

# 2014-2018

# Chapter

# 4,5&7:

# Forces and

# Motion

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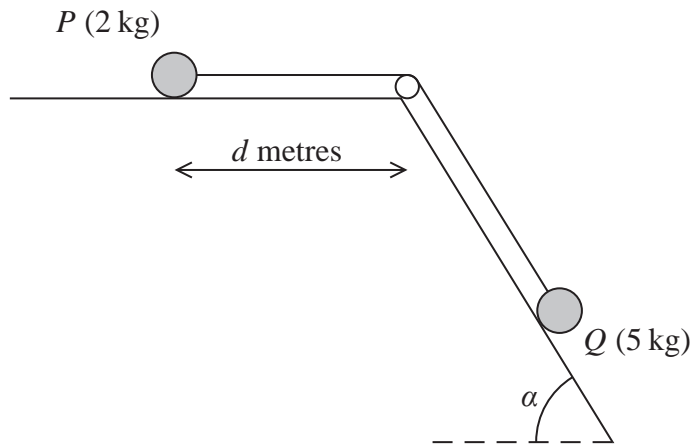


Figure 3

A particle  $P$  of mass 2 kg is attached to one end of a light inextensible string. A particle  $Q$  of mass 5 kg is attached to the other end of the string. The string passes over a small smooth light pulley. The pulley is fixed at a point on the intersection of a rough horizontal table and a fixed smooth inclined plane. The string lies along the table and also lies in a vertical plane which contains a line of greatest slope of the inclined plane. This plane is inclined to the horizontal at an angle  $\alpha$ , where  $\tan \alpha = \frac{3}{4}$ . Particle  $P$  is at rest on the table, a distance  $d$  metres from the pulley. Particle  $Q$  is on the inclined plane with the string taut, as shown in Figure 3. The coefficient of friction between  $P$  and the table is  $\frac{1}{4}$ .

The system is released from rest and  $P$  slides along the table towards the pulley.

Assuming that  $P$  has not reached the pulley and that  $Q$  remains on the inclined plane,

- (a) write down an equation of motion for  $P$ , (2)
- (b) write down an equation of motion for  $Q$ , (2)
- (c) (i) find the acceleration of  $P$ ,
- (ii) find the tension in the string. (5)

When  $P$  has moved a distance 0.5 m from its initial position, the string breaks. Given that  $P$  comes to rest just as it reaches the pulley,

- (d) find the value of  $d$ . (7)

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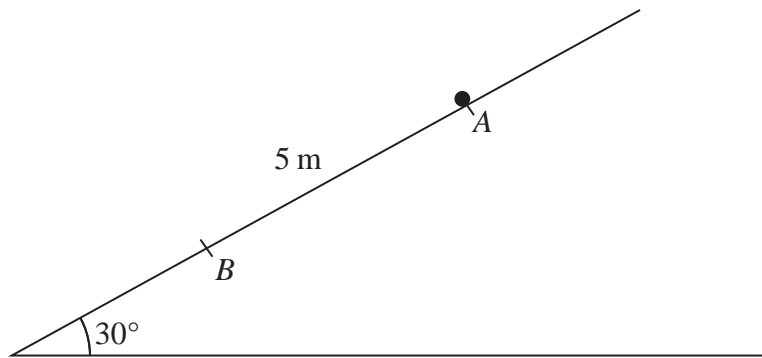


Figure 3

A particle  $P$  of mass  $4\text{ kg}$  is held at rest at the point  $A$  on a rough plane which is inclined at  $30^\circ$  to the horizontal. The point  $B$  lies on the line of greatest slope of the plane that passes through  $A$ . The point  $B$  is  $5\text{ m}$  down the plane from  $A$ , as shown in Figure 3. The coefficient of friction between the plane and  $P$  is  $0.3$

The particle is released from rest at  $A$  and slides down the plane.

(a) Find the speed of  $P$  at the instant it reaches  $B$ .

