 W.

(c) Find the magnitude of the tension in the coupling at the instant that the locomotive begins to accelerate. [5]

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2 A particle P of mass 0.4 kg is projected vertically upwards from horizontal ground with speed $10\,\mathrm{m\,s^{-1}}$.

(a) Find the greatest height above the ground reached by P. [2]

When P reaches the ground again, it bounces vertically upwards. At the first instant that it hits the ground, P loses 7.2 J of energy.

(b) Find the time between the first and second instants at which P hits the ground. [4]

7 A car of mass 1200 kg is travelling along a straight horizontal road. The power of the car's engine is constant and is equal to 16 kW. There is a constant resistance to motion of magnitude 500 N.

The car comes to the bottom of a straight hill of length 316 m, inclined at an angle to the horizontal of $\sin^{-1}\left(\frac{1}{60}\right)$. The power remains constant at 16 kW, but the magnitude of the resistance force is no longer constant and changes such that the work done against the resistance force in ascending the hill is 128 400 J. The time taken to ascend the hill is 15 s.

(c) Given that the car is travelling at a speed of $20\,\text{m}\,\text{s}^{-1}$ at the bottom of the hill, find its speed at the top of the hill. [6]

1 A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is $12\,\mathrm{m\,s^{-1}}$.

Find the work done against air resistance. [3]

2 Two particles A and B, of masses 3.2 kg and 2.4 kg respectively, lie on a smooth horizontal table. A moves towards B with a speed of $v\,\mathrm{m\,s^{-1}}$ and collides with B, which is moving towards A with a speed of $6\,\mathrm{m\,s^{-1}}$. In the collision the two particles come to rest.

(a) Find the value of v. [2]

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(b) Find the loss of kinetic energy of the system due to the collision. [2]

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4 An athlete of mass 84 kg is running along a straight road.

(a) Initially the road is horizontal and he runs at a constant speed of $3\,\text{m}\,\text{s}^{-1}$. The athlete produces a constant power of 60 W.

Find the resistive force which acts on the athlete. [1]

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(b) The athlete then runs up a 150 m section of the road which is inclined at 0.8° to the horizontal. The speed of the athlete at the start of this section of road is $3\,\text{m}\,\text{s}^{-1}$ and he now produces a constant driving force of 24 N. The total resistive force which acts on the athlete along this section of road has constant magnitude 13 N.

Use an energy method to find the speed of the athlete at the end of the 150 m section of road. [6]

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4 A lorry of mass 15 000 kg moves on a straight horizontal road in the direction from A to B. It passes A and B with speeds $20\,\mathrm{m\,s^{-1}}$ and $25\,\mathrm{m\,s^{-1}}$ respectively. The power of the lorry's engine is constant and there is a constant resistance to motion of magnitude 6000 N. The acceleration of the lorry at B is 0.5 times the acceleration of the lorry at A.

(a) Show that the power of the lorry's engine is 200 kW, and hence find the acceleration of the lorry when it is travelling at $20\,\mathrm{m\,s^{-1}}$. [5]

The lorry begins to ascend a straight hill inclined at $1°$ to the horizontal. It is given that the power of the lorry's engine and the resistance force do not change.

(b) Find the steady speed up the hill that the lorry could maintain. [2]

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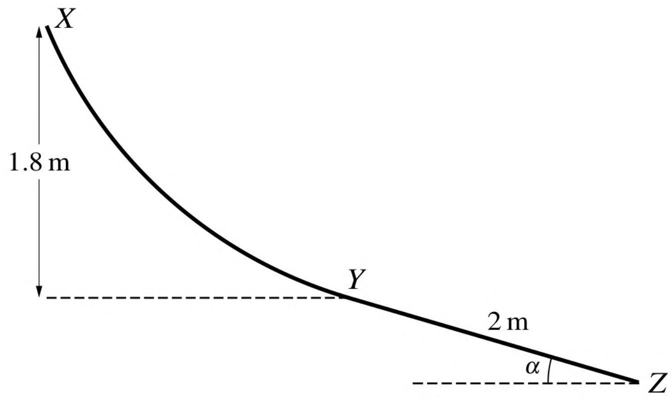

The diagram shows the vertical cross-section XYZ of a rough slide. The section YZ is a straight line of length 2 m inclined at an angle of α to the horizontal, where $\sin\alpha = 0.28$. The section YZ is tangential to the curved section XY at Y, and X is 1.8 m above the level of Y. A child of mass 25 kg slides down the slide, starting from rest at X. The work done by the child against the resistance force in moving from X to Y is 50 J.

