2014-2018 Chapter 4: Probability

Mr Faruk
Teacher of Mathematics
BSc/MSc/PGCE Mathematics
cieigcsesolutions@gmail.com

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5. A group of 100 students are asked if they like folk music, rock music or soul music.

All students who like folk music also like rock music

No students like both rock music and soul music

75 students do not like soul music

12 students who like rock music do not like folk music

30 students like folk music

(a) Draw a Venn diagram to illustrate this information.

(4)

(b) State two of these types of music that are mutually exclusive.

(1)

Find the probability that a randomly chosen student

(c) does not like folk music, rock music or soul music,

(1)

(d) likes rock music,

(1)

(e) likes folk music or soul music.

(1)

Given that a randomly chosen student likes rock music,

(f) find the probability that he or she also likes folk music.

(2)

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Question 5 continued

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7. In a large college, $\frac{3}{5}$ of the students are male, $\frac{3}{10}$ of the students are left handed and $\frac{1}{5}$ of the male students are left handed.

A student is chosen at random.

(a) Given that the student is left handed, find the probability that the student is male.

(2)

(b) Given that the student is female, find the probability that she is left handed.

(3)

(c) Find the probability that the randomly chosen student is male and right handed.

(2)

Two students are chosen at random.

(d) Find the probability that one student is left handed and one is right handed.

(2)

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Question 7 continued

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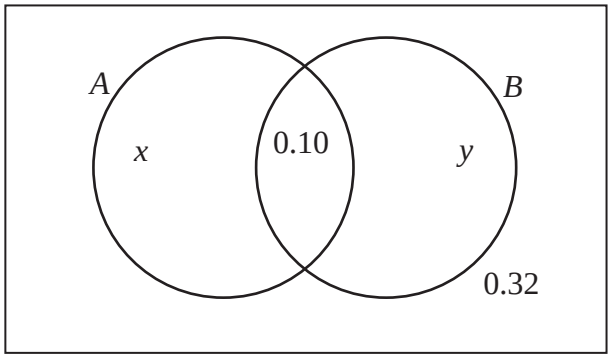
4. Events A and B are shown in the Venn diagram below

where x, y, 0.10 and 0.32 are probabilities.



(a) Find an expression in terms of x for

(i) P(A)

(ii) P($B \mid A$)

(3)

(b) Find an expression in terms of x and y for P($A \cup B$)

(1)

Given that P(A) = 2P(B)

(c) find the value of x and the value of y

(5)

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Question 4 continued

Q4

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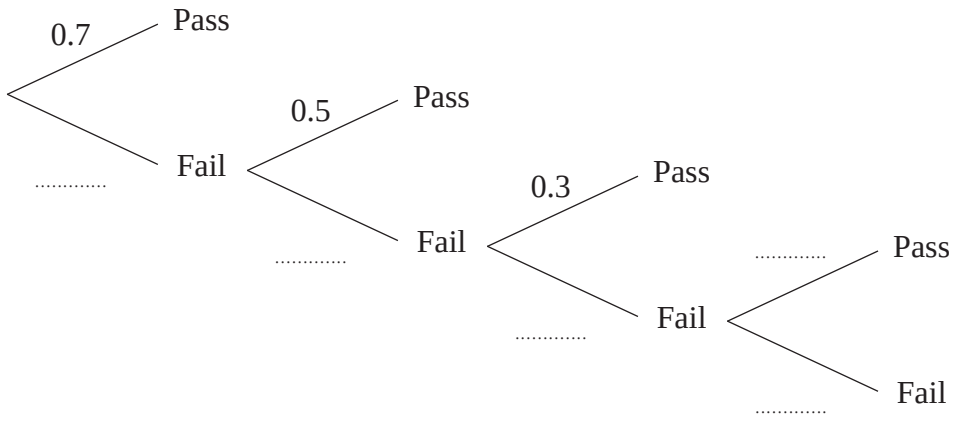
4. A training agency awards a certificate to each student who passes a test while completing a course.
Students failing the test will attempt the test again up to 3 more times, and, if they pass the test, will be awarded a certificate.
The probability of passing the test at the first attempt is 0.7, but the probability of passing reduces by 0.2 at each attempt.