(7)

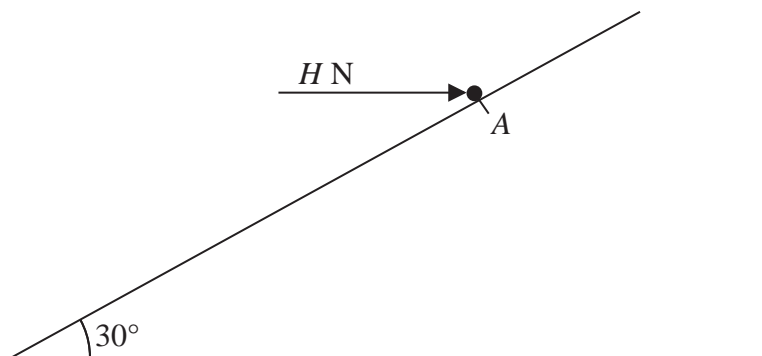


Figure 4

The particle is now returned to  $A$  and is held in equilibrium by a horizontal force of magnitude  $H$  newtons, as shown in Figure 4. The line of action of the force lies in the vertical plane containing the line of greatest slope of the plane through  $A$ . The particle is on the point of moving up the plane.

(b) Find the value of  $H$ .

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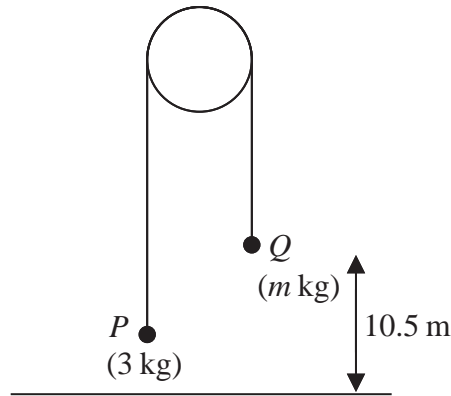


Figure 5

Two particles  $P$  and  $Q$  have masses  $3\text{ kg}$  and  $m\text{ kg}$  respectively ( $m > 3$ ). The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical. The particle  $Q$  is at a height of  $10.5\text{ m}$  above the horizontal ground, as shown in Figure 5. The system is released from rest and  $Q$  moves downwards. In the subsequent motion  $P$  does not reach the pulley. After the system is released, the tension in the string is  $33.6\text{ N}$ .

(a) Show that the magnitude of the acceleration of  $P$  is  $1.4\text{ m s}^{-2}$ . (3)

(b) Find the value of  $m$ . (3)

The system is released from rest at time  $t = 0$ . At time  $T_1$  seconds after release,  $Q$  strikes the ground and does not rebound. The string goes slack and  $P$  continues to move upwards.

(c) Find the value of  $T_1$  (3)

At time  $T_2$  seconds after release,  $P$  comes to instantaneous rest.

(d) Find the value of  $T_2$  (3)

At time  $T_3$  seconds after release ( $T_3 > T_1$ ) the string becomes taut again.

(e) Sketch a velocity-time graph for the motion of  $P$  in the interval  $0 \leq t \leq T_3$  (2)

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3. A particle  $P$  of mass  $1.5\text{ kg}$  is placed at a point  $A$  on a rough plane which is inclined at  $30^\circ$  to the horizontal. The coefficient of friction between  $P$  and the plane is  $0.6$

(a) Show that  $P$  rests in equilibrium at  $A$ .

(5)

A horizontal force of magnitude  $X$  newtons is now applied to  $P$ , as shown in Figure 1. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.

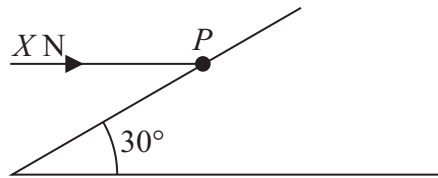


Figure 1

The particle is on the point of moving up the plane.

(b) Find

(i) the magnitude of the normal reaction of the plane on  $P$ ,

(ii) the value of  $X$ .

