3.
Forces, Friction and Static Particles

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3.Solutions

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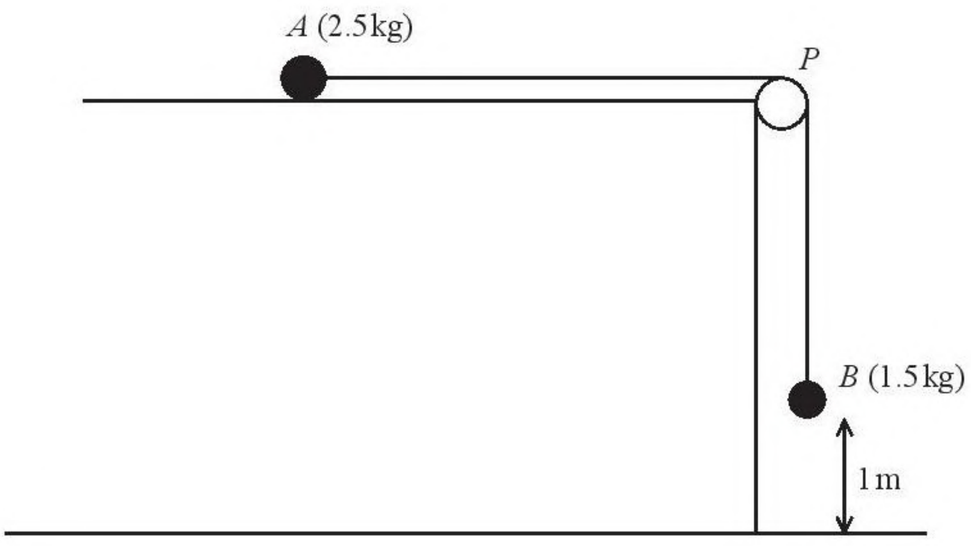

Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

It was found that it actually took 2.3 seconds for ball B to reach the floor.

(d) Using this information

(i) comment on the appropriateness of using the model to find the time it takes ball B to reach the floor, justifying your answer.

(ii) suggest one improvement that could be made in the model.

(2)

Question 9 continued

Question 9 continued

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

Question 7 continued

(Total for Question 7 is 8 marks)

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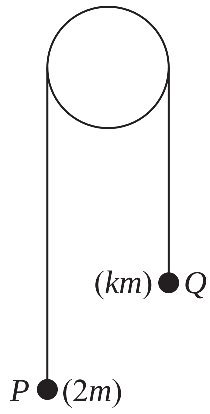

Figure 1

Two small balls, P and Q, have masses $2m$ and km respectively, where $k < 2$.
The balls are attached to the ends of a string that passes over a fixed pulley.
The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

The system is released from rest and, in the subsequent motion, P moves downwards with an acceleration of magnitude $\frac{5g}{7}$

The balls are modelled as particles moving freely.
The string is modelled as being light and inextensible.
The pulley is modelled as being small and smooth.

Using the model,

(a) find, in terms of m and g, the tension in the string, **(3)**

(b) explain why the acceleration of Q also has magnitude $\frac{5g}{7}$ **(1)**

(c) find the value of k. **(4)**

(d) Identify one limitation of the model that will affect the accuracy of your answer to part (c). **(1)**

Question 9 continued

7.

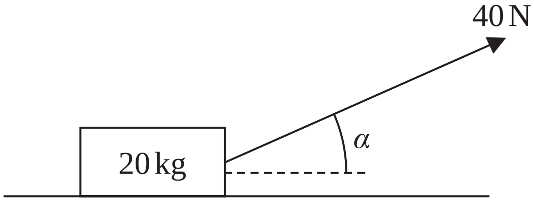

Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

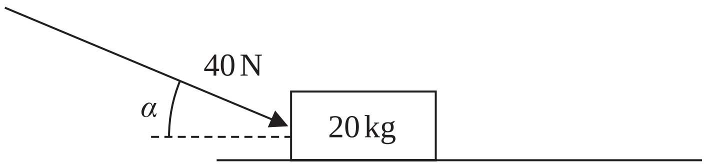

Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

Question 7 continued

2.