(a) Complete the tree diagram below to show this information.



(2)

A student who completed the course is selected at random.

(b) Find the probability that the student was awarded a certificate.

(2)

(c) Given that the student was awarded a certificate, find the probability that the student passed on the first or second attempt.

(3)

The training agency decides to alter the test taken by the students while completing the course, but will not allow more than 2 attempts. The agency requires the probability of passing the test at the first attempt to be p, and the probability of passing the test at the second attempt to be $(p - 0.2)$. The percentage of students who complete the course and are awarded a certificate is to be 95%

(d) Show that p satisfies the equation

$$p^2 - 2.2p + 1.15 = 0$$

(3)

(e) Hence find the value of p, giving your answer to 3 decimal places.

(3)

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Question 4 continued

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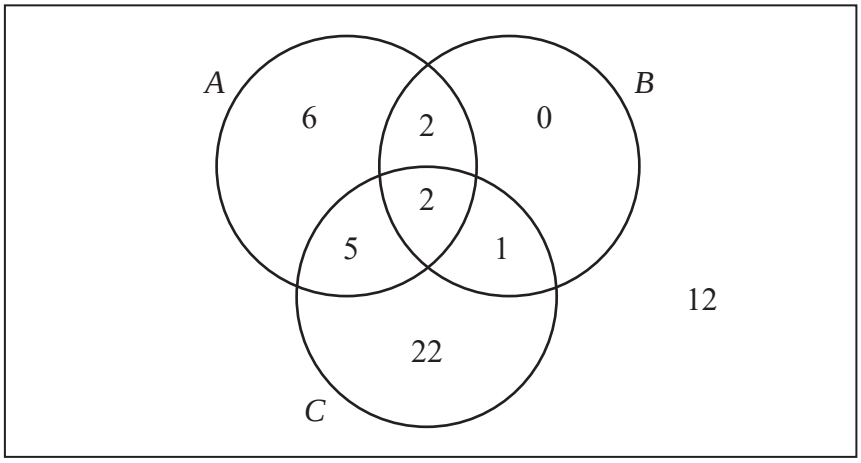
2. An integer is selected at random from the integers 1 to 50 inclusive.

A is the event that the integer selected is prime.
B is the event that the integer selected ends in a 3
C is the event that the integer selected is greater than 20

The Venn diagram shows the number of integers in each region for the events A, B and C



(a) Describe in words the event $(A \cap B)$

(1)

(b) Write down the probability that the integer selected is prime.

(1)

(c) Find $P([A \cup B \cup C]')$

(1)

Given that the integer selected is greater than 20

(d) find the probability that it is prime.

(2)

Using your answers to (b) and (d),

(e) state, with a reason, whether or not the events A and C are statistically independent.

(2)

Given that the integer selected is greater than 20 and prime,

(f) find the probability that it ends in a 3

(2)

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Question 2 continued

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5. In a survey, people were asked if they use a computer every day.

Of those people under 50 years old, 80% said they use a computer every day.
Of those people aged 50 or more, 55% said they use a computer every day.

The proportion of people in the survey under 50 years old is p

(a) Draw a tree diagram to represent this information.

(2)

In the survey, 70% of all people said they use a computer every day.

(b) Find the value of p

(2)

One person is selected at random. Given that this person uses a computer every day,

(c) find the probability that this person is under 50 years old.

(2)

Question 5 continued

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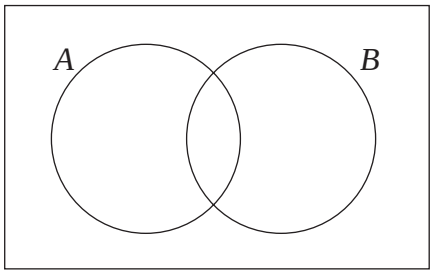
2. (a) Shade the region representing the event $A \cup B'$ on the Venn diagram below.