(7)

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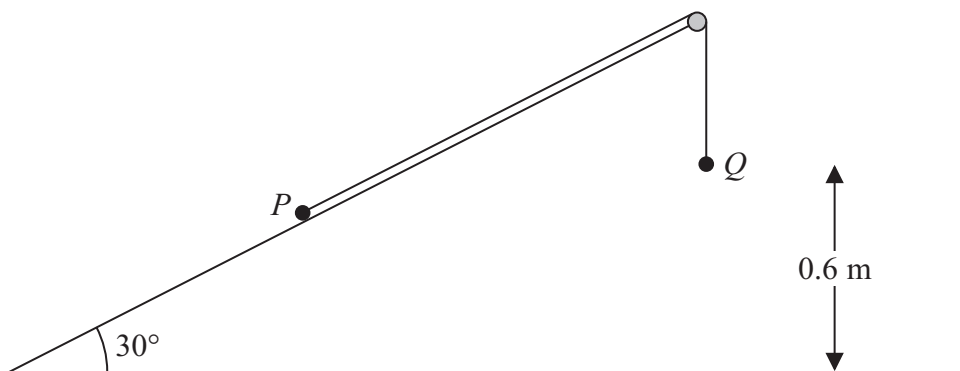


Figure 3

Two particles  $P$  and  $Q$ , of mass 2 kg and 3 kg respectively, are connected by a light inextensible string. Initially  $P$  is held at rest on a fixed smooth plane inclined at  $30^\circ$  to the horizontal. The string passes over a small smooth fixed pulley at the top of the plane. The particle  $Q$  hangs freely below the pulley and 0.6 m above the ground, as shown in Figure 3. The part of the string from  $P$  to the pulley is parallel to a line of greatest slope of the plane. The system is released from rest with the string taut.

For the motion before  $Q$  hits the ground,

- (a) (i) show that the acceleration of  $Q$  is  $\frac{2g}{5}$ ,
- (ii) find the tension in the string. (8)

On hitting the ground  $Q$  is immediately brought to rest by the impact.

- (b) Find the speed of  $P$  at the instant when  $Q$  hits the ground. (2)

In its subsequent motion  $P$  does not reach the pulley.

- (c) Find the total distance moved up the plane by  $P$  before it comes to instantaneous rest. (4)
- (d) Find the length of time between  $Q$  hitting the ground and  $P$  first coming to instantaneous rest. (2)

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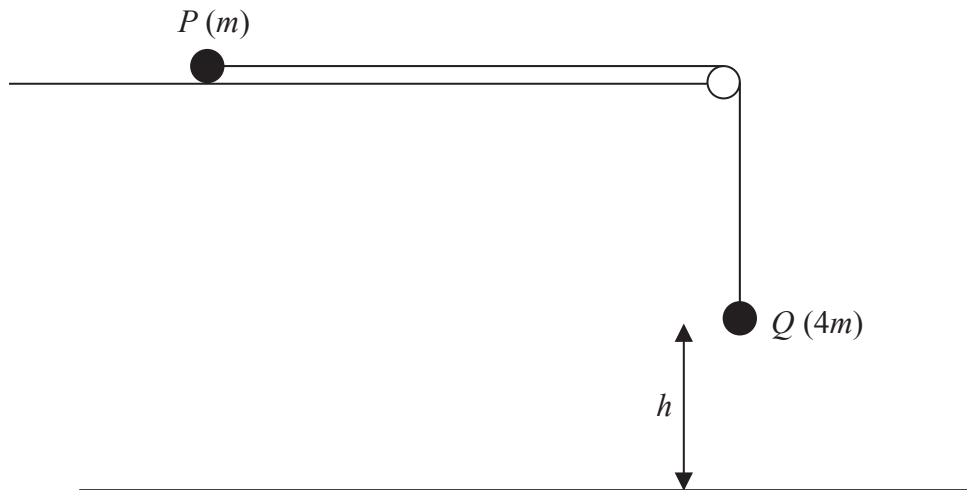


Figure 3

Two particles  $P$  and  $Q$  have masses  $m$  and  $4m$  respectively. The particles are attached to the ends of a light inextensible string. Particle  $P$  is held at rest on a rough horizontal table. The string lies along the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle  $Q$  hangs at rest vertically below the pulley, at a height  $h$  above a horizontal plane, as shown in Figure 3. The coefficient of friction between  $P$  and the table is  $0.5$ . Particle  $P$  is released from rest with the string taut and slides along the table.

