7. Functions and Graphs

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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, \ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

11.

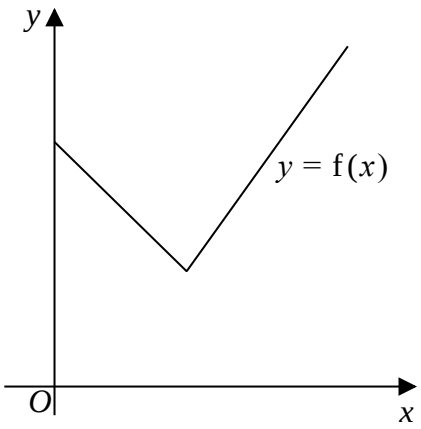

Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

Question 11 continued

(Total for Question 11 is 6 marks)

Answer ALL questions. Write your answers in the spaces provided.

1.

$$g(x) = \frac{2x + 5}{x - 3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

6.

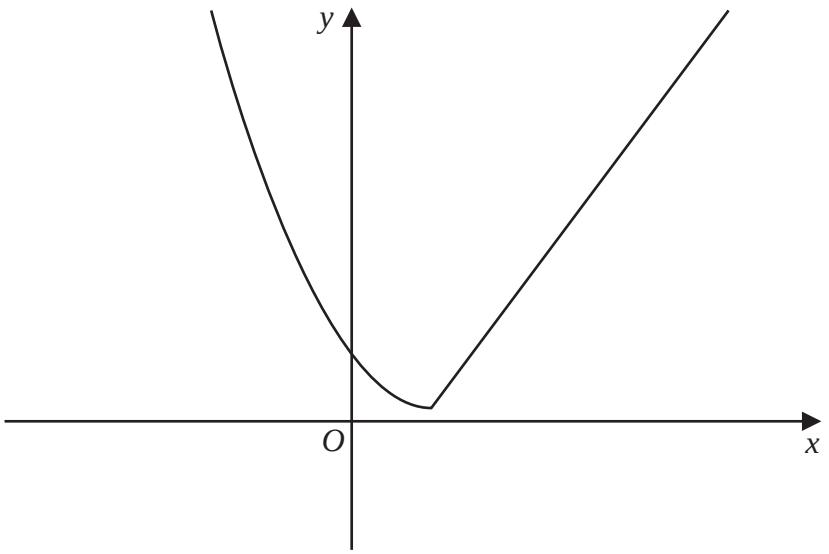

Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $gg(0)$.

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

Question 6 continued

4. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \frac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

9.

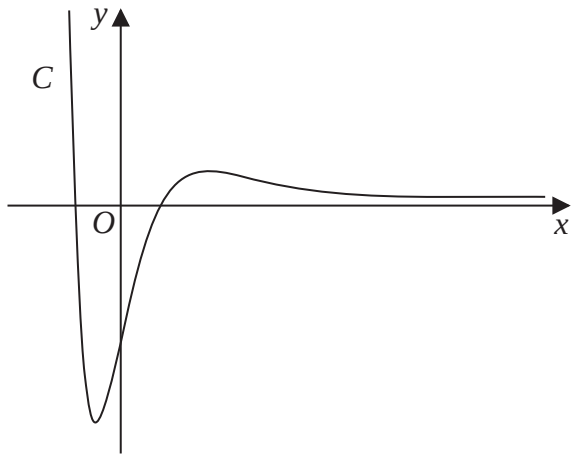

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4\left(x^2 - 2\right)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8\left(2 + x - x^2\right)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

Question 9 continued

11.

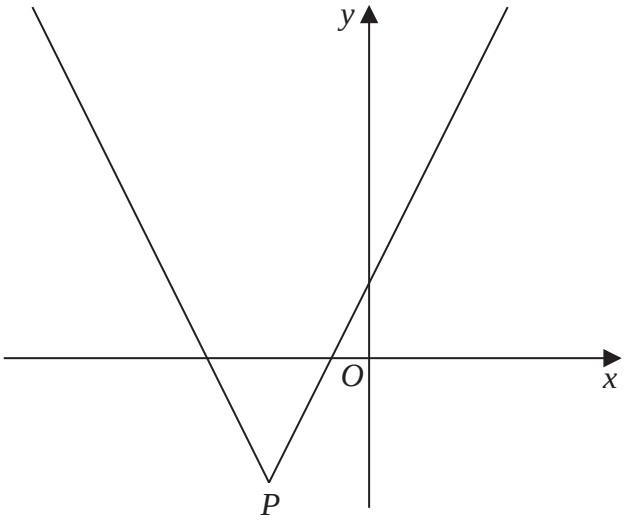

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

Question 11 continued

2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find $gf(1.8)$

(2)

(c) Find $g^{-1}(x)$

(2)

11.