(1)

The two events C and D are mutually exclusive.

Given that $\mathrm{P}(C) = \frac{1}{5}$ and $\mathrm{P}(D) = \frac{3}{10}$

find

(b) (i) $\mathrm{P}(C \cup D)$

(ii) $\mathrm{P}(C \mid D)$

(3)

The two events F and G are independent.

Given that $\mathrm{P}(F) = \frac{1}{6}$ and $\mathrm{P}(F \cup G) = \frac{3}{8}$

find

(c) (i) $\mathrm{P}(G)$

(ii) $\mathrm{P}(F \mid G')$

(4)

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Question 2 continued

(Total 8 marks)

Q2

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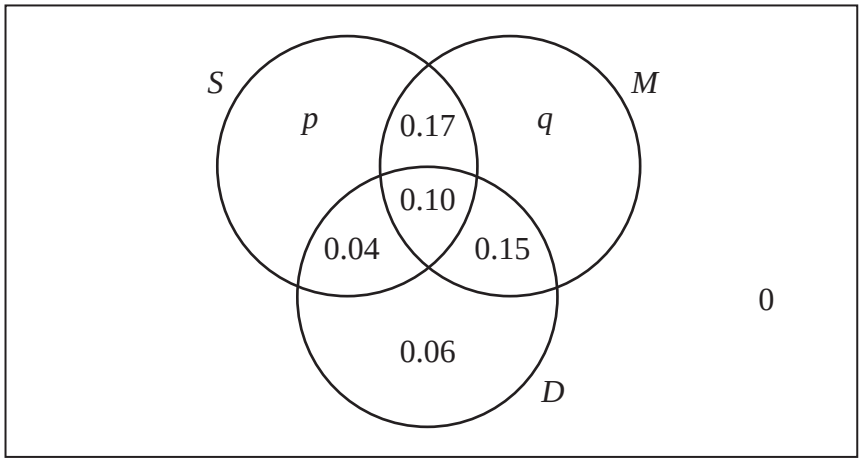
6. The Venn diagram below shows the probabilities of customers having various combinations of a starter, main course or dessert at Polly's restaurant.

S = the event a customer has a starter.

M = the event a customer has a main course.

D = the event a customer has a dessert.



Given that the events S and D are statistically independent

(a) find the value of p.

(4)

(b) Hence find the value of q.

(2)

(c) Find

(i) $P(D \mid M \cap S)$

(ii) $P(D \mid M \cap S')$

(4)

One evening 63 customers are booked into Polly's restaurant for an office party. Polly has asked for their starter and main course orders before they arrive.

Of these 63 customers

27 ordered a main course and a starter,

36 ordered a main course without a starter.

(d) Estimate the number of desserts that these 63 customers will have.

(2)

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Question 6 continued

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4. A bag contains 19 red beads and 1 blue bead only.

Linda selects a bead at random from the bag. She notes its colour and replaces the bead in the bag. She then selects a second bead at random from the bag and notes its colour.

Find the probability that

(a) both beads selected are blue,

(1)

(b) exactly one bead selected is red.

(2)

In another bag there are 9 beads, 4 of which are green and the rest are yellow.

Linda selects 3 beads from this bag at random without replacement.

(c) Find the probability that 2 of these beads are yellow and 1 is green.

(3)

Linda replaces the 3 beads and then selects another 4 at random without replacement.

(d) Find the probability that at least 1 of the beads is green.

