

Chapter 5: Connected Particles

Mr Faruk

Teacher of Mathematics
BSc/MSc/PGCE Mathematics

✉ ciegcsolutions@gmail.com



6 On a straight horizontal test track, driverless vehicles (with no passengers) are being tested. A car of mass 1600 kg is towing a trailer of mass 700 kg along the track. The brakes are applied, resulting in a deceleration of 12 m s^{-2} . The braking force acts on the car only. In addition to the braking force there are constant resistance forces of 600 N on the car and of 200 N on the trailer.

(a) Find the magnitude of the force in the tow-bar. [2]

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(b) Find the braking force. [2]

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- (c) At the instant when the brakes are applied, the car has speed 22 m s^{-1} . At this instant the car is 17.5 m away from a stationary van, which is directly in front of the car.

Show that the car hits the van at a speed of 8 m s^{-1} . [2]

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- (d) After the collision, the van starts to move with speed 5 m s^{-1} and the car and trailer continue moving in the same direction with speed 2 m s^{-1} .

Find the mass of the van. [3]

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- 2 A car of mass 1800 kg is towing a trailer of mass 400 kg along a straight horizontal road. The car and trailer are connected by a light rigid tow-bar. The car is accelerating at 1.5 m s^{-2} . There are constant resistance forces of 250 N on the car and 100 N on the trailer.

(a) Find the tension in the tow-bar. [2]

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(b) Find the power of the engine of the car at the instant when the speed is 20 m s^{-1} . [3]

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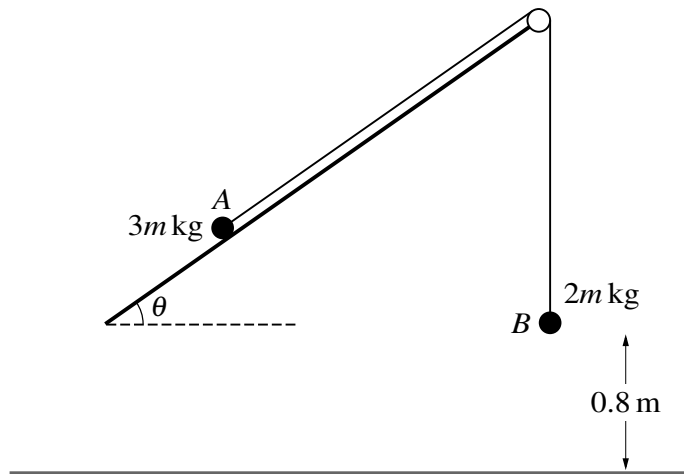
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Two particles A and B , of masses $3m$ kg and $2m$ kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the edge of a plane. The plane is inclined at an angle θ to the horizontal. A lies on the plane and B hangs vertically, 0.8 m above the floor, which is horizontal. The string between A and the pulley is parallel to a line of greatest slope of the plane (see diagram). Initially A and B are at rest.

- (a) Given that the plane is smooth, find the value of θ for which A remains at rest. [3]

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It is given instead that the plane is rough, $\theta = 30^\circ$ and the acceleration of A up the plane is 0.1 m s^{-2} .

- (b) Show that the coefficient of friction between A and the plane is $\frac{1}{10}\sqrt{3}$. [5]

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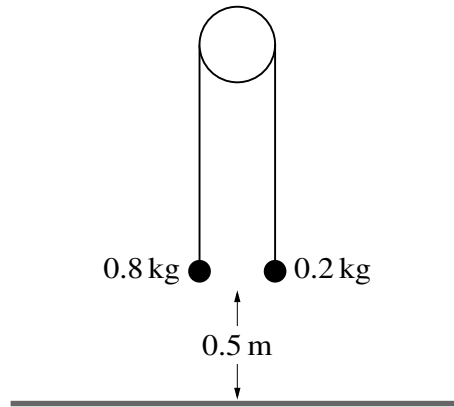
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Two particles of masses 0.8 kg and 0.2 kg are connected by a light inextensible string that passes over a fixed smooth pulley. The system is released from rest with both particles 0.5 m above a horizontal floor (see diagram). In the subsequent motion the 0.2 kg particle does not reach the pulley.

- (a) Show that the magnitude of the acceleration of the particles is 6 m s^{-2} and find the tension in the string. [4]

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(b) When the 0.8 kg particle reaches the floor it comes to rest.

Find the greatest height of the 0.2 kg particle above the floor.