- (a) Find, in terms of  $mg$ , the tension in the string while both particles are moving. (8)

The particle  $P$  does not reach the pulley before  $Q$  hits the plane.

- (b) Show that the speed of  $Q$  immediately before it hits the plane is  $\sqrt{1.4gh}$  (2)

When  $Q$  hits the plane,  $Q$  does not rebound and  $P$  continues to slide along the table. Given that  $P$  comes to rest before it reaches the pulley,

- (c) show that the total length of the string must be greater than  $2.4h$  (6)

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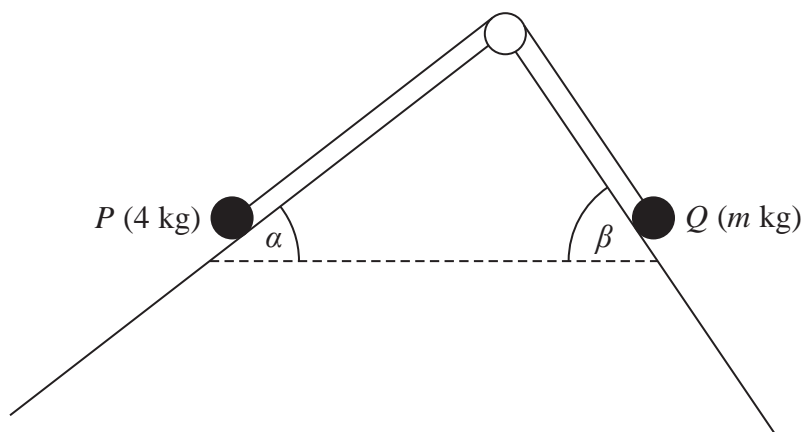


Figure 3

A particle  $P$  of mass 4 kg is attached to one end of a light inextensible string. A particle  $Q$  of mass  $m$  kg is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. The first plane is inclined to the horizontal at an angle  $\alpha$ , where  $\tan \alpha = \frac{3}{4}$  and the second plane is inclined to the horizontal at an angle  $\beta$ , where  $\tan \beta = \frac{4}{3}$ . Particle  $P$  is on the first plane and particle  $Q$  is on the second plane with the string taut, as shown in Figure 3.

The first plane is rough and the coefficient of friction between  $P$  and the plane is  $\frac{1}{4}$ . The second plane is smooth. The system is in limiting equilibrium.

Given that  $P$  is on the point of slipping down the first plane,

- (a) find the value of  $m$ , (10)
- (b) find the magnitude of the force exerted on the pulley by the string, (4)
- (c) find the direction of the force exerted on the pulley by the string. (1)

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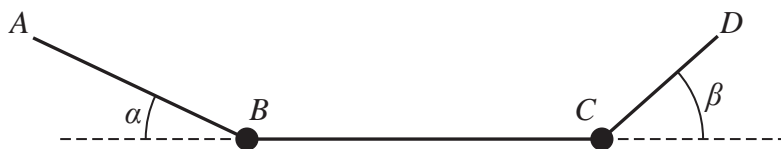


Figure 2

A washing line  $ABCD$  is fixed at the points  $A$  and  $D$ . There are two heavy items of clothing hanging on the washing line, one fixed at  $B$  and the other fixed at  $C$ . The washing line is modelled as a light inextensible string, the item at  $B$  is modelled as a particle of mass  $3$  kg and the item at  $C$  is modelled as a particle of mass  $M$  kg. The section  $AB$  makes an angle  $\alpha$  with the horizontal, where  $\tan \alpha = \frac{3}{4}$ , the section  $BC$  is horizontal and the section  $CD$  makes an angle  $\beta$  with the horizontal, where  $\tan \beta = \frac{12}{5}$ , as shown in Figure 2. The system is in equilibrium.

