Chapter 1: Velocity and Acceleration

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4 A cyclist travels along a straight road with constant acceleration. He passes through points A, B and C. The cyclist takes 2 seconds to travel along each of the sections AB and BC and passes through B with speed $4.5\,\text{m s}^{-1}$. The distance AB is $\frac{4}{5}$ of the distance BC.

(a) Find the acceleration of the cyclist. [5]

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(b) Find AC. [2]

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3 A particle P is projected vertically upwards with speed $5\,\text{m}\,\text{s}^{-1}$ from a point A which is 2.8 m above horizontal ground.

(a) Find the greatest height above the ground reached by P. [3]

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(b) Find the length of time for which P is at a height of more than 3.6 m above the ground. [4]

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1 A tram starts from rest and moves with uniform acceleration for 20 s. The tram then travels at a constant speed, V m s^{-1}, for 170 s before being brought to rest with a uniform deceleration of magnitude twice that of the acceleration. The total distance travelled by the tram is 2.775 km.

(a) Sketch a velocity-time graph for the motion, stating the total time for which the tram is moving. [2]

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(b) Find V. [2]

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(c) Find the magnitude of the acceleration. [2]

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4 A car starts from rest and moves in a straight line with constant acceleration $a\,\text{m s}^{-2}$ for a distance of 50 m. The car then travels with constant velocity for 500 m for a period of 25 s, before decelerating to rest. The magnitude of this deceleration is $2a\,\text{m s}^{-2}$.

(a) Sketch the velocity-time graph for the motion of the car. [1]

(b) Find the value of a. [3]

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(c) Find the total time for which the car is in motion. [3]

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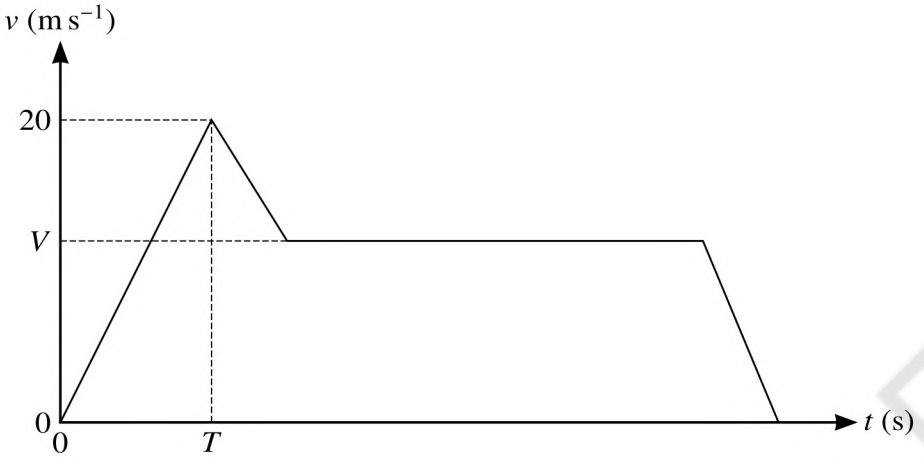

The diagram shows a velocity-time graph which models the motion of a car. The graph consists of four straight line segments. The car accelerates at a constant rate of $2\,m\,s^{-2}$ from rest to a speed of $20\,m\,s^{-1}$ over a period of T s. It then decelerates at a constant rate for 5 seconds before travelling at a constant speed of $V\,m\,s^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T. [1]

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(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V. [4]

5 A particle is projected vertically upwards with speed $40\,\text{m s}^{-1}$ alongside a building of height h m.

(a) Given that the particle is above the level of the top of the building for 4 s, find h. [4]

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed $20\,\text{m}\,\text{s}^{-1}$.

Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [4]

1 A particle P is projected vertically upwards with speed v m s^{-1} from a point on the ground. P reaches its greatest height after 3 s.