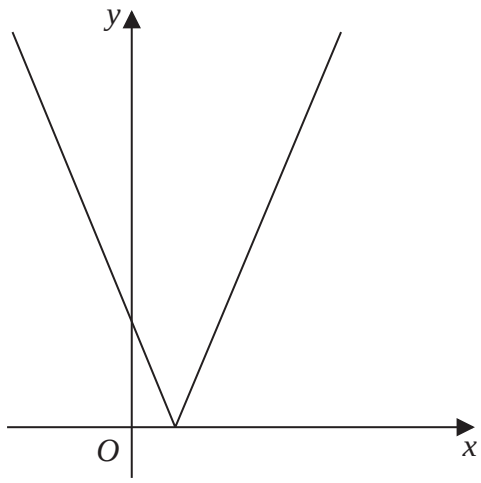

Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

Question 11 continued

1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x), \quad x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(x) + 2$

(1)

(b) $y = |f(x)|$

(1)

(c) $y = 3f(x - 2) + 2$

(2)

Question 1 continued

(Total for Question 1 is 4 marks)

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

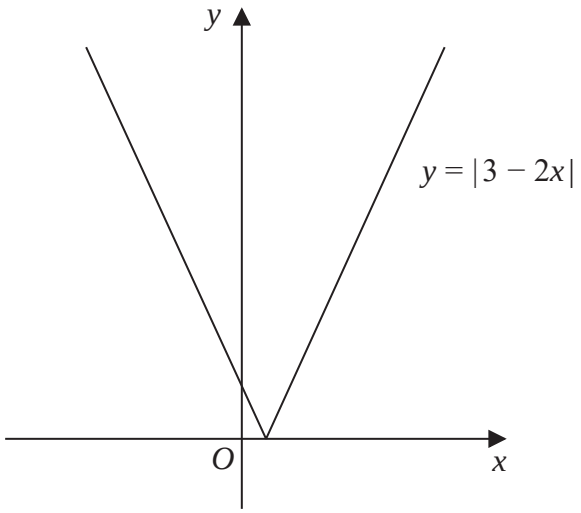

Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

10. The function f is defined by

$$f(x) = \frac{8x + 5}{2x + 3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$f(x) = A + \frac{B}{2x + 3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of $f g^{-1}$

(3)

Question 10 continued

7. The function f is defined by

$$f(x) = 3 + \sqrt{x - 2} \qquad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x - 3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(c) Find gf(6)

(2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

Question 7 continued

12.

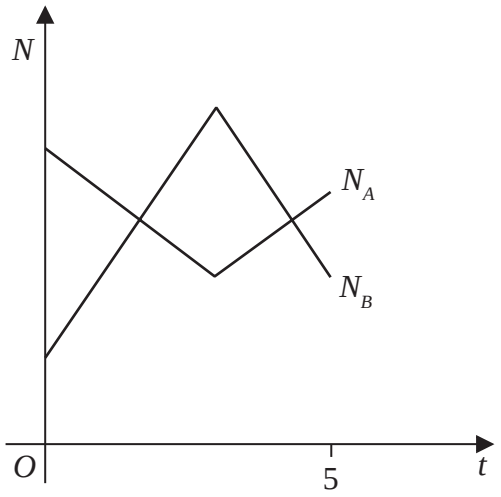

Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5)

(d) State a limitation of the model used for **company B**.

(1)

Question 12 continued

6.

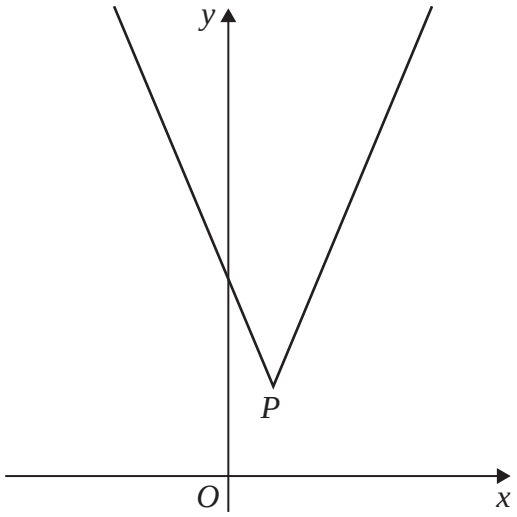

Figure 1

Figure 1 shows a sketch of the graph with equation

$$y = 3|x - 2| + 5$$

The vertex of the graph is at the point P, shown in Figure 1.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$16 - 4x = 3|x - 2| + 5$$

(2)

A line l has equation $y = kx + 4$ where k is a constant.

Given that l intersects $y = 3|x - 2| + 5$ at 2 distinct points,

(c) find the range of values of k.

(2)

Question 6 continued

(Total for Question 6 is 6 marks)

8. The functions f and g are defined by

$$f(x) = 4 - 3x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x - 9} \qquad x \in \mathbb{R},\ x \neq \frac{9}{2}$$

(a) Find fg(2)

(2)

(b) Find g^{-1}

(3)

(c) (i) Find gf(x), giving your answer as a simplified fraction.

(ii) Deduce the range of gf(x).

(3)

The function h is defined by

$$h(x) = 2x^2 - 6x + k \qquad x \in \mathbb{R}$$

where k is a constant.

(d) Find the range of values of k for which the equation

$$f(x) = h(x)$$

has no real solutions.

(3)

Question 8 continued