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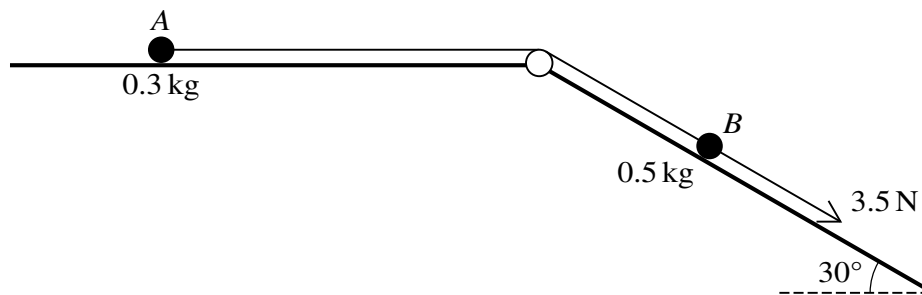
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Two particles A and B , of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane (see diagram).

(a) Given that both planes are smooth, find the tension in the string and the acceleration of B . [5]

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6 A car of mass 1600 kg is pulling a caravan of mass 800 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 400 N and 250 N respectively.

(a) The car and caravan are travelling along a straight horizontal road.

(i) Given that the car and caravan have a constant speed of 25 m s^{-1} , find the power of the car's engine. [2]

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(ii) The engine's power is now suddenly increased to 39 kW. Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

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(b) The car and caravan now travel up a straight hill, inclined at an angle of $\sin^{-1} 0.05$ to the horizontal, at a constant speed of $v \text{ m s}^{-1}$. The car's engine is working at 32.5 kW.

Find v . [3]

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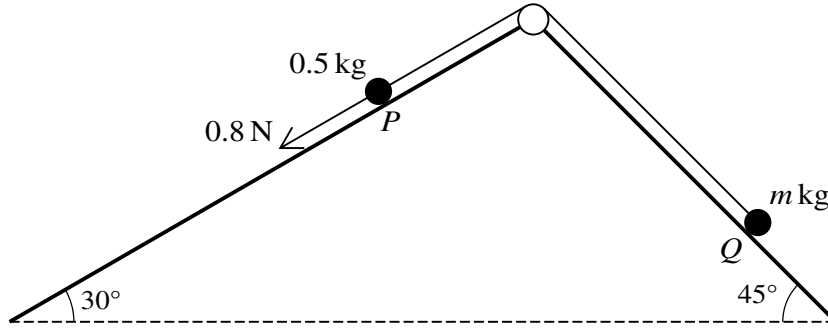
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Two particles P and Q of masses 0.5 kg and $m\text{ kg}$ respectively are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the top of two inclined planes. The particles are initially at rest with P on a smooth plane inclined at 30° to the horizontal and Q on a plane inclined at 45° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes. A force of magnitude 0.8 N is applied to P acting down the plane, causing P to move down the plane (see diagram).

(a) It is given that $m = 0.3$, and that the plane on which Q rests is smooth.

Find the tension in the string.

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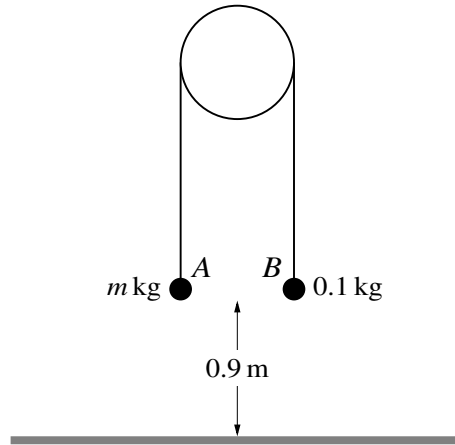
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Two particles A and B have masses m kg and 0.1 kg respectively, where $m > 0.1$. The particles are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley and the particles hang vertically below it. Both particles are at a height of 0.9 m above horizontal ground (see diagram). The system is released from rest, and while both particles are in motion the tension in the string is 1.5 N. Particle B does not reach the pulley.

(a) Find m . [4]

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(b) Find the speed at which A reaches the ground. [2]

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- 2 A van of mass 3600 kg is towing a trailer of mass 1200 kg along a straight horizontal road using a light horizontal rope. There are resistance forces of 700 N on the van and 300 N on the trailer.

- (a) The driving force exerted by the van is 2500 N.

Find the tension in the rope.