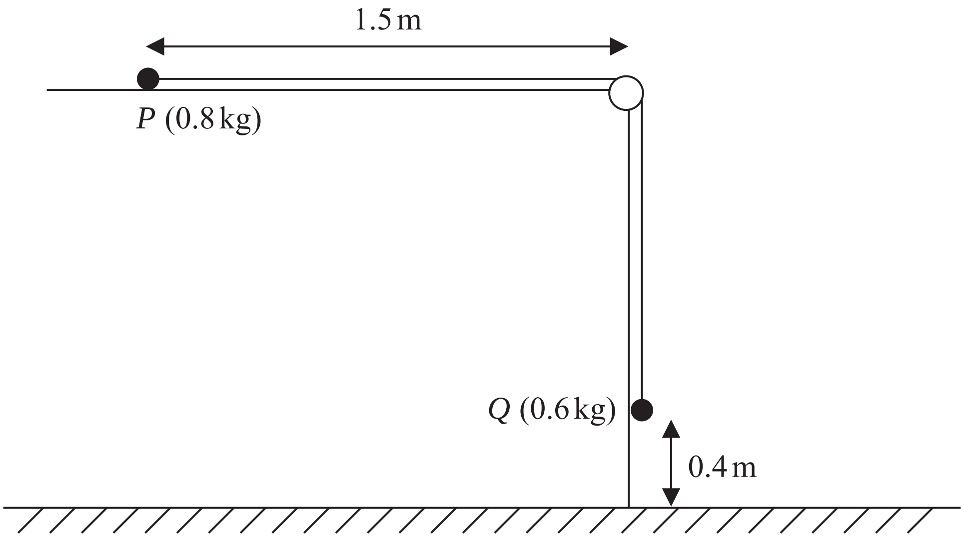

Figure 1

A small ball, P, of mass 0.8 kg, is held at rest on a smooth horizontal table and is attached to one end of a thin rope.

The rope passes over a pulley that is fixed at the edge of the table.

The other end of the rope is attached to another small ball, Q, of mass 0.6 kg, that hangs freely below the pulley.

Ball P is released from rest, with the rope taut, with P at a distance of 1.5 m from the pulley and with Q at a height of 0.4 m above the horizontal floor, as shown in Figure 1.

Ball Q descends, hits the floor and does not rebound.

The balls are modelled as particles, the rope as a light and inextensible string and the pulley as small and smooth.

Using this model,

(a) show that the acceleration of Q, as it falls, is $4.2\,\text{m}\,\text{s}^{-2}$

(5)

(b) find the time taken by P to hit the pulley from the instant when P is released.

(6)

(c) State one limitation of the model that will affect the accuracy of your answer to part (a).

(1)

Question 2 continued

3.

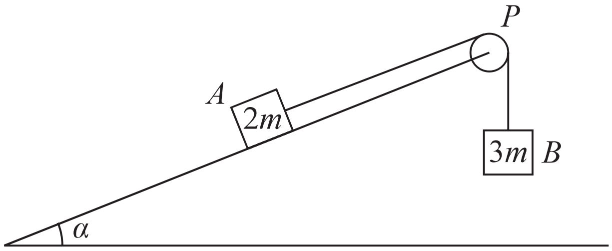

Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

Question 3 continued

2.

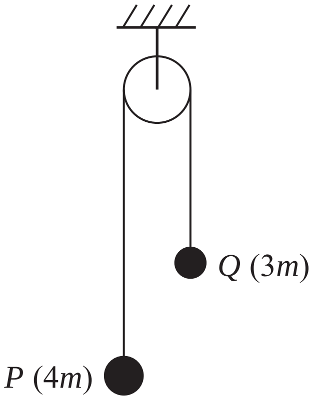

Figure 1

One end of a string is attached to a small ball P of mass $4m$.

The other end of the string is attached to another small ball Q of mass $3m$.

The string passes over a fixed pulley.

Ball P is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

Ball P is released.

The string is modelled as being light and inextensible, the balls are modelled as particles, the pulley is modelled as being smooth and air resistance is ignored.

(a) Using the model, find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while P is falling and before Q hits the pulley.

(8)

(b) State one limitation of the model, apart from ignoring air resistance, that will affect the accuracy of your answer to part (a).

(1)

Question 2 continued

1. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P

(2)

(b) show that $\mu = \frac{3}{4}$

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane.

(1)

Brick Q is now projected with speed $0.5\,\text{m s}^{-1}$ down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.

(2)

Question 1 continued

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Question 1 continued

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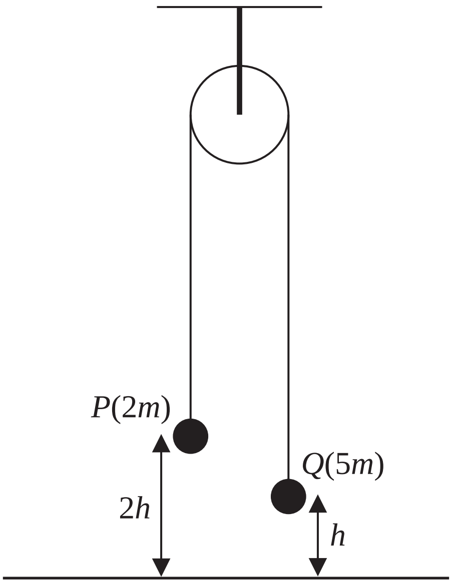

Figure 1

A ball P of mass $2m$ is attached to one end of a string.

