# Chapter 4: Friction

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**2** A particle $P$ of mass 0.4 kg is on a rough horizontal floor. The coefficient of friction between $P$ and the floor is $\mu$. A force of magnitude 3 N is applied to $P$ upwards at an angle $\alpha$ above the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is initially at rest and accelerates at $2\,\text{m s}^{-2}$.

**(a)** Find the time it takes for $P$ to travel a distance of 1.44 m from its starting point. [2]

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**(b)** Find $\mu$. [4]

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**(b)** Given instead that the four forces are in equilibrium, find the values of $F$ and $\alpha$. [5]

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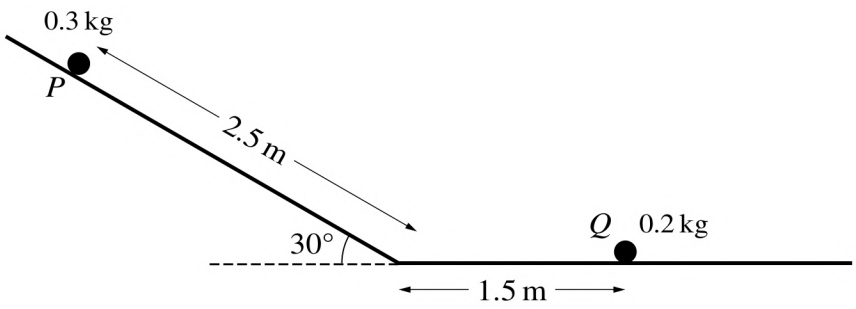


A particle $P$ of mass 0.3 kg, lying on a smooth plane inclined at 30° to the horizontal, is released from rest. $P$ slides down the plane for a distance of 2.5 m and then reaches a horizontal plane. There is no change in speed when $P$ reaches the horizontal plane. A particle $Q$ of mass 0.2 kg lies at rest on the horizontal plane 1.5 m from the end of the inclined plane (see diagram). $P$ collides directly with $Q$.

**(a)** It is given that the horizontal plane is smooth and that, after the collision, $P$ continues moving in the same direction, with speed 2 m s$^{-1}$.

Find the speed of $Q$ after the collision. [5]

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**(b)** It is given instead that the horizontal plane is rough and that when $P$ and $Q$ collide, they coalesce and move with speed $1.2\,\text{m}\,\text{s}^{-1}$.

Find the coefficient of friction between $P$ and the horizontal plane. [5]

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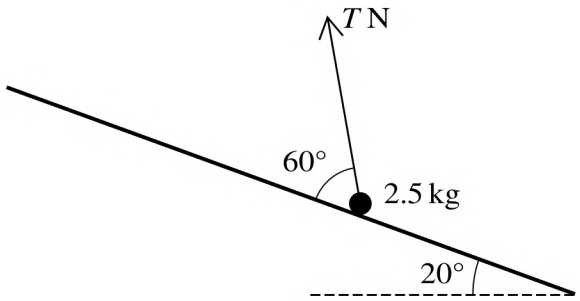


A particle of mass 2.5 kg is held in equilibrium on a rough plane inclined at 20° to the horizontal by a force of magnitude $T$ N making an angle of 60° with a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.3.

Find the greatest and least possible values of $T$. [8]

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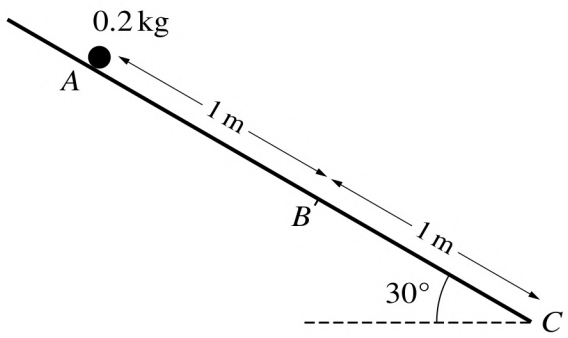


Three points $A$, $B$ and $C$ lie on a line of greatest slope of a plane inclined at an angle of 30° to the horizontal, with $AB = 1$ m and $BC = 1$ m, as shown in the diagram. A particle of mass 0.2 kg is released from rest at $A$ and slides down the plane. The part of the plane from $A$ to $B$ is smooth. The part of the plane from $B$ to $C$ is rough, with coefficient of friction $\mu$ between the plane and the particle.