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The driving force is now removed and the van driver applies a braking force which acts only on the van. The resistance forces remain unchanged.

- (b) Find the least possible value of the braking force which will cause the rope to become slack. [2]

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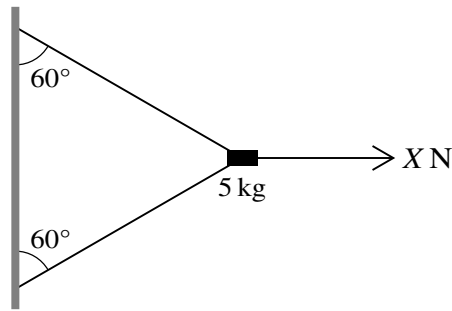
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A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

(a) Given that $X = 100$, find the tension in the lower string. [4]

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2 A particle of mass 8 kg is suspended in equilibrium by two light inextensible strings which make angles of 60° and 45° above the horizontal.

(a) Draw a diagram showing the forces acting on the particle. [1]

(b) Find the tensions in the strings. [6]

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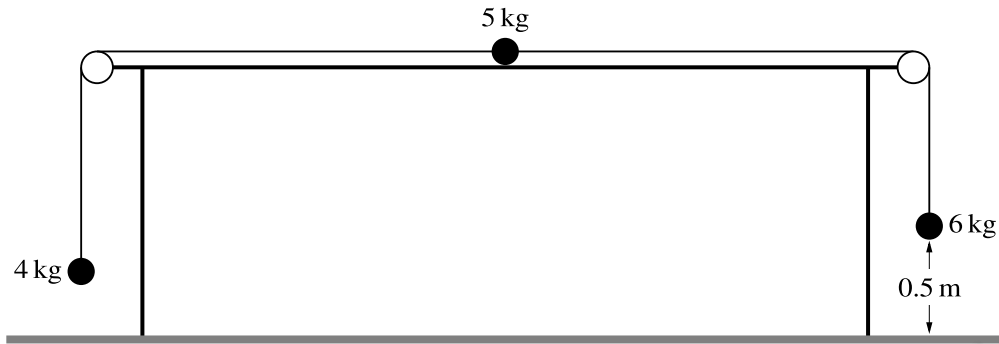
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The diagram shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.

- (a) Show that the coefficient of friction between the 5 kg particle and the table is 0.4. [2]

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The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest.

- (b) Find the acceleration of the 4 kg particle and the tensions in the strings. [5]

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(c) In the subsequent motion the 8 kg particle hits the ground and does not rebound.

Find the time that elapses after the 8 kg particle hits the ground before the other two particles come to instantaneous rest. (You may assume this occurs before either particle reaches a pulley.) [5]

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(b) Find the greatest height of B above the plane. [3]

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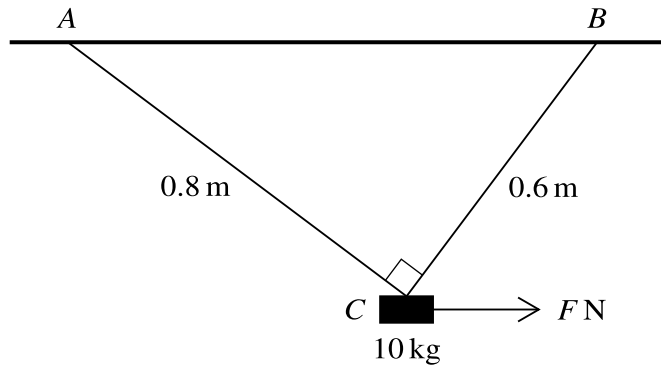
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The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC , of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90^\circ$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. [5]

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- (b) It is given instead that the plane BC is rough. A force of magnitude 3 N is applied to Q directly up the plane along a line of greatest slope of the plane.

Find the least value of the coefficient of friction between Q and the plane BC for which the particles remain at rest. [5]

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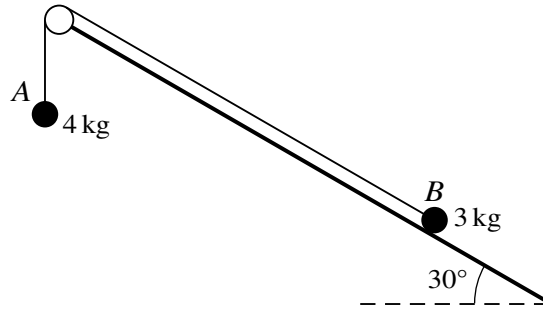


Fig. 6.1

Fig. 6.1 shows particles A and B , of masses 4 kg and 3 kg respectively, attached to the ends of a light inextensible string that passes over a small smooth pulley. The pulley is fixed at the top of a plane which is inclined at an angle of 30° to the horizontal. A hangs freely below the pulley and B is on the inclined plane. The string is taut and the section of the string between B and the pulley is parallel to a line of greatest slope of the plane.