The other end of the string is attached to a ball Q of mass $5m$.

The string passes over a fixed pulley.

The system is held at rest with the balls hanging freely and the string taut.

The hanging parts of the string are vertical with P at a height $2h$ above horizontal ground and with Q at a height h above the ground, as shown in Figure 1.

The system is released from rest.

In the subsequent motion, Q does not rebound when it hits the ground and P does not hit the pulley.

The balls are modelled as particles.

The string is modelled as being light and inextensible.

The pulley is modelled as being small and smooth.

Air resistance is modelled as being negligible.

Using this model,

(a) (i) write down an equation of motion for P,

(ii) write down an equation of motion for Q,

(4)

(b) find, in terms of h only, the height above the ground at which P first comes to instantaneous rest.

(7)

(c) State one limitation of modelling the balls as particles that could affect your answer to part (b).

(1)

In reality, the string will not be inextensible.

(d) State how this would affect the accelerations of the particles.

(1)

Question 3 continued

2.

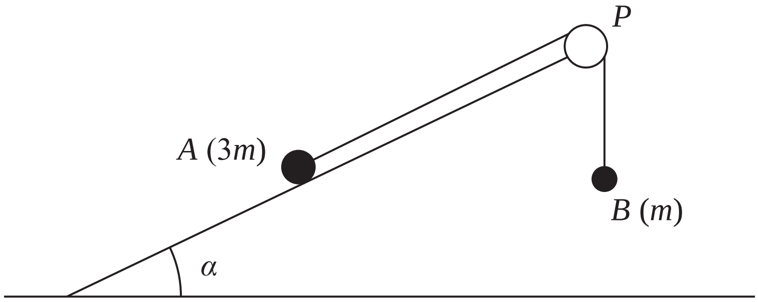

Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

In reality, the string is not light.

(d) State how this would affect the working in part (b).

(1)

Question 2 continued

Question 2 continued

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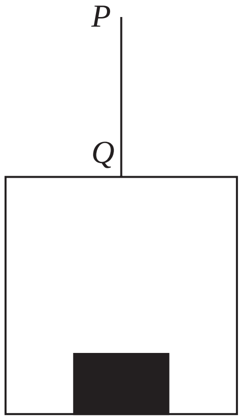

Figure 1

A vertical rope PQ has its end Q attached to the top of a small lift cage.

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in Figure 1.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration $0.2\,\text{m}\,\text{s}^{-2}$

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(a) find the tension in the rope PQ

(3)

(b) find the magnitude of the force exerted on the block by the lift cage.

(3)

Question 4 continued

2.

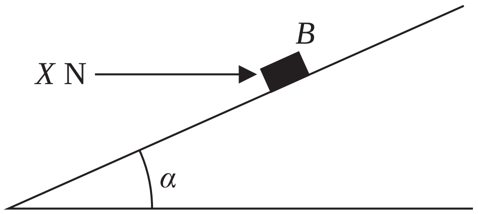

Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane.

(6)

Question 2 continued

3. [*In this question*, **i** *and* **j** *are horizontal unit vectors*.]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})$ N, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

Question 3 continued

4.

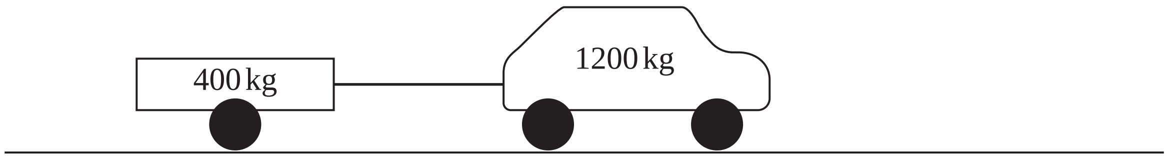

Figure 2

A car of mass 1200 kg is towing a trailer of mass 400 kg along a straight horizontal road using a tow rope, as shown in Figure 2.
The rope is horizontal and parallel to the direction of motion of the car.

- The resistance to motion of the car is modelled as a constant force of magnitude $2R$ newtons
- The resistance to motion of the trailer is modelled as a constant force of magnitude R newtons
- The rope is modelled as being light and inextensible
- The acceleration of the car is modelled as $a\,\text{m s}^{-2}$

The driving force of the engine of the car is 7400 N and the tension in the tow rope is 2400 N.