(a) Find v. [1]

(b) Find the greatest height of P above the ground. [2]

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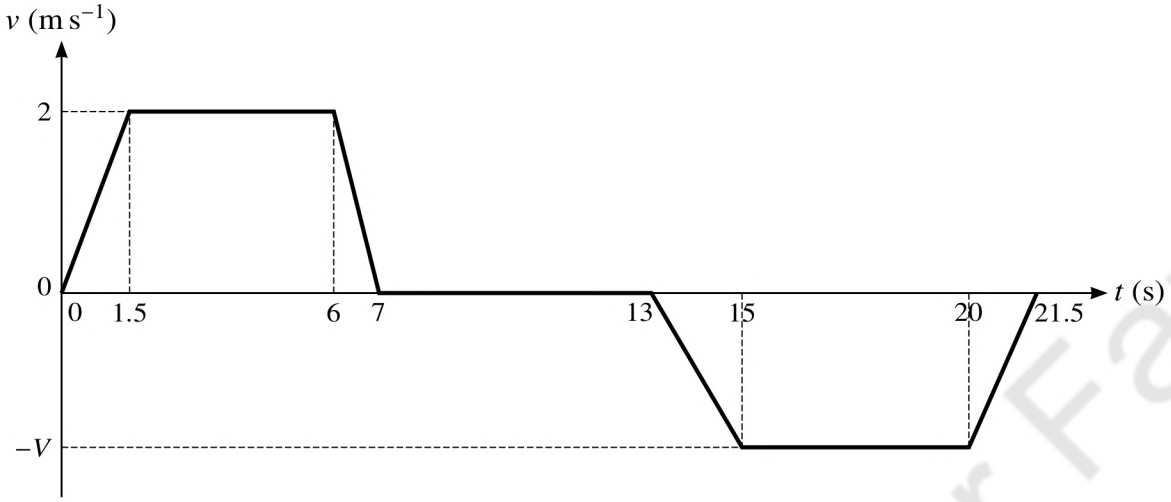

An elevator moves vertically, supported by a cable. The diagram shows a velocity-time graph which models the motion of the elevator. The graph consists of 7 straight line segments.

The elevator accelerates upwards from rest to a speed of 2 m s^{-1} over a period of 1.5 s and then travels at this speed for 4.5 s, before decelerating to rest over a period of 1 s.

The elevator then remains at rest for 6 s, before accelerating to a speed of V m s^{-1} downwards over a period of 2 s. The elevator travels at this speed for a period of 5 s, before decelerating to rest over a period of 1.5 s.

(a) Find the acceleration of the elevator during the first 1.5 s. [1]

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(b) Given that the elevator starts and finishes its journey on the ground floor, find V. [2]

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(c) The combined weight of the elevator and passengers on its upward journey is 1500 kg. Assuming that there is no resistance to motion, find the tension in the elevator cable on its upward journey when the elevator is decelerating. [3]

4 Two cyclists, Isabella and Maria, are having a race. They both travel along a straight road with constant acceleration, starting from rest at point A.

Isabella accelerates for 5 s at a constant rate $a\,\text{m s}^{-2}$. She then travels at the constant speed she has reached for 10 s, before decelerating to rest at a constant rate over a period of 5 s.

Maria accelerates at a constant rate, reaching a speed of $5\,\text{m s}^{-1}$ in a distance of 27.5 m. She then maintains this speed for a period of 10 s, before decelerating to rest at a constant rate over a period of 5 s.

(a) Given that $a = 1.1$, find which cyclist travels further. [5]

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(b) Find the value of a for which the two cyclists travel the same distance. [2]

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6 A particle A is projected vertically upwards from level ground with an initial speed of $30\,\text{m}\,\text{s}^{-1}$. At the same instant a particle B is released from rest 15 m vertically above A. The mass of one of the particles is twice the mass of the other particle. During the subsequent motion A and B collide and coalesce to form particle C.

Find the difference between the two possible times at which C hits the ground. [8]

4 A particle is projected vertically upwards with speed u m s^{-1} from a point on horizontal ground. After 2 seconds, the height of the particle above the ground is 24 m.

(a) Show that $u = 22$. [2]

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(b) The height of the particle above the ground is more than h m for a period of 3.6 s.

Find h. [4]

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1 A bus moves from rest with constant acceleration for 12 s. It then moves with constant speed for 30 s before decelerating uniformly to rest in a further 6 s. The total distance travelled is 585 m.

