

15.

Numerical Methods

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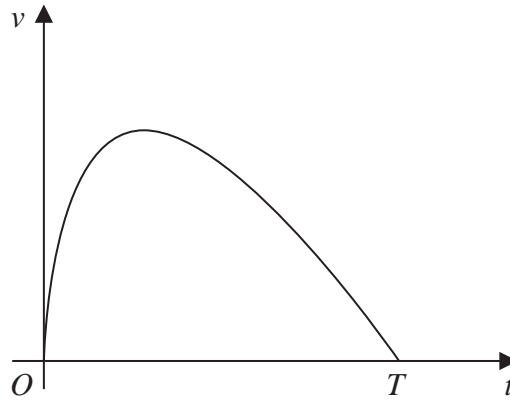


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v \text{ ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t) \ln(t + 1) \quad 0 \leq t \leq T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T (1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1 \tag{4}$$

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,
 (ii) find, by repeated iteration, the time taken for the car to reach maximum speed. (3)

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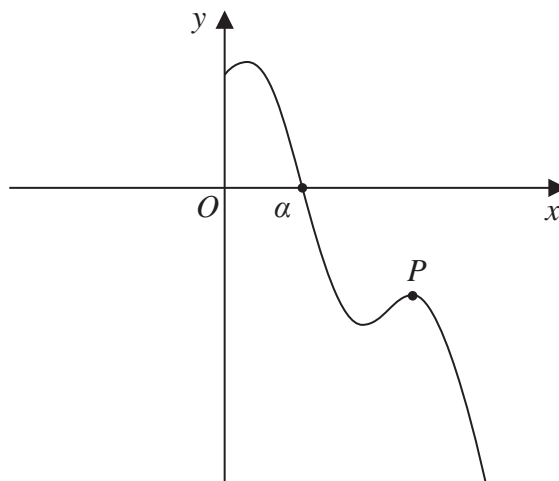


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8 \sin\left(\frac{1}{2}x\right) - 3x + 9 \quad x > 0$$

and x is measured in radians.

The point P , shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

- (a) find the x coordinate of P , giving your answer to 3 significant figures. (4)

The curve crosses the x -axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

- (b) explain why α must lie in the interval $[4, 5]$ (1)

- (c) Taking $x_0 = 5$ as a first approximation to α , apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α .

Show your method and give your answer to 3 significant figures. (2)

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15. A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β (1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2
 (ii) the value of β (3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places. (2)

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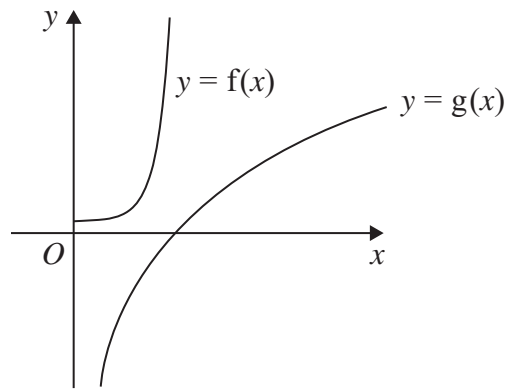


Figure 1

Figure 1 shows a sketch of the curves with equations $y = f(x)$ and $y = g(x)$ where

$$f(x) = e^{4x^2-1} \quad x > 0$$

$$g(x) = 8 \ln x \quad x > 0$$

Given that $f'(x) = g'(x)$ at $x = \alpha$

The iterative formula

$$x_{n+1} = \sqrt{\frac{1 - 2 \ln x_n}{4}}$$

is used with $x_1 = 0.6$ to find an approximate value for α

(c) Calculate, giving each answer to 4 decimal places,

- (i) the value of x_2
- (ii) the value of α

(3)

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Question 6 continued

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