2014-2018 Chapter 6: Random Variables

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4. A discrete random variable X has the probability distribution given in the table below, where a and b are constants.

x	–1	0	1	2	3
$P(X = x)$	a	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	b

Given $E(X) = \frac{9}{5}$

(a) (i) find two simultaneous equations for a and b,

(ii) show that $a = \frac{1}{20}$ and find the value of b.

(4)

(b) Specify the cumulative distribution function $F(x)$ for $x = –1, 0, 1, 2$ and 3

(2)

(c) Find $P(X < 2.5)$.

(1)

(d) Find $\text{Var}(3 – 2X)$.

(4)

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Question 4 continued

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1. The discrete random variable X has probability function $p(x)$ and cumulative distribution function $F(x)$ given in the table below.

x	1	2	3	4	5
$p(x)$	0.10	a	0.28	c	0.24
$F(x)$	0.10	0.26	b	0.76	d

(a) Write down the value of d

(1)

(b) Find the values of a, b and c

(3)

(c) Write down the value of $P(X > 4)$

(1)

Two independent observations, X_1 and X_2, are taken from the distribution of X.

(d) Find the probability that X_1 and X_2 are both odd.

(2)

Given that X_1 and X_2 are both odd,

(e) find the probability that the sum of X_1 and X_2 is 6
Give your answer to 3 significant figures.

(3)

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Question 1 continued

Q1

(Total 10 marks)

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6. The random variable X has probability function

$$P(X = x) = \frac{x^2}{k} \qquad x = 1, 2, 3, 4$$

(a) Show that $k = 30$

(2)

(b) Find $P(X \neq 4)$

(2)

(c) Find the exact value of $E(X)$

(2)

(d) Find the exact value of $\text{Var}(X)$

(4)

Given that $Y = 3X - 1$

(e) find $E(Y^2)$

(4)

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Question 6 continued

Q6

(Total 14 marks)

Leave blank

1. The discrete random variable X has the probability distribution given in the table below.

x	–2	1	3	4	6
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{6}$

(a) Write down the value of F(5)

(1)

(b) Find $E(X)$

(2)

(c) Find $\text{Var}(X)$

(3)

The random variable $Y = 7 - 2X$

(d) Find

(i) $E(Y)$

(ii) $\text{Var}(Y)$

(iii) $P(Y > X)$

(6)

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Question 1 continued

Q1

(Total 12 marks)

Leave blank

4. In a game, the number of points scored by a player in the first round is given by the random variable X with probability distribution

x	5	6	7	8
$P(X = x)$	0.13	0.21	0.29	0.37

Find

(a) $E(X)$

(2)

(b) $\text{Var}(X)$

(3)

(c) $\text{Var}(3 - 2X)$

(2)

The number of points scored by a player in the second round is given by the random variable Y and is independent of the number of points scored in the first round.

The random variable Y has probability function

$$P(Y = y) = \frac{1}{4} \qquad \text{for } y = 5, 6, 7, 8$$

(d) Write down the value of $E(Y)$

(1)

(e) Find $P(X = Y)$

(2)

(f) Find the probability that the number of points scored by a player in the first round is greater than the number of points scored by the player in the second round.

(3)

Question 4 continued

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7. The discrete random variable X can take only the values 1, 2, 3 and 4. For these values, the probability function is given by

$$P(X = x) = \frac{ax + b}{60} \qquad \text{for } x = 1, 2, 3, 4$$

where a and b are constants.

(a) Show that $5a + 2b = 30$

(2)

Given that $F(3) = \frac{13}{20}$

(b) find the value of a and the value of b

(4)

Given also that $Y = X^2$

(c) find the cumulative distribution function of Y

(4)

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Question 7 continued

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4. The discrete random variable X has probability distribution

x	–4	–3	1	2	5
$P(X = x)$	a	b	a	b	0.2

(a) Find $E(X)$ in terms of a and b

(1)

For this probability distribution, $Var(X) = E(X^2)$

(b) (i) Write down the value of $E(X)$

(ii) Find the value of a and the value of b

(5)

(c) Find $Var(1 - 3X)$

(3)

Given that $Y = 1 - X$, find

(d) (i) $P(Y < 0)$

(ii) the largest possible value of k such that $P(Y < k) = 0.2$

(4)

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Question 4 continued

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6. Anju has a bag that contains 5 socks of which 2 are blue.