(a) Find the constant speed of the bus. [2]

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(b) Find the magnitude of the deceleration. [1]

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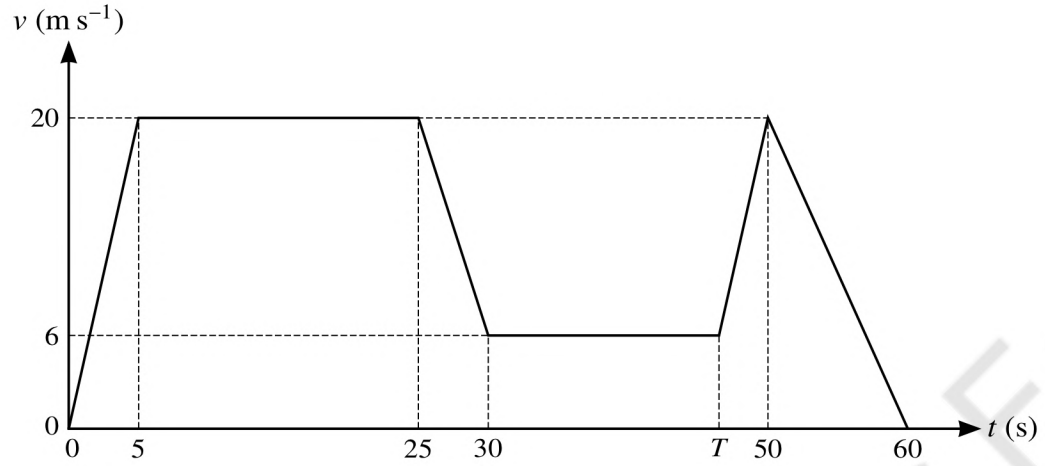

The diagram shows a velocity-time graph which models the motion of a car. The graph consists of six straight line segments. The car accelerates from rest to a speed of $20\,\text{m s}^{-1}$ over a period of 5 s, and then travels at this speed for a further 20 s. The car then decelerates to a speed of $6\,\text{m s}^{-1}$ over a period of 5 s. This speed is maintained for a further $(T - 30)$ s. The car then accelerates again to a speed of $20\,\text{m s}^{-1}$ over a period of $(50 - T)$ s, before decelerating to rest over a period of 10 s.

(a) Given that during the two stages of the motion when the car is accelerating, the accelerations are equal, find the value of T. [2]

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(b) Find the total distance travelled by the car during the motion. [2]

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3 A ball of mass 1.6 kg is released from rest at a point 5 m above horizontal ground. When the ball hits the ground it instantaneously loses 8 J of kinetic energy and starts to move upwards.

(a) Use an energy method to find the greatest height that the ball reaches after hitting the ground. [3]

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(b) Find the total time taken, from the initial release of the ball until it reaches this greatest height. [3]

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2 A particle P is projected vertically upwards from horizontal ground with speed $u\,\text{m s}^{-1}$. P reaches a maximum height of 20 m above the ground.

(a) Find the value of u. [2]

(b) Find the total time for which P is at least 15 m above the ground. [3]

1 A car starts from rest and moves in a straight line with constant acceleration for a distance of 200 m, reaching a speed of 25 m s^{-1}. The car then travels at this speed for 400 m, before decelerating uniformly to rest over a period of 5 s.

(a) Find the time for which the car is accelerating. [2]

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(b) Sketch the velocity–time graph for the motion of the car, showing the key points. [2]

(c) Find the average speed of the car during its motion. [2]

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4 A particle A, moving along a straight horizontal track with constant speed $8\,\text{m s}^{-1}$, passes a fixed point O. Four seconds later, another particle B passes O, moving along a parallel track in the same direction as A. Particle B has speed $20\,\text{m s}^{-1}$ when it passes O and has a constant deceleration of $2\,\text{m s}^{-2}$. B comes to rest when it returns to O.

(a) Find expressions, in terms of t, for the displacement from O of each particle t seconds after B passes O. [3]

(b) Find the values of t when the particles are the same distance from O. [3]