Using the model,

(a) find the value of a

(5)

In a refined model, the rope is modelled as having mass and the acceleration of the car is found to be $a_1\,\text{m s}^{-2}$

(b) State how the value of a_1 compares with the value of a

(1)

(c) State one limitation of the model used for the resistance to motion of the car.

(1)

Question 4 continued

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Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\text{m}\,\text{s}^{-2}$

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

Question 2 continued

(Total for Question 2 is 4 marks)

3. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle P is moving on a smooth horizontal surface under the action of two forces.

Given that

- the mass of P is 2 kg
- the two forces are $(2\mathbf{i} + 4\mathbf{j})$ N and $(c\mathbf{i} - 2\mathbf{j})$ N, where c is a constant
- the magnitude of the acceleration of P is $\sqrt{5}\,\mathrm{m\,s^{-2}}$

find the two possible values of c.

(5)

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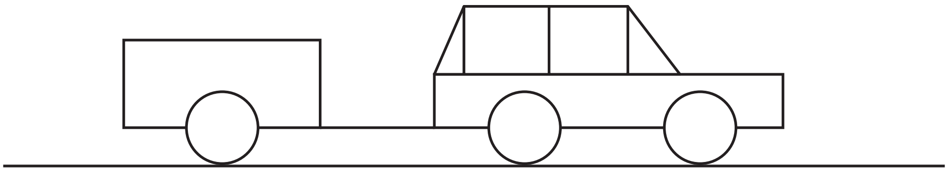

Figure 2

Figure 2 shows a car towing a trailer along a straight horizontal road.

The mass of the car is 800 kg and the mass of the trailer is 600 kg.

The trailer is attached to the car by a towbar which is parallel to the road and parallel to the direction of motion of the car and the trailer.

The towbar is modelled as a light rod.

The resistance to the motion of the car is modelled as a constant force of magnitude 400 N.

The resistance to the motion of the trailer is modelled as a constant force of magnitude *R* newtons.

The engine of the car is producing a constant driving force that is horizontal and of magnitude 1740 N.

The acceleration of the car is $0.6\,\text{m}\,\text{s}^{-2}$ and the tension in the towbar is *T* newtons.

Using the model,

(a) show that $R = 500$ **(3)**

(b) find the value of *T*. **(3)**

At the instant when the speed of the car and the trailer is $12.5\,\text{m}\,\text{s}^{-1}$, the towbar breaks.

The trailer moves a further distance *d* metres before coming to rest.

The resistance to the motion of the trailer is modelled as a constant force of magnitude 500 N.

Using the model,

(c) show that, after the towbar breaks, the deceleration of the trailer is $\frac{5}{6}\,\text{m}\,\text{s}^{-2}$ **(1)**

(d) find the value of *d*. **(3)**

In reality, the distance *d* metres is likely to be different from the answer found in part (d).

(e) Give two **different** reasons why this is the case. **(2)**

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Question 4 continued

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Question 4 continued

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1.

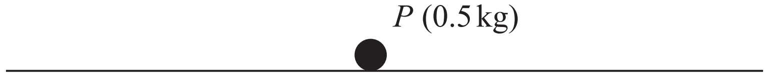

Figure 1

Figure 1 shows a particle P of mass 0.5 kg at rest on a rough horizontal plane.

(a) Find the magnitude of the normal reaction of the plane on P.

(1)

The coefficient of friction between P and the plane is $\frac{2}{7}$

A horizontal force of magnitude X newtons is applied to P.

Given that P is now in limiting equilibrium,

(b) find the value of X.

(2)

3.

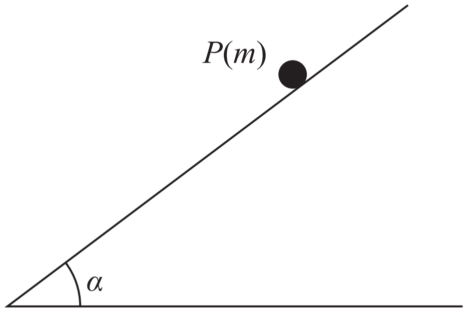

Figure 3

A particle P of mass m is held at rest at a point on a rough inclined plane, as shown in Figure 3.

It is given that

- the plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$
- the coefficient of friction between P and the plane is μ, where $\mu < \frac{5}{12}$

The particle P is released from rest and slides down the plane.
Air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on P,

(2)

(b) show that, as P slides down the plane, the acceleration of P down the plane is

$$\frac{1}{13}g(5 - 12\mu)$$

(4)

(c) State what would happen to P if it is released from rest but $\mu \geqslant \frac{5}{12}$

(1)

Question 3 continued

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