**(a)** Given that $\mu = \frac{1}{2}\sqrt{3}$, find the speed of the particle at $C$. [8]

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**(b)** Given instead that the particle comes to rest at $C$, find the exact value of $\mu$. [4]

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**6** A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is $\mu$.

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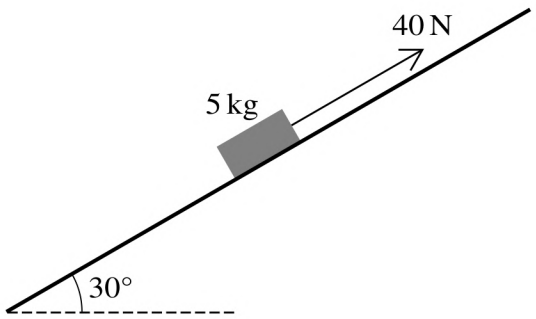


**Fig. 6.1**

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$. [4]

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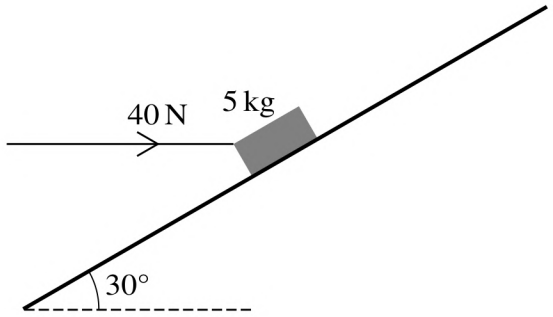


**Fig. 6.2**

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of $\mu$ is 0.152. [4]

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**3** A string is attached to a block of mass 4 kg which rests in limiting equilibrium on a rough horizontal table. The string makes an angle of 24° above the horizontal and the tension in the string is 30 N.

**(a)** Draw a diagram showing all the forces acting on the block. [1]

**(b)** Find the coefficient of friction between the block and the table. [5]

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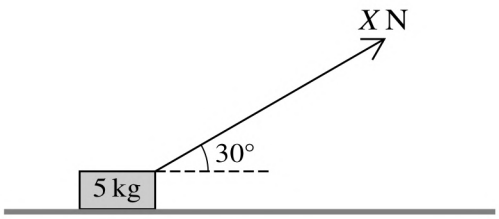


A block of mass 5 kg is being pulled along a rough horizontal floor by a force of magnitude $X$ N acting at 30° above the horizontal (see diagram). The block starts from rest and travels 2 m in the first 5 s of its motion.

**(a)** Find the acceleration of the block. [2]

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**(b)** Given that the coefficient of friction between the block and the floor is 0.4, find $X$. [4]

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The block is now placed on a part of the floor where the coefficient of friction between the block and the floor has a different value. The value of $X$ is changed to 25, and the block is now in limiting equilibrium.

**(c)** Find the value of the coefficient of friction between the block and this part of the floor. [3]

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**4** A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A pulling force of magnitude $P$ N acts at an angle of 8° above a line of greatest slope of the plane. This force is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.3.

Find the greatest possible value of $P$. [6]

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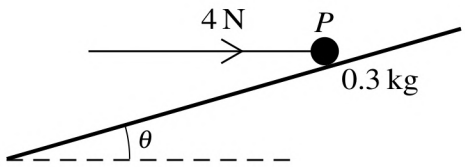


A particle $P$ of mass 0.3 kg rests on a rough plane inclined at an angle $\theta$ to the horizontal, where $\sin\theta = \frac{7}{25}$. A horizontal force of magnitude 4 N, acting in the vertical plane containing a line of greatest slope of the plane, is applied to $P$ (see diagram). The particle is on the point of sliding up the plane.