- (a) Find the tension in  $AB$ . (4)
- (b) Find the tension in  $BC$ . (3)
- (c) Find the value of  $M$ . (5)

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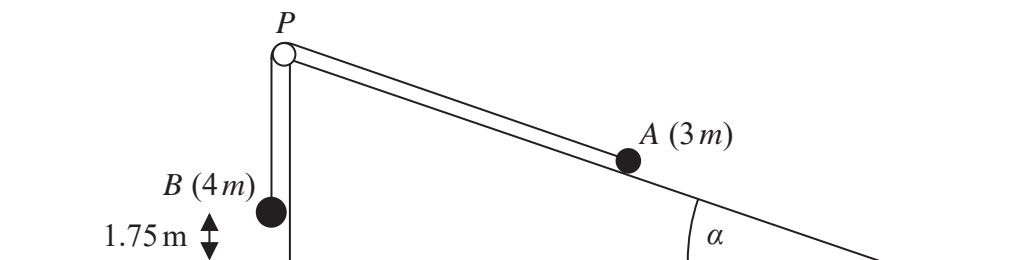


Figure 2

Figure 2 shows two particles  $A$  and  $B$ , of masses  $3m$  and  $4m$  respectively, attached to the ends of a light inextensible string. Initially  $A$  is held at rest on the surface of a fixed rough inclined plane. The plane is inclined to the horizontal at an angle  $\alpha$  where  $\tan \alpha = \frac{3}{4}$ . The coefficient of friction between  $A$  and the plane is  $\frac{1}{4}$ . The string passes over a small smooth light pulley  $P$  which is fixed at the top of the plane. The part of the string from  $A$  to  $P$  is parallel to a line of greatest slope of the plane. The particle  $B$  hangs freely and is vertically below  $P$ . The system is released from rest with the string taut and with  $B$  at a height of  $1.75\text{ m}$  above the ground. In the subsequent motion,  $A$  does not hit the pulley.

For the period before  $B$  hits the ground,

(a) write down an equation of motion for each particle. (4)

(b) Hence show that the acceleration of  $B$  is  $\frac{8}{35}g$ . (5)

(c) Explain how you have used the fact that the string is inextensible in your calculation. (1)

When  $B$  hits the ground,  $B$  does not rebound and comes immediately to rest.

(d) Find the distance travelled by  $A$  from the instant when the system is released to the instant when  $A$  first comes to rest. (7)

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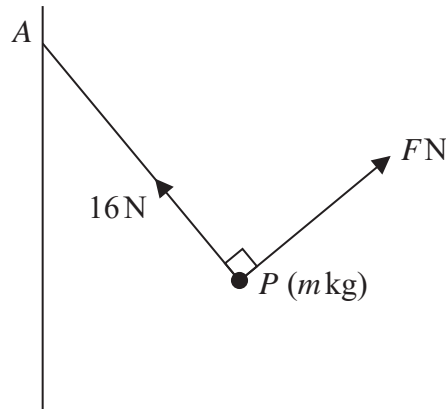


Figure 2

A particle  $P$  of mass  $m$  kg is attached to one end of a light inextensible string of length 2.5 m. The other end of the string is attached to a fixed point  $A$  on a vertical wall. The tension in the string is 16 N. The particle is held in equilibrium by a force of magnitude  $F$  newtons, acting in the vertical plane which is perpendicular to the wall and contains the string. This force acts in a direction perpendicular to the string, as shown in Figure 2.

Given that the horizontal distance of  $P$  from the wall is 1.5 m, find

- (i) the value of  $F$ ,
- (ii) the value of  $m$ .

(7)

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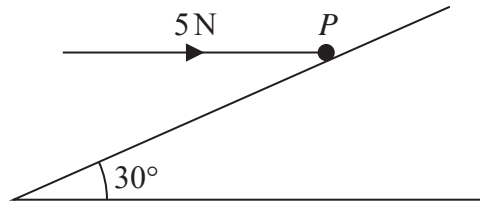


Figure 4

A rough plane is inclined at  $30^\circ$  to the horizontal. A particle  $P$  of mass  $0.5\text{ kg}$  is held at rest on the plane by a horizontal force of magnitude  $5\text{ N}$ , as shown in Figure 4. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The particle is on the point of moving up the plane.

(a) Find the magnitude of the normal reaction of the plane on  $P$ . (4)

(b) Find the coefficient of friction between  $P$  and the plane. (5)

The force of magnitude  $5\text{ N}$  is now removed and  $P$  accelerates from rest down the plane.

(c) Find the speed of  $P$  after it has travelled  $3\text{ m}$  down the plane. (8)

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