(3)

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Question 4 continued

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6. Three events A, B and C are such that

$$P(A) = \frac{2}{5} \qquad P(C) = \frac{1}{2} \qquad P(A \cup B) = \frac{5}{8}$$

Given that A and C are mutually exclusive find

(a) $P(A \cup C)$

(1)

Given that A and B are independent

(b) show that $P(B) = \frac{3}{8}$

(4)

(c) Find $P(A \mid B)$

(1)

Given that $P(C' \cap B') = 0.3$

(d) draw a Venn diagram to represent the events A, B and C

(5)

Question 6 continued

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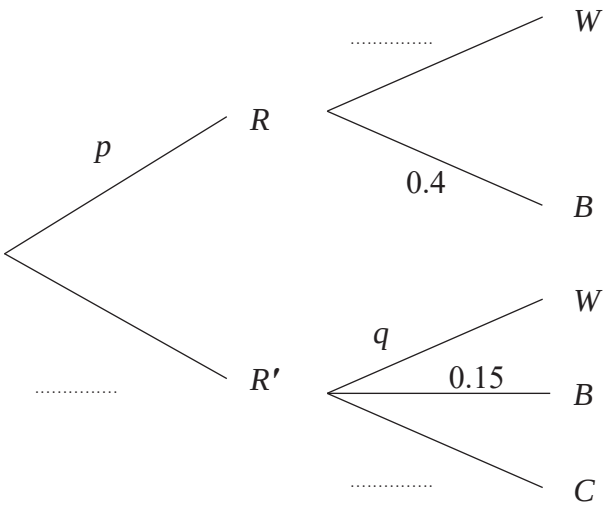
4. The partially completed tree diagram, where p and q are probabilities, gives information about Andrew's journey to work each day.



R represents the event that it is raining

W represents the event that Andrew walks to work

B represents the event that Andrew takes the bus to work

C represents the event that Andrew cycles to work

Given that $P(B) = 0.26$

(a) find the value of p

(3)

Given also that $P(R' \mid W) = 0.175$

(b) find the value of q

(4)

(c) Find the probability that Andrew cycles to work.

(2)

Given that Andrew did not cycle to work on Friday,

(d) find the probability that it was raining on Friday.

(3)

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Question 4 continued

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3. A manufacturer of electric generators buys engines for its generators from three companies, *R*, *S* and *T*.

Company *R* supplies 40% of the engines.

Company *S* supplies 25% of the engines.

The rest of the engines are supplied by company *T*.

It is known that 2% of the engines supplied by company *R* are faulty, 1% of the engines supplied by company *S* are faulty and 2% of the engines supplied by company *T* are faulty.

An engine is chosen at random.

(a) Draw a tree diagram to show all the possible outcomes and the associated probabilities.

(2)

(b) Calculate the probability that the engine is from company *R* and is not faulty.

(1)

(c) Calculate the probability that the engine is faulty.

(2)

Given that the engine is faulty,

(d) find the probability that the engine did **not** come from company *S*.

(2)

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Question 3 continued

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7. Events A and B are such that

$$P(A) = 0.5 \qquad P(A|B) = \frac{2}{3} \qquad P(A' \cup B') = 0.6$$

(a) Find $P(B)$

(3)

(b) Find $P(A'|B')$

(2)

The event C has $P(C) = 0.15$

The events A and C are mutually exclusive.

The events B and C are independent.

(c) Find $P(B \cap C)$

(2)

(d) Draw a Venn diagram to illustrate the events A, B and C and the probabilities for each region.

(5)

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Question 7 continued

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3. Hugo recorded the purchases of 80 customers in the ladies fashion department of a large store. His results were as follows

20 customers bought a coat
12 customers bought a coat and a scarf
23 customers bought a pair of gloves
13 customers bought a pair of gloves and a scarf
no customer bought a coat and a pair of gloves
14 customers did not buy a coat nor a scarf nor a pair of gloves.

(a) Draw a Venn diagram to represent all of this information.

(4)

(b) One of the 80 customers is selected at random.

(i) Find the probability that the customer bought a scarf.

(1)

(ii) Given that the customer bought a coat, find the probability that the customer also bought a scarf.

(2)

(iii) State, giving a reason, whether or not the event 'the customer bought a coat' and the event 'the customer bought a scarf ' are statistically independent.

(2)

Hugo had asked the member of staff selling coats and the member of staff selling gloves to encourage customers also to buy a scarf.