**(a)** Show that the coefficient of friction between the particle and the plane is $\frac{3}{4}$. [4]

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The force acting horizontally is replaced by a force of magnitude 4 N acting up the plane parallel to a line of greatest slope.

**(b)** Find the acceleration of $P$. [3]

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**(c)** Starting with $P$ at rest, the force of 4 N parallel to the plane acts for 3 seconds and is then removed.

Find the total distance travelled until $P$ comes to instantaneous rest. [3]

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**7** A bead, $A$, of mass 0.1 kg is threaded on a long straight rigid wire which is inclined at $\sin^{-1}\left(\frac{7}{25}\right)$ to the horizontal. $A$ is released from rest and moves down the wire. The coefficient of friction between $A$ and the wire is $\mu$. When $A$ has travelled 0.45 m down the wire, its speed is $0.6\,\text{m}\,\text{s}^{-1}$.

**(a)** Show that $\mu = 0.25$. [6]

Another bead, $B$, of mass 0.5 kg is also threaded on the wire. At the point where $A$ has travelled 0.45 m down the wire, it hits $B$ which is instantaneously at rest on the wire. $A$ is brought to instantaneous rest in the collision. The coefficient of friction between $B$ and the wire is 0.275.

**(b)** Find the time from when the collision occurs until $A$ collides with $B$ again. [6]

**3** A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude $X$ N, acting at an angle $\alpha$ above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of $X$ for which the crate remains at rest. [5]

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**(b)** Find the greatest height of $B$ above the plane. [3]

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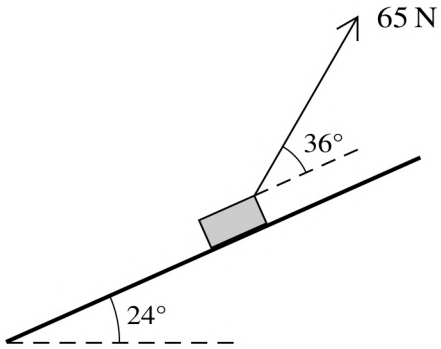


A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is $\mu$. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find $\mu$. [6]

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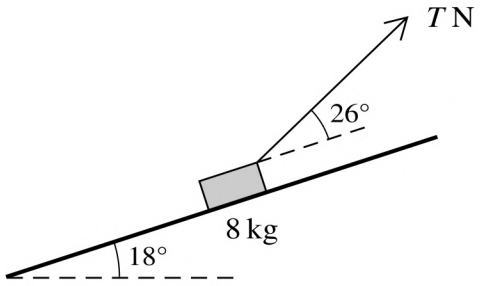


A block of mass 8 kg is placed on a rough plane which is inclined at an angle of 18° to the horizontal. The block is pulled up the plane by a light string that makes an angle of 26° above a line of greatest slope. The tension in the string is $T$ N (see diagram). The coefficient of friction between the block and plane is 0.65.

**(a)** The acceleration of the block is $0.2\,\text{m}\,\text{s}^{-2}$.

Find $T$. [7]

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**(b)** The block is initially at rest.

Find the distance travelled by the block during the fourth second of motion. [2]

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**2** A particle $P$ of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

**(a)** Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$. [3]

A force of magnitude 7.2 N is now applied to $P$ directly up a line of greatest slope of the plane.

**(b)** Given that $P$ starts from rest, find the time that it takes for $P$ to move 1 m up the plane. [4]

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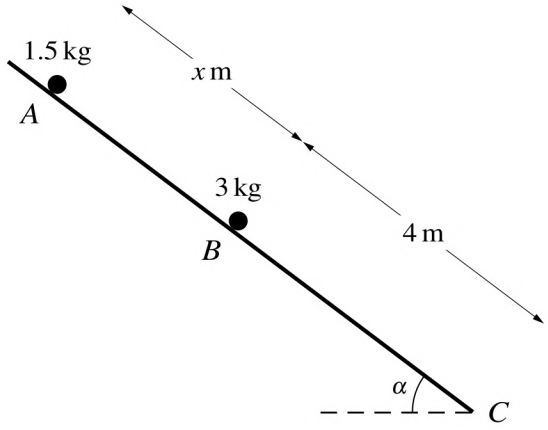