(a) It is given that the plane is rough and the particles are in limiting equilibrium.

Find the coefficient of friction between B and the plane. [6]

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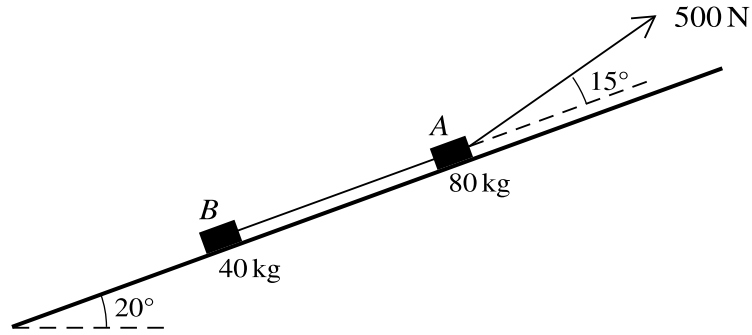
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A block A of mass 80 kg is connected by a light, inextensible rope to a block B of mass 40 kg. The rope joining the two blocks is taut and is parallel to a line of greatest slope of a plane which is inclined at an angle of 20° to the horizontal. A force of magnitude 500 N inclined at an angle of 15° above the same line of greatest slope acts on A (see diagram). The blocks move up the plane and there is a resistance force of 50 N on B, but no resistance force on A.

- (a) Find the acceleration of the blocks and the tension in the rope. [5]

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(b) Find the time that it takes for the blocks to reach a speed of 1.2 m s^{-1} from rest. [2]

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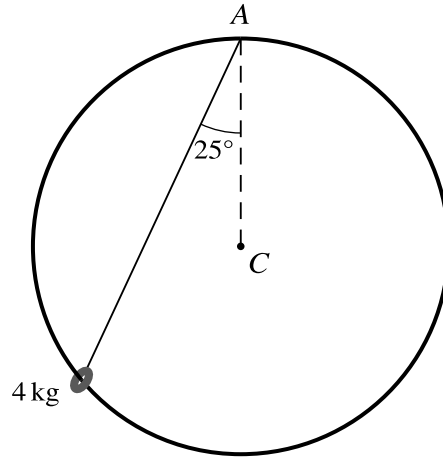
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A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre C . The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A , the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

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1 A crate of mass 200 kg is being pulled at constant speed along horizontal ground by a horizontal rope attached to a winch. The winch is working at a constant rate of 4.5 kW and there is a constant resistance to the motion of the crate of magnitude 600 N.

(a) Find the time that it takes for the crate to move a distance of 15 m. [2]

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The rope breaks after the crate has moved 15 m.

(b) Find the time taken, after the rope breaks, for the crate to come to rest. [3]

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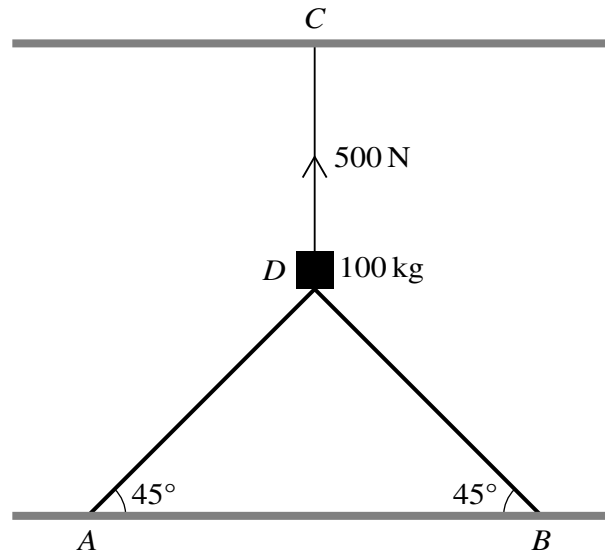
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The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD , each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

- (a) Find the magnitude of the force in each of the struts AD and BD . [3]