(c) By considering suitable conditional probabilities, determine whether the member of staff selling coats or the member of staff selling gloves has the better performance at selling scarves to their customers. Give a reason for your answer.

(3)

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Question 3 continued

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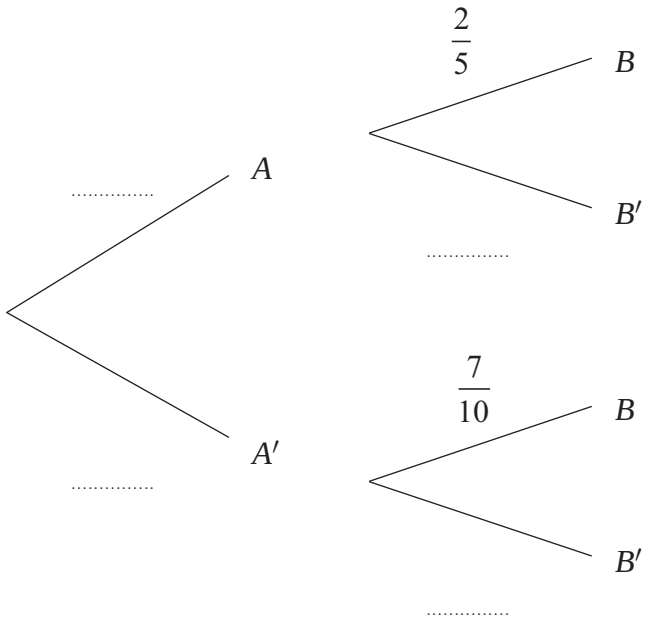
4. The following incomplete tree diagram shows the relationships between the event A and the event B.



Given that $P(B) = \frac{9}{20}$

(a) find $P(A)$ and complete the tree diagram,

(6)

(b) find $P(A' \mid B')$.

(2)

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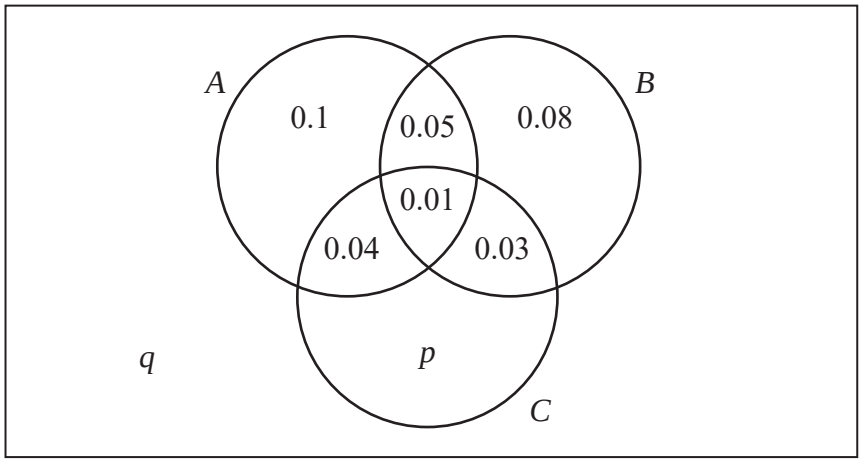
4. Pieces of wood cladding are produced by a timber merchant. There are three types of fault, *A*, *B* and *C*, that can appear in each piece of wood cladding.

The Venn diagram shows the probabilities of a piece of wood cladding having the various types of fault.



A piece of wood cladding is chosen at random.

(a) Find the probability that the piece of wood cladding has more than one type of fault.

(1)

Fault types *A* and *C* occur independently.

(b) Find the probability that the piece of wood cladding has no faults.

(4)

Given that the piece of wood cladding has fault *A*,

(c) find the probability that it also has fault *B* but not fault *C*.

(2)

Two pieces of the wood cladding are selected at random.

(d) Find the probability that both have exactly 2 types of fault.

(3)

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Question 4 continued

(Total 10 marks)

Q4