(a) Find the speed of the child at Y. [4]

It is given that the child comes to rest at Z.

(b) Use an energy method to find the coefficient of friction between the child and YZ, giving your answer as a fraction in its simplest form. [6]

1 A particle of mass 1.6 kg is projected with a speed of $20\,\text{m}\,\text{s}^{-1}$ up a line of greatest slope of a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

Use an energy method to find the distance the particle moves up the plane before coming to instantaneous rest. [3]

4 Two particles P and Q, of masses 6 kg and 2 kg respectively, lie at rest 12.5 m apart on a rough horizontal plane. The coefficient of friction between each particle and the plane is 0.4. Particle P is projected towards Q with speed $20\,\text{m s}^{-1}$.

(a) Show that the speed of P immediately before the collision with Q is $10\sqrt{3}\,\text{m s}^{-1}$. [3]

In the collision P and Q coalesce to form particle R.

(b) Find the loss of kinetic energy due to the collision. [4]

6 A car of mass 1300 kg is moving on a straight road.

(a) On a horizontal section of the road, the car has a constant speed of $30\,\mathrm{m\,s^{-1}}$ and there is a constant force of 650 N resisting the motion.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

(ii) Given that this power is suddenly increased **by** 9 kW, find the instantaneous acceleration of the car. [3]

(b) On a section of the road inclined at $\sin^{-1} 0.08$ to the horizontal, the resistance to the motion of the car is $(1000 + 20v)$ N when the speed of the car is $v \text{ m s}^{-1}$. The car travels downwards along this section of the road at constant speed with the engine working at 11.5 kW.

Find this constant speed. [4]

1 A block of mass 15 kg slides down a line of greatest slope of an inclined plane. The top of the plane is at a vertical height of 1.6 m above the level of the bottom of the plane. The speed of the block at the top of the plane is $2\,m\,s^{-1}$ and the speed of the block at the bottom of the plane is $4\,m\,s^{-1}$.

Find the work done against the resistance to motion of the block. [4]

6 A railway engine of mass 120 000 kg is towing a coach of mass 60 000 kg up a straight track inclined at an angle of α to the horizontal where $\sin\alpha = 0.02$. There is a light rigid coupling, parallel to the track, connecting the engine and coach. The driving force produced by the engine is 125 000 N and there are constant resistances to motion of 22 000 N on the engine and 13 000 N on the coach.

At an instant when the engine is travelling at $30\,\text{m}\,\text{s}^{-1}$, it comes to a section of track inclined upwards at an angle β to the horizontal. The power produced by the engine is now 4 500 000 W and, as a result, the engine maintains a constant speed.

(b) Assuming that the resistance forces remain unchanged, find the value of β. [4]

4 A car has mass 1600 kg.

(a) The car is moving along a straight horizontal road at a constant speed of $24\,\mathrm{m\,s^{-1}}$ and is subject to a constant resistance of magnitude 480 N.

Find, in kW, the rate at which the engine of the car is working. [2]

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The car now moves down a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.09$. The engine of the car is working at a constant rate of 12 kW. The speed of the car is $24\,\mathrm{m\,s^{-1}}$ at the top of the hill. Ten seconds later the car has travelled 280 m down the hill and has speed $32\,\mathrm{m\,s^{-1}}$.

(b) Given that the resistance is not constant, use an energy method to find the total work done against the resistance during the ten seconds. [5]

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