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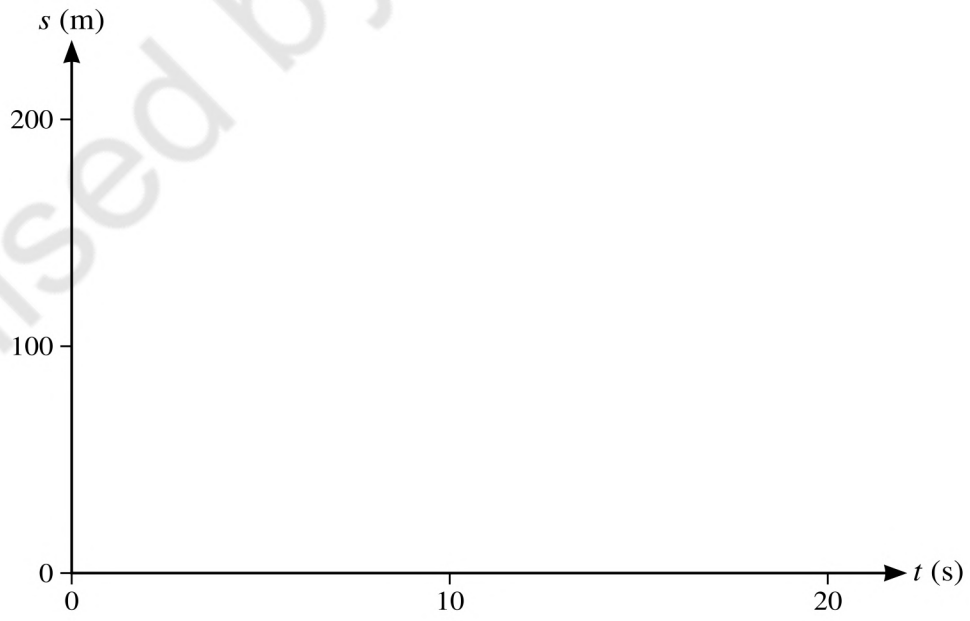
(c) On the given axes, sketch the displacement-time graphs for both particles, for values of t from 0 to 20. [3]



2 A particle P is projected vertically upwards from horizontal ground. P reaches a maximum height of 45 m. After reaching the ground, P comes to rest without rebounding.

(a) Find the speed at which P was projected. [2]

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(b) Find the total time for which the speed of P is at least $10\,\text{m}\,\text{s}^{-1}$. [3]

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1 A particle P is projected vertically upwards with speed $u\,\text{m s}^{-1}$ from a point on the ground. P reaches its greatest height after 3 s.

(a) Find u. [1]

(b) Find the greatest height of P above the ground. [2]

2 A particle P is projected vertically upwards from horizontal ground with speed $15\,\text{m}\,\text{s}^{-1}$.

(a) Find the speed of P when it is 10 m above the ground. [2]

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At the same instant that P is projected, a second particle Q is dropped from a height of 18 m above the ground in the same vertical line as P.

(b) Find the height above the ground at which the two particles collide. [3]

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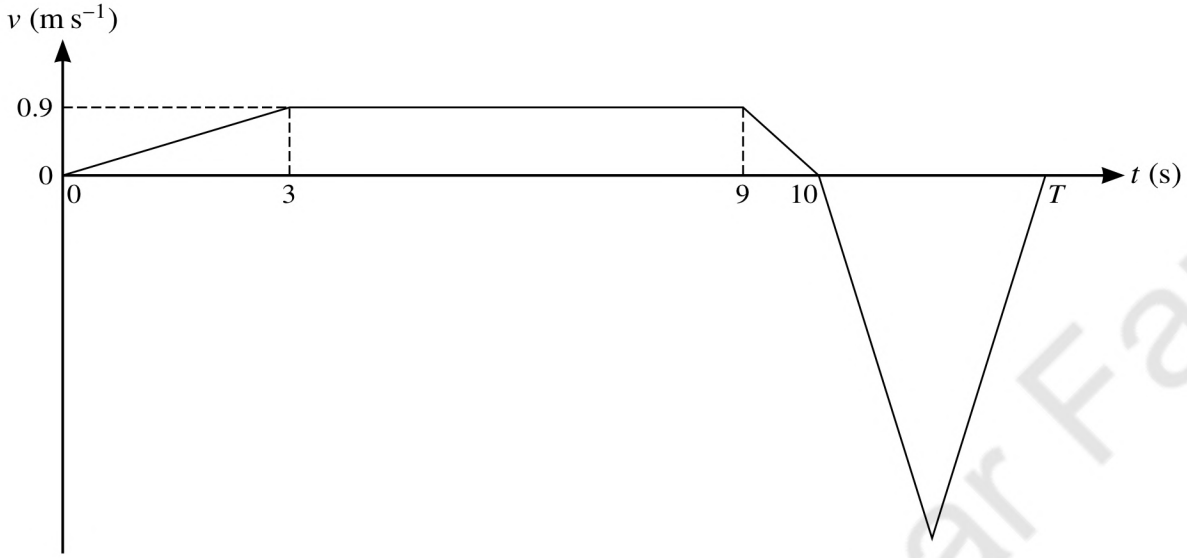

The velocity of a particle at time t s after leaving a fixed point O is v m s^{-1}. The diagram shows a velocity-time graph which models the motion of the particle. The graph consists of 5 straight line segments. The particle accelerates to a speed of 0.9 m s^{-1} in a period of 3 s, then travels at constant speed for 6 s, and then comes instantaneously to rest 1 s later. The particle then moves back and returns to rest at O at time T s.

