3.
Statistical Distributions

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5. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geqslant 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

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Question 5 continued

(Total for Question 5 is 9 marks)

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

3. For a particular type of bulb, 36% grow into plants with blue flowers and the remainder grow into plants with white flowers. Bulbs are sold in mixed bags of 40

Russell selects a random sample of 5 bags of bulbs.

(a) Find the probability that fewer than 2 of these bags will contain more bulbs that grow into plants with blue flowers than grow into plants with white flowers.

(4)

3. Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games,

(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

5. A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 2)$ for $r = 1, 2$

Given that $P(X = 2) = 0.35$

(a) find the complete probability distribution of X.

(2)

Ambroh spins the spinner 60 times.

(b) Find the probability that more than half of the spins land on the number 4
Give your answer to 3 significant figures.

(3)

The random variable $Y = \frac{12}{X}$

(c) Find $P(Y - X \leqslant 4)$

(3)

Question 5 continued

Answer ALL questions. Write your answers in the spaces provided.

1. Helen believes that the random variable C, representing cloud cover from the large data set, can be modelled by a discrete uniform distribution.

(a) Write down the probability distribution for C.

(2)

(b) Using this model, find the probability that cloud cover is less than 50%

(1)

Helen used all the data from the large data set for Hurn in 2015 and found that the proportion of days with cloud cover of less than 50% was 0.315

(c) Comment on the suitability of Helen's model in the light of this information.

(1)

(d) Suggest an appropriate refinement to Helen's model.

(1)

Question 1 continued

(Total for Question 1 is 5 marks)

3. In an experiment a group of children each repeatedly throw a dart at a target.
For each child, the random variable H represents the number of times the dart hits the target in the first 10 throws.

Peta models H as B(10, 0.1)

(a) State two assumptions Peta needs to make to use her model.

(2)

(b) Using Peta's model, find P($H \geqslant 4$)

(1)

For each child the random variable F represents the number of the throw on which the dart first hits the target.

Using Peta's assumptions about this experiment,

(c) find P($F = 5$)

(2)

Thomas assumes that in this experiment no child will need more than 10 throws for the dart to hit the target for the first time. He models P($F = n$) as

$$P(F = n) = 0.01 + (n - 1) \times \alpha$$

where α is a constant.

(d) Find the value of α

(4)

(e) Using Thomas' model, find P($F = 5$)

(1)

(f) Explain how Peta's and Thomas' models differ in describing the probability that a dart hits the target in this experiment.

(1)

Question 3 continued

3. A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find $P(4 \leqslant X < 8)$

(5)

5. Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$H_0 : p = 0.15$

$H_1 : p \geqslant 0.15$

Let X = the number of customers who buy chocolate.

$X \sim B(30, 0.15)$

$P(X = 8) = 0.0420$

$0.0420 < 0.05$ so reject H_0

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

(a) Identify the first two errors that Julie has made in her hypothesis test.

(2)

(b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer.

(1)

(c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05

(2)

(d) Find the actual level of significance of this test.

(1)

Question 5 continued

4. Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim \text{B}(8, 0.76)$

Using Magali's model,

(b) (i) find $\text{P}(X \geqslant 6)$

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(2)

(c) Explain whether or not your answers to part (b) support the use of Magali's model.

(1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

(e) Comment on Magali's model in light of your answer to part (d).

(2)

Question 4 continued

3. In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants.

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

(6)

5. Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

Question 5 continued

4. The discrete random variable D has the following probability distribution

d	10	20	30	40	50
$P(D = d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where k is a constant.

(a) Show that the value of k is $\frac{600}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(b) Find $P(D_1 + D_2 = 80)$
Give your answer to 3 significant figures.

(3)

A single observation of D is made.

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(c) Find the exact probability that the smallest angle of Q is more than 50°

(5)

Question 4 continued

4. A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

(1)

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,

(ii) there are at least 4 red beads in Aliya's bracelet.

