3.
Graphs and Transformation

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13. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

Question 13 continued

(Total for Question 13 is 7 marks)

9. $g(x) = 4x^3 - 12x^2 - 15x + 50$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

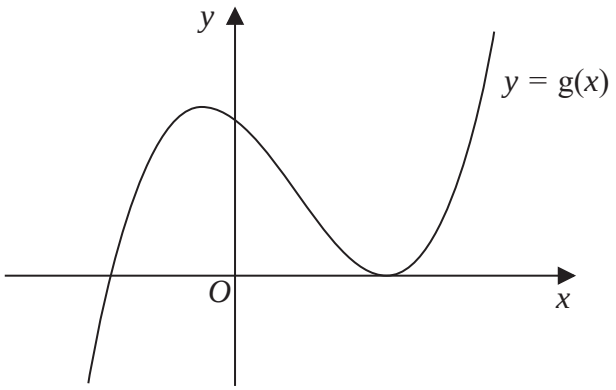

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

Question 9 continued

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

Question 7 continued

(Total for Question 7 is 8 marks)

5. $f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

Question 5 continued

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

6.

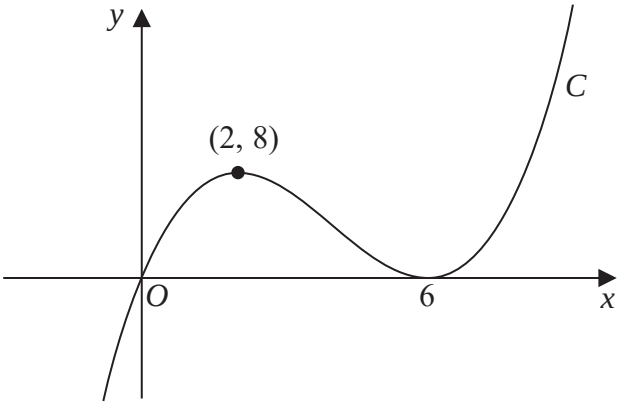

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

Question 6 continued

(Total for Question 6 is 6 marks)

4. (a) Sketch the curve with equation

$$y = \frac{k}{x} \qquad x \neq 0$$

where k is a positive constant.

(2)

(b) Hence or otherwise, solve

$$\frac{16}{x} \leqslant 2$$

(3)

5. The curve C_1 has equation

$$y = \frac{6}{x} + 3$$

(a) (i) Sketch C_1 stating the coordinates of any points where the curve cuts the coordinate axes.

(ii) State the equations of any asymptotes to the curve C_1

(3)

The curve C_2 has equation

$$y = 3x^2 - 4x - 10$$

(b) Show that C_1 and C_2 intersect when

$$3x^3 - 4x^2 - 13x - 6 = 0$$

(2)

Given that the x coordinate of one of the points of intersection is $-\frac{2}{3}$

(c) use algebra to find the x coordinates of the other points of intersection between C_1 and C_2

(*Solutions relying on calculator technology are not acceptable*.)

(4)

Question 5 continued

3. The point $P(3, -2)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the coordinates of the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x - 2)$

(ii) $y = f(2x)$

(iii) $y = 3f(-x) + 5$

(4)