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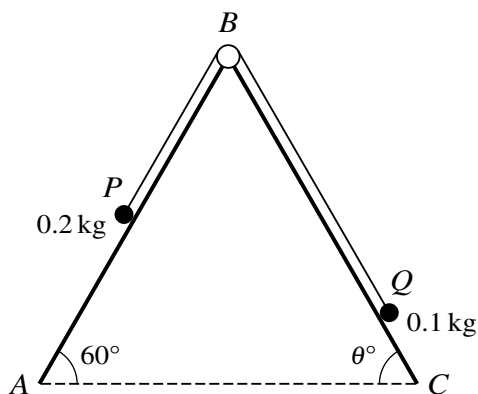
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Two particles P and Q , of masses 0.2 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. Particle P lies on a smooth plane AB which is inclined at 60° to the horizontal. Particle Q lies on a plane BC which is inclined at an angle of θ° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

- (a) It is given that $\theta = 60$, the plane BC is rough and the coefficient of friction between Q and the plane BC is 0.7. The particles are released from rest.

Determine whether the particles move. [4]

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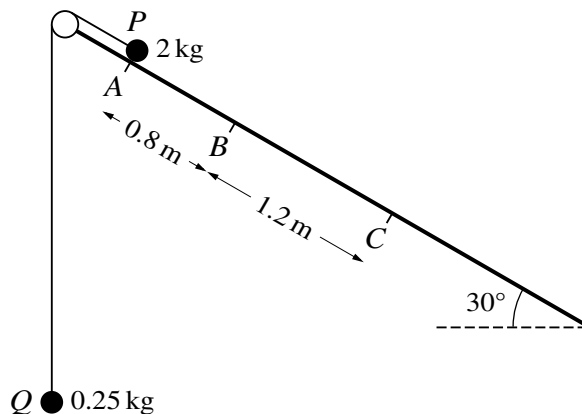
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Two particles P and Q , of masses 2 kg and 0.25 kg respectively, are connected by a light inextensible string that passes over a fixed smooth pulley. Particle P is on an inclined plane at an angle of 30° to the horizontal. Particle Q hangs below the pulley. Three points A , B and C lie on a line of greatest slope of the plane with $AB = 0.8\text{ m}$ and $BC = 1.2\text{ m}$ (see diagram).

Particle P is released from rest at A with the string taut and slides down the plane. During the motion of P from A to C , Q does not reach the pulley. The part of the plane from A to B is rough, with coefficient of friction 0.3 between the plane and P . The part of the plane from B to C is smooth.

- (a) (i) Find the acceleration of P between A and B . [4]

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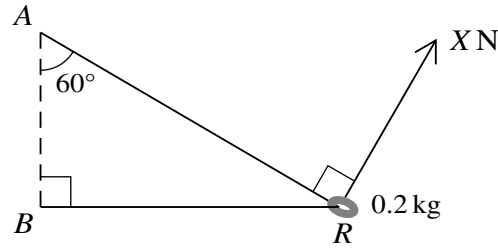
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A smooth ring R of mass 0.2 kg is threaded on a light string ARB . The ends of the string are attached to fixed points A and B with A vertically above B . The string is taut and angle $ABR = 90^\circ$. The angle between the part AR of the string and the vertical is 60° . The ring is held in equilibrium by a force of magnitude $X\text{ N}$, acting on the ring in a direction perpendicular to AR (see diagram).

Calculate the tension in the string and the value of X . [5]

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- 6** An elevator is pulled vertically upwards by a cable. The elevator accelerates at 0.4 m s^{-2} for 5 s, then travels at constant speed for 25 s. The elevator then decelerates at 0.2 m s^{-2} until it comes to rest.

The mass of the elevator is 1200 kg and there is a crate of mass m kg resting on the floor of the elevator.