(3)

(c) Comment on the suitability of a binomial distribution to model this situation.

(1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

(4)

(e) Find the p-value in this case.

(1)

Question 4 continued

1. (a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geqslant 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

(1)

A random sample of 50 students is taken from the university.

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

Question 1 continued.

(Total for Question 1 is 7 marks)

6. The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers $(a < b < c)$
- all the probabilities are greater than zero

(a) Find

(i) the value of a

(ii) the value of b

(iii) the value of c

Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

Question 6 continued.

2. A manufacturer of sweets knows that 8% of the bags of sugar delivered from supplier *A* will be damp.
A random sample of 35 bags of sugar is taken from supplier *A*.

(a) Using a suitable model, find the probability that the number of bags of sugar that are damp is

(i) exactly 2

(ii) more than 3

(3)

Supplier *B* claims that when it supplies bags of sugar, the proportion of bags that are damp is less than 8%

The manufacturer takes a random sample of 70 bags of sugar from supplier *B* and finds that only 2 of the bags are damp.

(b) Carry out a suitable test to assess supplier *B*'s claim.
You should state your hypotheses clearly and use a 10% level of significance.

(4)

Question 2 continued

(Total for Question 2 is 7 marks)

5. Manon has two biased spinners, one red and one green.

The random variable R represents the score when the red spinner is spun.
The random variable G represents the score when the green spinner is spun.

The probability distributions for R and G are given below.

r	2	3
$P(R = r)$	$\frac{1}{4}$	$\frac{3}{4}$

g	1	4
$P(G = g)$	$\frac{2}{3}$	$\frac{1}{3}$

Manon spins each spinner once and adds the two scores.

(a) Find the probability that

(i) the sum of the two scores is 7

(ii) the sum of the two scores is less than 4

(3)

The random variable $X = mR + nG$ where m and n are integers.

$$P(X = 20) = \frac{1}{6} \quad \text{and} \quad P(X = 50) = \frac{1}{4}$$

(b) Find the value of m and the value of n

(5)

Question 5 continued

1. George throws a ball at a target 15 times.
Each time George throws the ball, the probability of the ball hitting the target is 0.48

The random variable X represents the number of times George hits the target in 15 throws.

(a) Find

(i) $P(X = 3)$

(ii) $P(X \geqslant 5)$

(3)

4. A dentist knows from past records that 10% of customers arrive late for their appointment.

A new manager believes that there has been a change in the proportion of customers who arrive late for their appointment.

A random sample of 50 of the dentist's customers is taken.

(a) Write down

- a null hypothesis corresponding to no change in the proportion of customers who arrive late
- an alternative hypothesis corresponding to the manager's belief

(1)

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis in (a)
You should state the probability of rejection in each tail, which should be less than 0.025

(3)

(c) Find the actual level of significance of the test based on your critical region from part (b)

(1)

The manager observes that 15 of the 50 customers arrived late for their appointment.

(d) With reference to part (b), comment on the manager's belief.

(1)

Question 4 continued

(Total for Question 4 is 6 marks)

4. Past information shows that 25% of adults in a large population have a particular allergy.

Rylan believes that the proportion that has the allergy differs from 25%

He takes a random sample of 50 adults from the population.

Rylan carries out a test of the null hypothesis H_0: $p = 0.25$ using a 5% level of significance.

(a) Write down the alternative hypothesis for Rylan's test.

(1)

(b) Find the critical region for this test.
You should state the probability associated with each tail, which should be as close to 2.5% as possible.

(4)

(c) State the actual probability of incorrectly rejecting H_0 for this test.

(1)

Rylan finds that 10 of the adults in his sample have the allergy.

(d) State the conclusion of Rylan's hypothesis test.

(1)

Question 4 continued

(Total for Question 4 is 7 marks)

5. Julia selects 3 letters at random, one at a time without replacement, from the word

V A R I A N C E

The discrete random variable X represents the number of times she selects a letter A.