(a) Find the distance travelled by the particle in the first 10 s of its motion. [2]

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(b) Given that $T = 12$, find the minimum velocity of the particle. [2]

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(c) Given instead that the greatest speed of the particle is $3\,\text{m}\,\text{s}^{-1}$, find the value of T and hence find the average speed of the particle for the whole of the motion. [4]

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6 An elevator is pulled vertically upwards by a cable. The elevator accelerates at $0.4\,\text{m}\,\text{s}^{-2}$ for 5 s, then travels at constant speed for 25 s. The elevator then decelerates at $0.2\,\text{m}\,\text{s}^{-2}$ until it comes to rest.

(a) Find the greatest speed of the elevator and hence draw a velocity-time graph for the motion of the elevator. [3]

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(b) Find the total distance travelled by the elevator. [2]

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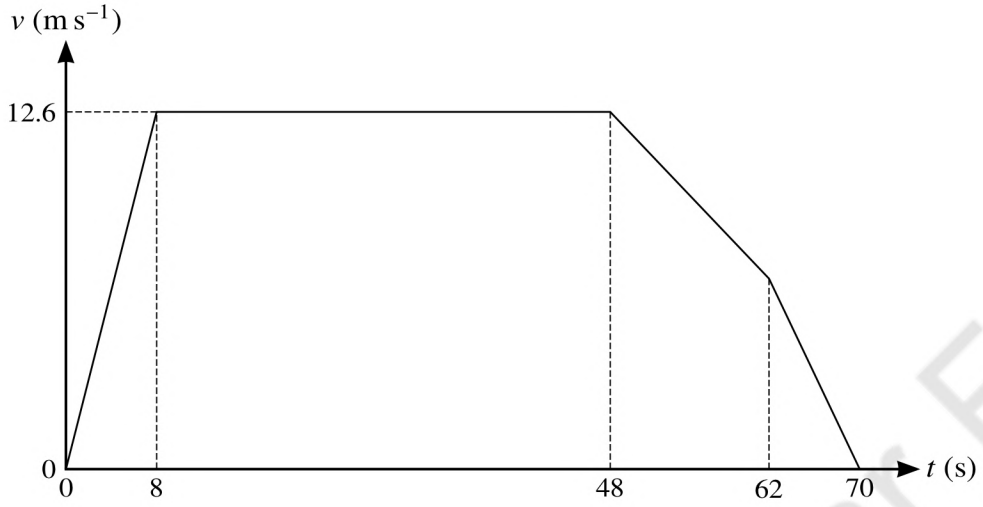

The diagram shows the velocity-time graph for the motion of a bus. The bus starts from rest and accelerates uniformly for 8 seconds until it reaches a speed of $12.6\,\text{m s}^{-1}$. The bus maintains this speed for 40 seconds. It then decelerates uniformly in two stages. Between 48 and 62 seconds the bus decelerates at $a\,\text{m s}^{-2}$ and between 62 and 70 seconds it decelerates at $2a\,\text{m s}^{-2}$ until coming to rest.

(a) Find the distance covered by the bus in the first 8 seconds. [1]

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(b) Find the value of a. [3]

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(c) Find the average speed of the bus for the whole journey. [4]

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5 A particle A of mass 0.5 kg is projected vertically upwards from horizontal ground with speed 25 m s^{-1}.

(a) Find the speed of A when it reaches a height of 20 m above the ground. [2]

When A reaches a height of 20 m, it collides with a particle B of mass 0.3 kg which is moving downwards in the same vertical line as A with speed 32.5 m s^{-1}. In the collision between the two particles, B is brought to instantaneous rest.

(b) Show that the velocity of A immediately after the collision is 4.5 m s^{-1} downwards. [2]

(c) Find the time interval between A and B reaching the ground. You should assume that A does not bounce when it reaches the ground. [4]

1 A particle is projected vertically upwards from horizontal ground with a speed of $u\,\text{m s}^{-1}$. The particle has height $s\,\text{m}$ above the ground at times 3 seconds and 4 seconds after projection.

Find the value of u and the value of s. [3]