Anju randomly selects socks from the bag, one sock at a time. She does not replace any socks but continues to select socks at random until she has both blue socks.

The discrete random variable S represents the total number of socks that Anju has selected.

(a) Write down the value of $P(S = 1)$

(1)

(b) Find $P(S > 2)$

(3)

(c) Find $P(S = 3)$

(2)

(d) Given that the second sock selected is blue, find the probability that Anju selects exactly 3 socks.

(2)

(e) Find $P(S = 5)$

(3)

Question 6 continued

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3. A biased four-sided die has faces marked 1, 3, 5 and 7. The random variable X represents the score on the die when it is rolled. The cumulative distribution function of X, $F(x)$, is given in the table below.

x	1	3	5	7
$F(x)$	0.2	0.5	0.9	1

(a) Find the probability distribution of X

(4)

(b) Find $P(2 < X \leqslant 6)$

(2)

(c) Write down the value of $F(4)$

(1)

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5. The discrete random variable X has the following probability distribution

x	–2	0	2	4
$P(X = x)$	a	b	a	c

where a, b and c are probabilities.

Given that $E(X) = 0.8$

(a) find the value of c.

(2)

Given also that $E(X^2) = 5$ find

(b) the value of a and the value of b,

(4)

(c) $\text{Var}(X)$

(2)

The random variable $Y = 5 - 3X$

Find

(d) $E(Y)$

(1)

(e) $\text{Var}(Y)$

(2)

(f) $P(Y \geqslant 0)$

(4)

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Question 5 continued

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1. The discrete random variable X can only take the values 1, 2, 3 and 4
For these values the cumulative distribution function is defined by

$$F(x) = kx^2 \quad \text{for} \quad x = 1, 2, 3, 4$$

where k is a constant.

(a) Find the value of k.

(2)

(b) Find the probability distribution of X.

(2)

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6. The random variable X has a discrete uniform distribution and takes the values 1, 2, 3, 4

Find

(a) F(3), where F(x) is the cumulative distribution function of X,

(1)

(b) E(X).

(1)

(c) Show that Var(X) = $\frac{5}{4}$

(3)

The random variable Y has a discrete uniform distribution and takes the values

$$3,\ 3 + k,\ 3 + 2k,\ 3 + 3k$$

where k is a constant.

(d) Write down P($Y = y$) for $y = 3,\ 3 + k,\ 3 + 2k,\ 3 + 3k$

(1)

The relationship between X and Y may be written in the form $Y = kX + c$ where c is a constant.

(e) Find Var(Y) in terms of k.

(2)

(f) Express c in terms of k.

(1)

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Question 6 continued

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3. The discrete random variable X has probability distribution

$$P(X = x) = \frac{1}{5} \qquad x = 1, 2, 3, 4, 5$$

(a) Write down the name given to this distribution.

(1)

Find

(b) $P(X = 4)$

(1)

(c) $F(3)$

(1)

(d) $P(3X - 3 > X + 4)$

(2)

(e) Write down $E(X)$

(1)

(f) Find $E(X^2)$

(2)

(g) Hence find $\text{Var}(X)$

(2)

Given that $E(aX - 3) = 11.4$

(h) find $\text{Var}(aX - 3)$

(4)

Question 3 continued

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5. A biased tetrahedral die has faces numbered 0, 1, 2 and 3. The die is rolled and the number face down on the die, X, is recorded. The probability distribution of X is

x	0	1	2	3
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

If $X = 3$ then the final score is 3
If $X \neq 3$ then the die is rolled again and the final score is the sum of the two numbers.

The random variable T is the final score.