(a) Find the complete probability distribution of X.

(5)

Yuki selects 10 letters at random, one at a time **with** replacement, from the word

D E V I A T I O N

(b) Find the probability that he selects the letter E at least 4 times.

(3)

Question 5 continued

2. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.

The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $\mathrm{B}(40, \frac{1}{7})$

(1)

A box is selected at random.

(b) Using $T \sim \mathrm{B}(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

Question 2 continued

5. Tisam is playing a game.
She uses a ball, a cup and a spinner.

The random variable X represents the number the spinner lands on when it is spun.
The probability distribution of X is given in the following table

x	20	50	80	100
$P(X = x)$	a	b	c	d

where a, b, c and d are probabilities.

To play the game

- the spinner is spun to obtain a value of x
- Tisam then stands x cm from the cup and tries to throw the ball into the cup

The event S represents the event that Tisam successfully throws the ball into the cup.

To model this game Tisam assumes that

- $P(S \mid \{X = x\}) = \frac{k}{x}$ where k is a constant
- $P(S \cap \{X = x\})$ should be the same whatever value of x is obtained from the spinner

Using Tisam's model,

(a) show that $c = \frac{8}{5}b$

(2)

(b) find the probability distribution of X

(5)

Nav tries, a large number of times, to throw the ball into the cup from a distance of 100 cm.
He successfully gets the ball in the cup 30% of the time.

(c) State, giving a reason, why Tisam's model of this game is not suitable to describe Nav playing the game for all values of X

(1)

Question 5 continued

4. The random variable $X \sim B(27, 0.35)$

(a) Find (i) $P(X = 10)$

(ii) $P(12 \leqslant X < 15)$

(3)

Historical records show that the proportion of defective items produced by a machine is 0.12

Following a maintenance service of the machine, a random sample of 60 items is taken and 3 defective items are found.

(b) Carry out a suitable test to determine whether the proportion of defective items produced by the machine has decreased following the maintenance service.
You should state your hypotheses clearly and use a 5% level of significance.

(4)

(c) Write down the *p*-value for your test in part (b)

(1)

Question 4 continued

5. A biased 4-sided spinner has the numbers 6, 7, 8 and 10 on it.

The discrete random variable X represents the score when the spinner is spun once and has the following probability distribution,

x	6	7	8	10
$P(X = x)$	0.5	0.2	q	q

where q is a probability.

(a) Find the value of q

(1)

Karen spins the spinner repeatedly until she **either** gets a 7 **or** she has taken 4 spins.

(b) Show that the probability that Karen stops after taking her 3rd spin is 0.128

(2)

The random variable S represents the number of spins Karen takes.

(c) Find the probability distribution for S

(4)

The random variable N represents the number of times Karen gets a 7

(d) Find $P(S > N)$

(1)

Question 5 continued

1. Xian rolls a fair die 10 times.

The random variable X represents the number of times the die lands on a six.

(a) Using a suitable distribution for X, find

(i) $P(X = 3)$

(ii) $P(X < 3)$

(3)

Xian repeats this experiment each day for 60 days and records the number of days when $X = 3$

(b) Find the probability that there were at least 12 days when $X = 3$

(3)

(c) Find an estimate for the total number of sixes that Xian will roll during these 60 days.

(1)

Question 1 continued

4. The proportion of left-handed adults in a country is 10%
Freya believes that the proportion of left-handed adults under the age of 25 in this country is different from 10%
She takes a random sample of 40 adults under the age of 25 from this country to investigate her belief.

(a) Find the critical region for a suitable test to assess Freya's belief.
You should
- state your hypotheses clearly
- use a 5% level of significance
- state the probability of rejection in each tail

(4)

(b) Write down the actual significance level of your test in part (a)

(1)

In Freya's sample 7 adults were left-handed.

(c) With reference to your answer in part (a) comment on Freya's belief.

(1)

Question 4 continued

(Total for Question 4 is 6 marks)