- (c) Given that the tension in the cable when the elevator is decelerating is 12 250 N, find the value of m . [3]

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- (d) Find the greatest magnitude of the force exerted on the crate by the floor of the elevator, and state its direction. [3]

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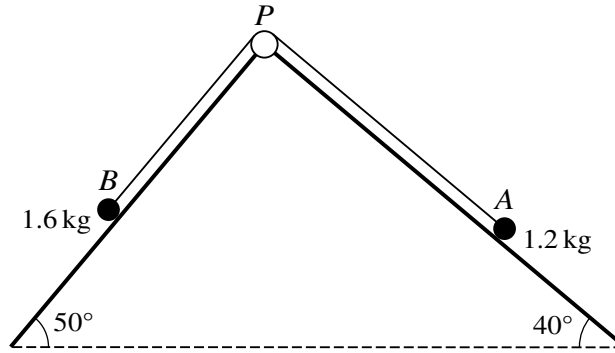
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The diagram shows a particle A , of mass 1.2 kg , which lies on a plane inclined at an angle of 40° to the horizontal and a particle B , of mass 1.6 kg , which lies on a plane inclined at an angle of 50° to the horizontal. The particles are connected by a light inextensible string which passes over a small smooth pulley P fixed at the top of the planes. The parts AP and BP of the string are taut and parallel to lines of greatest slope of the respective planes. The two planes are rough, with the same coefficient of friction, μ , between the particles and the planes.

Find the value of μ for which the system is in limiting equilibrium.

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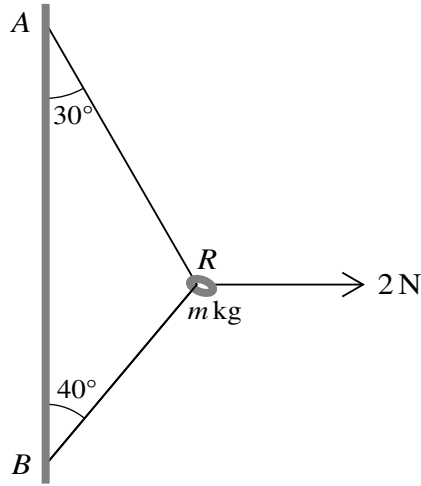
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The diagram shows a smooth ring R , of mass m kg, threaded on a light inextensible string. A horizontal force of magnitude 2 N acts on R . The ends of the string are attached to fixed points A and B on a vertical wall. The part AR of the string makes an angle of 30° with the vertical, the part BR makes an angle of 40° with the vertical and the string is taut. The ring is in equilibrium.

Find the tension in the string and find the value of m . [5]

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At an instant when the engine is travelling at 30 m s^{-1} , it comes to a section of track inclined upwards at an angle β to the horizontal. The power produced by the engine is now $4\,500\,000 \text{ W}$ and, as a result, the engine maintains a constant speed.

- (b) Assuming that the resistance forces remain unchanged, find the value of β . [4]

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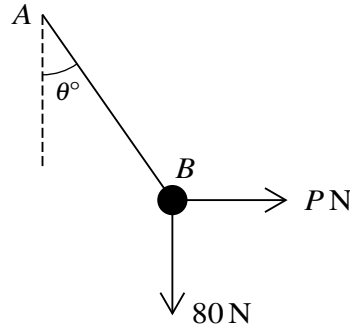
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A light string AB is fixed at A and has a particle of weight 80 N attached at B . A horizontal force of magnitude P N is applied at B such that the string makes an angle θ° to the vertical (see diagram).

(a) It is given that $P = 32$ and the system is in equilibrium.

Find the tension in the string and the value of θ .

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(b) It is given instead that the tension in the string is 120N and that the particle attached at B still has weight 80N.

Find the value of P and the value of θ .

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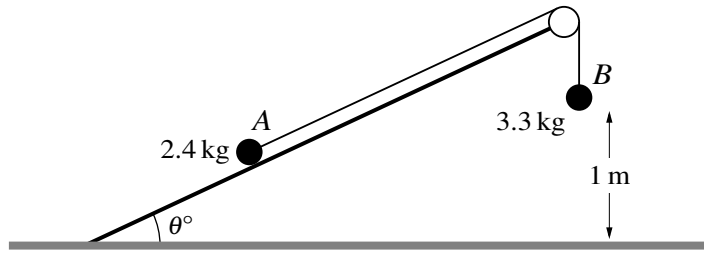
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Particles A and B , of masses 2.4 kg and 3.3 kg respectively, are connected by a light inextensible string that passes over a smooth pulley which is fixed to the top of a rough plane. The plane makes an angle of θ° with horizontal ground. Particle A is on the plane and the section of the string between A and the pulley is parallel to a line of greatest slope of the plane. Particle B hangs vertically below the pulley and is 1 m above the ground (see diagram). The coefficient of friction between the plane and A is μ .

- (a) It is given that $\theta = 30$ and the system is in equilibrium with A on the point of moving directly up the plane.

Show that $\mu = 1.01$ correct to 3 significant figures.

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