(a) Find $P(T = 2)$

(2)

(b) Find $P(T = 3)$

(3)

(c) Given that the die is rolled twice, find the probability that the final score is 3

(3)

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6. A biased coin has probability 0.4 of showing a head. In an experiment, the coin is spun until a head appears. If a head has not appeared after 4 spins, the coin is not spun again. The random variable X represents the number of times the coin is spun.

For example, $X = 3$ if the first two spins do not show a head but the third spin does show a head. The coin would not then be spun a fourth time since the coin has already shown a head.

(a) Show that $P(X = 3) = 0.144$

(1)

The table gives some values for the probability distribution of X

x	1	2	3	4
$P(X = x)$		0.24	0.144	

(b) (i) Write down the value of $P(X = 1)$

(ii) Find $P(X = 4)$

(3)

(c) Find $E(X)$

(2)

(d) Find $Var(X)$

(3)

The random variable H represents the number of heads obtained when the coin is spun in the experiment.

(e) Explain why H can only take the values 0 and 1 and find the probability distribution of H.

(2)

(f) Write down the value of

(i) $P(\{X = 3\} \cap \{H = 0\})$

(ii) $P(\{X = 4\} \cap \{H = 0\})$

(2)

The random variable $S = X + H$

(g) Find the probability distribution of S

(4)

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Question 6 continued

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4. A discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(2 - x) & x = 0, 1 \\ k(3 - x) & x = 2, 3 \\ k(x + 1) & x = 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$

(2)

Find the exact value of

(b) $P(1 \leqslant X < 4)$

(1)

(c) $E(X)$

(2)

(d) $E(X^2)$

(2)

(e) $\text{Var}(3X + 1)$

(3)

Question 4 continued

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2. The discrete random variable X has probability distribution

x	–2	–1	1	2	3
$P(X = x)$	b	a	a	b	$\frac{1}{5}$

where a and b are constants.

(a) Write down an equation for a and b.

(1)

(b) Calculate $E(X)$.

(2)

Given that $E(X^2) = 3.5$

(c) (i) find a second equation in a and b,

(ii) hence find the value of a and the value of b.

(4)

(d) Find $Var(X)$.

(2)

The random variable $Y = 5 - 3X$

(e) Find $P(Y > 0)$.

(3)

(f) Find

(i) $E(Y)$,

(ii) $Var(Y)$.

(3)

Question 2 continued

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6. The discrete random variable D with the following probability distribution represents the score when a 4-sided die is rolled.

d	1	2	3	4
$P(D = d)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(a) Write down the name of this distribution.

(1)

The die is used to play a game and the random variable X represents the number of points scored. The die is rolled once and if $D = 2$, 3 or 4 then $X = D$. If $D = 1$ the die is rolled a second time and $X = 0$ if $D = 1$ again, otherwise X is the sum of the two scores on the die.

(b) Show that the probability of scoring 3 points in this game is $\frac{5}{16}$

(2)

(c) Find the probability of scoring 0 in this game.

(1)

The table below shows the probability distribution for the remaining values of X.

x	0	2	3	4	5
$P(X = x)$		$\frac{1}{4}$		$\frac{5}{16}$	$\frac{1}{16}$

(d) Find $E(X)$

(2)

(e) Find $\text{Var}(X)$

(3)

The discrete random variable R represents the number of times the die is rolled in the game.

(f) Write down the probability distribution of R.

(2)

The random variable $Y = 2R + 0.5$

(g) Show that $E(Y) = E(X)$

(3)

The game is played once.

(h) Find $P(X > Y)$

(3)

Question 6 continued

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5. The discrete random variable X is defined by the cumulative distribution function

x	1	2	3	4	5
$F(x)$	$\frac{3k}{2}$	$4k$	$\frac{15k}{2}$	$12k$	$\frac{35k}{2}$

where k is a constant.

(a) Find the probability distribution of X.

(3)

(b) Find $P(1.5 < X \leqslant 3.5)$

(2)

The random variable $Y = 12 - 7X$

(c) Calculate $\text{Var}(Y)$

(6)

(d) Calculate $P(4X \leqslant |Y|)$

(3)

Question 5 continued

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