Particles of masses 1.5 kg and 3 kg lie on a plane which is inclined at an angle of $\alpha$ to the horizontal, where $\tan\alpha = \frac{3}{4}$. The section of the plane from $A$ to $B$ is smooth and the section of the plane from $B$ to $C$ is rough. The 1.5 kg particle is held at rest at $A$ and the 3 kg particle is in limiting equilibrium at $B$. The distance $AB$ is $x$ m and the distance $BC$ is 4 m (see diagram).

**(a)** Show that the coefficient of friction between the particle at $B$ and the plane is 0.75. [3]

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The 1.5 kg particle is released from rest. In the subsequent motion the two particles collide and coalesce. The time taken for the combined particle to travel from $B$ to $C$ is 2 s. The coefficient of friction between the combined particle and the plane is still 0.75.

**(b)** Find $x$. [6]

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**(c)** Find the total loss of energy of the particles from the time the 1.5 kg particle is released until the combined particle reaches $C$. [3]

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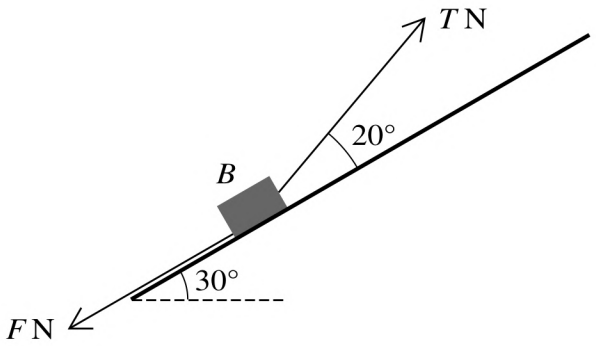


A block $B$, of mass 2 kg, lies on a rough inclined plane sloping at 30° to the horizontal. A light rope, inclined at an angle of 20° above a line of greatest slope, is attached to $B$. The tension in the rope is $T$ N. There is a friction force of $F$ N acting on $B$ (see diagram). The coefficient of friction between $B$ and the plane is $\mu$.

**(a)** It is given that $F = 5$ and that the acceleration of $B$ up the plane is $1.2\,\text{m s}^{-2}$.

**(i)** Find the value of $T$. [3]

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**(ii)** Find the value of $\mu$. [3]

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**(b)** It is given instead that $\mu = 0.8$ and $T = 15$.

Determine whether $B$ will move up the plane. [3]

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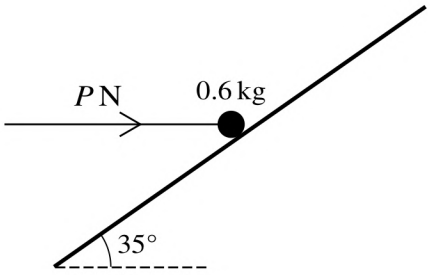


A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude $P$ N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of $P$. [6]

**4** A particle $P$ of mass 0.2 kg lies at rest on a rough horizontal plane. A horizontal force of 1.2 N is applied to $P$.

**(a)** Given that $P$ is in limiting equilibrium, find the coefficient of friction between $P$ and the plane. [3]

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**(b)** Given instead that the coefficient of friction between $P$ and the plane is 0.3, find the distance travelled by $P$ in the third second of its motion. [4]

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**3** A block of mass 8 kg slides down a rough plane inclined at 30° to the horizontal, starting from rest. The coefficient of friction between the block and the plane is $\mu$. The block accelerates uniformly down the plane at $2.4\,\text{m}\,\text{s}^{-2}$.

**(a)** Draw a diagram showing the forces acting on the block. [1]

**(b)** Find the value of $\mu$. [4]

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**(c)** Find the speed of the block after it has moved 3 m down the plane. [1]

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